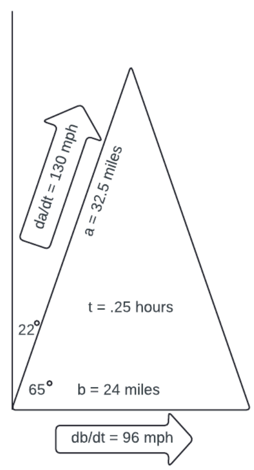
The first step I take in solving the problem is to draw a diagram and identify variables. This will help me identify which law or identity I need to derive. The diagram and variables are shown in the diagram below. The side lengths are extrapolated from the time (15 minutes) at the rate of change (130 miles per hour and 96 miles per hour for a and b respectively).



As there are no right angles in this triangle, I'm going to need the Law of Sines or the Law of Cosines. I'm able to identify which through the angles and sides that I know, which means this problem will require the Law of Cosines. The Law of Cosines says that if we have a non-right angle triangle, and we know two sides and an angle between those two sides, we can calculate the third side with this information. The equation is $c = \sqrt{a^2 + b^2 - 2abcos\gamma}$, where a and b are the two known sides and γ is the known angle between the two sides.

Now that all information is known or extrapolated, the second step is to calculate c, as it will be used shortly in our calculation of $\frac{dc}{dt}$. This is shown in the following steps:

$$c = \sqrt{a^2 + b^2 - 2abcos\gamma}$$

$$c = \sqrt{(32.5)^2 + (24)^2 - (2)(32.5)(24)(cos(65^\circ))}$$

$$c \approx 31.1924$$

The third step is to differentiate the Law of Cosines.

$$c = \sqrt{a^2 + b^2 - 2abcos\gamma}$$

Differentiation via the Chain Rule:

$$\frac{dc}{dt} = \frac{1}{2}(a^2 + b^2 - 2abcos\gamma)^{-\frac{1}{2}}(2aa' + 2bb' + (2abcos\gamma)')$$

Differentiation via nested Product Rule:

$$(2abcos\gamma)' = 2(abcos\gamma)' = 2(ab)(-sin\gamma) + 2(ab' + a'b)(cos\gamma)$$

Simplification:

$$\frac{dc}{dt} = \frac{2aa'+2bb'+2absin\gamma-2ab'cos\gamma-2a'bcos\gamma}{2\sqrt{a^2+b^2-2abcos\gamma}} = \frac{2aa'+2bb'+2absin\gamma-2ab'cos\gamma-2a'bcos\gamma}{2c}$$

Plugging in:

$$\frac{dc}{dt} = \frac{(2)(32.5)(130)+(2)(24)(96)+(2)(32.5)(24)(sin(65^\circ))-(2)(32.5)(96)(cos(65^\circ))-(2)(130)(24)(cos(65^\circ))}{2(31.1924)}$$

Solving:

$$\frac{dc}{dt} = 147.4327759 \text{ miles per hour}$$

Given this, I can conclude that the rate of change of distance, over time, between the two aircraft is approximately 147.43 miles per hour.