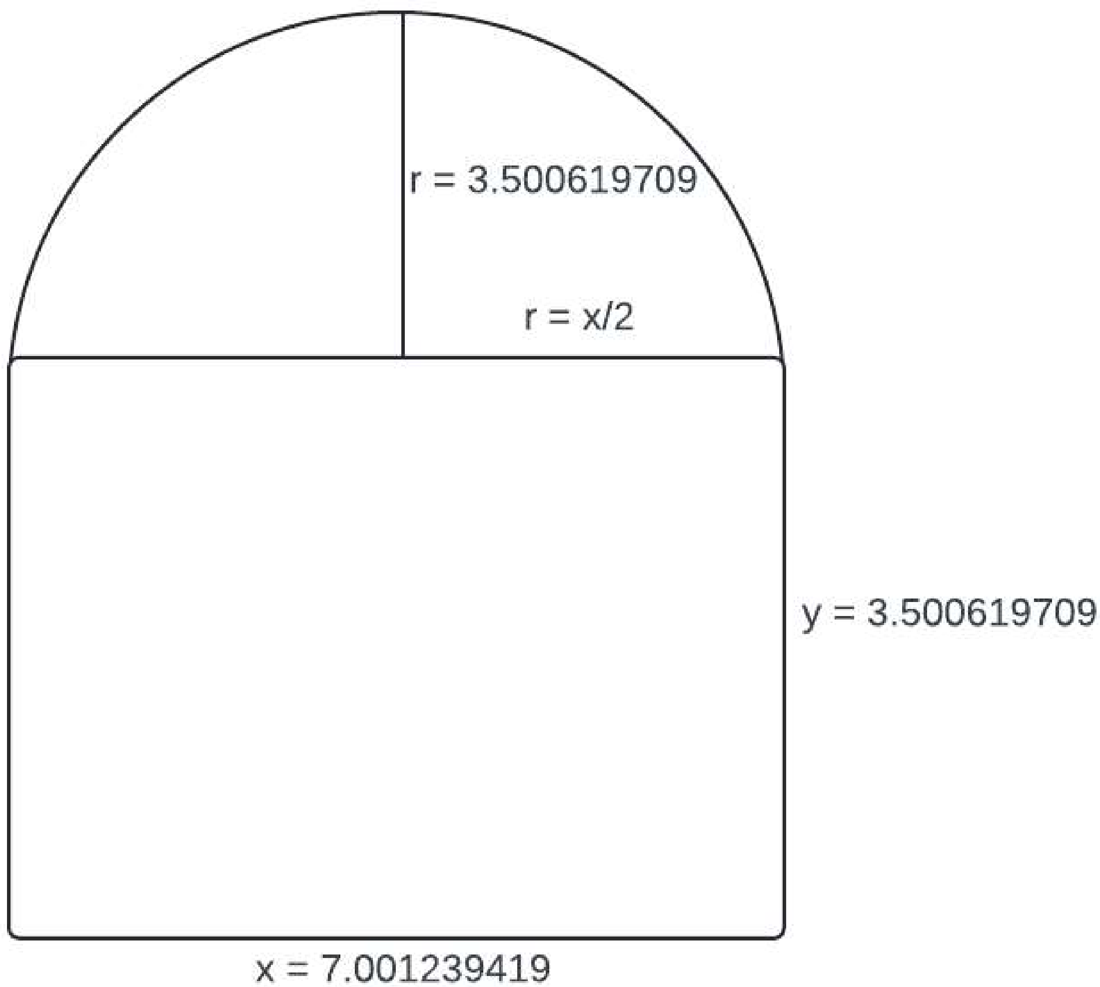
The first step I take in solving the problem is to draw a diagram and identify variables. This will help me identify which formulas to use. In this case, four are needed: the perimeter of a rectangle, the perimeter of a semicircle, the area of a rectangle, and the area of a semicircle. These will be combined to form the shape of a Norman window, which is given a perimeter of 25 feet.



The perimeter of a circle is $P_C = 2\pi r$, meaning the perimeter of a semicircle is $P_S = \pi r$, which does not include the line cutting the circle in half. The perimeter of a rectangle without the top line (which is instead the semicircle) is $P_r = 2y + x$. Combining these two equations into a single perimeter equation results in $P = 2y + x + \pi r$. However, we can deduce that $r = \frac{x}{2}$, as the center of the semicircle is the center of x. Therefore, we can write our perimeter equation as $P = 2y + x + \frac{\pi x}{2} = 25$. The area of a circle is $A_C = \pi r^2$, thus the area of a semicircle is $A_S = \frac{\pi r^2}{2}$. The area of a rectangle is $A_R = xy$. Therefore, the combined area is $A = \frac{\pi r^2}{2} + xy$. Inserting $r = \frac{x}{2}$, we are left with $A = \frac{\pi x^2}{8} + xy$.

$$A = \frac{\pi x^2}{8} + xy$$

$$P = 2y + x + \frac{\pi x}{2} = 25$$

Our next step is solve for the y variable.

$$25 = 2y + x + \frac{\pi x}{2}$$

$$-2y = -25 + x + \frac{\pi x}{2}$$

$$y = \frac{-25 + x + \frac{\pi x}{2}}{-2}$$

Now, we find the maximum area. This is interpreted by finding the point where the rate of change equals 0, or in other words setting the derivative equal to 0. First I will simplify the area equation.

$$A = \frac{\pi x^2}{8} + x\left(\frac{-\pi x}{4} - \frac{x}{2} + \frac{25}{2}\right)$$

$$A = \frac{\pi x^2}{8} - \frac{\pi x^2}{4} - \frac{x^2}{2} + \frac{25x}{2}$$

$$A = \frac{\pi x^2}{8} - \frac{2\pi x^2}{8} - \frac{4x^2}{8} + \frac{25x}{2}$$

$$A = \frac{\pi x^2 - 2\pi x^2 - 4x^2}{8} + \frac{25x}{2}$$

$$A = \frac{-\pi x^2 - 4x^2}{8} + \frac{25x}{2}$$

$$A = (-x^2)\frac{\pi + 4}{8} + \frac{25x}{2}$$

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Now that we have the area equation solely in terms of x, it is time to take the derivative and solve for x.

$$A' = (-2x)\frac{\pi + 4}{8} + \frac{25}{2}$$

$$A' = \frac{-2\pi x - 8x}{8} + \frac{25}{2}$$

$$0 = \frac{-2\pi x - 8x}{8} + \frac{25}{2}$$

$$-\frac{25}{2} = \frac{-2\pi x - 8x}{8}$$

$$-\frac{200}{2} = -2\pi x - 8x$$

$$-100 = -2x(\pi - 4)$$

$$\frac{-100}{\pi - 4} = -2x$$

$$x = \frac{\frac{-100}{\pi - 4}}{-2}$$

$$x \approx 7.001239419$$

The next step is to plug in x to find the value of y.

$$y = \frac{-25+x+\frac{\pi x}{2}}{-2}$$

$$y = \frac{-25+7.001239419+\frac{7.001239419\pi}{2}}{-2}$$

$$y \approx 3.500619709$$

Next, we find the value of r using the value of x.

$$r = \frac{x}{2}$$

$$r = \frac{7.001239419}{2}$$

$$r \approx 3.500619709$$

The final step is to check my work. We'll use the perimeter equation and plug in all of these values.

$$P = 2y + x + \pi r = 25$$

$$P = 2(3.500619709) + 7.001239419 + (3.500619709)\pi = 25$$

Therefore, the dimensions of the Norman window that maximizes its area (letting in the greatest amount of light), given a 25 foot perimeter, is:

$$x \approx 7.001239419\, feet$$

$$y \approx 3.500619709\, feet$$

$$r \approx 3.500619709\, feet$$