Machine learning

By - Andrew NG

Thu 14 Jul

Week 1

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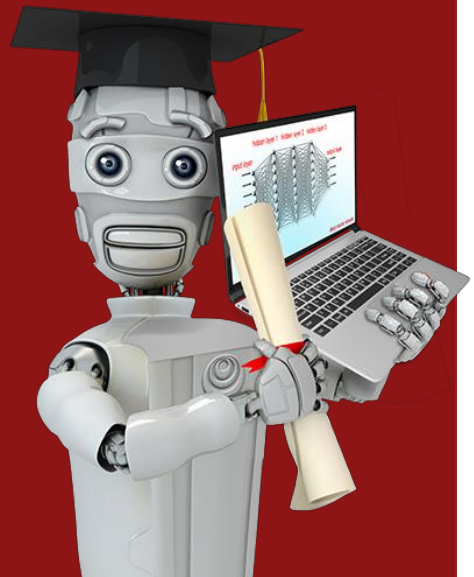
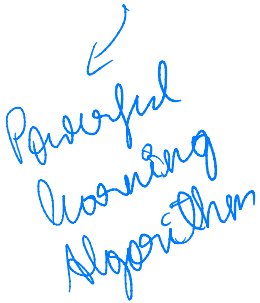


Advanced Learning Algorithms

Welcome!

Advanced learning algorithms

Neural Networks 
  inference (prediction)
  training
Practical advice for building machine learning systems 
Decision Trees 



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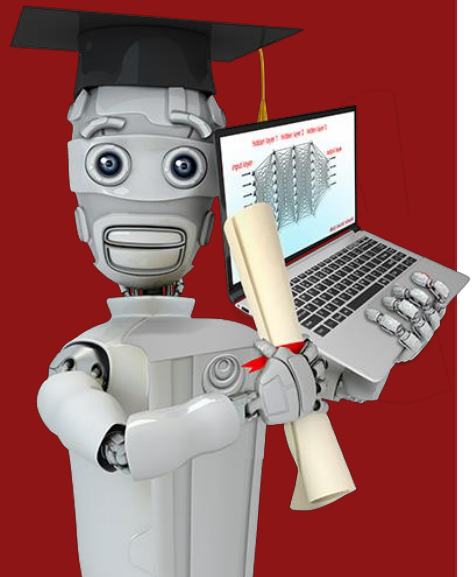


Neural Networks Intuition

Neurons and the brain

Neural networks

Origins: Algorithms that try to mimic the brain.

Used in the 1980's and early 1990's.
Fell out of favor in the late 1990's.

Resurgence from around 2005.

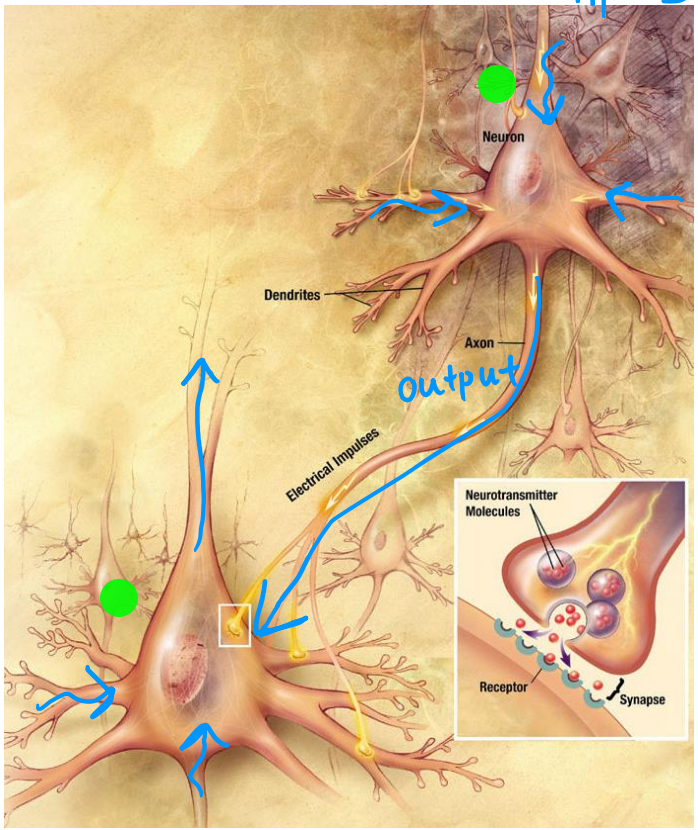
speech → images → text (NLP) → ...

speech recognition

images: Computer vision

text (NLP): Natural Language processing

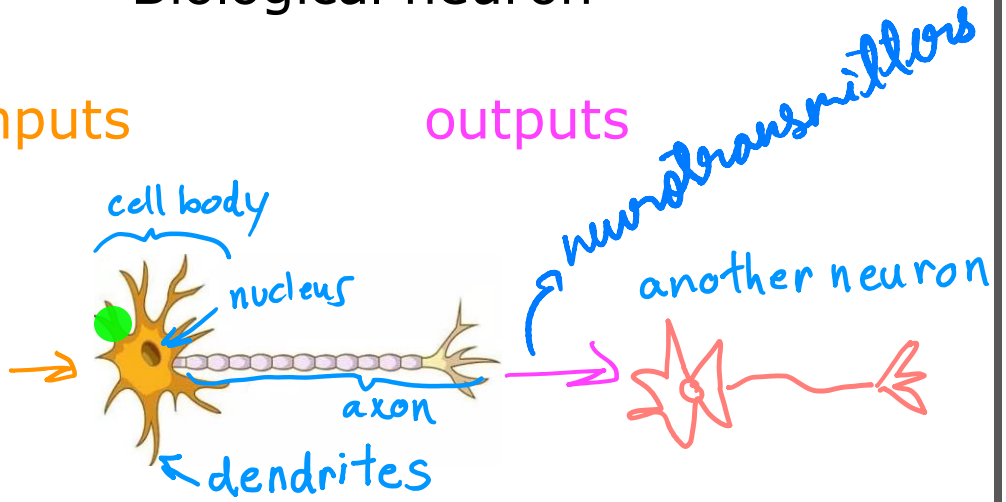
Neurons in the brain



[Credit: US National Institutes of Health, National Institute on Aging]

Biological neuron

inputs outputs



Simplified mathematical model of a neuron

inputs outputs

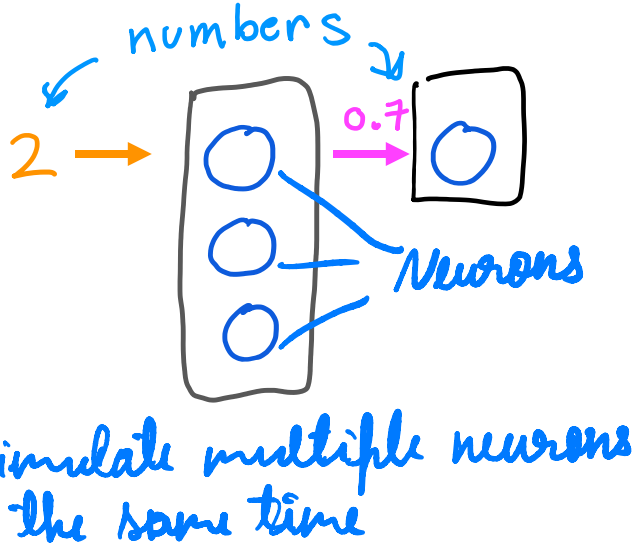
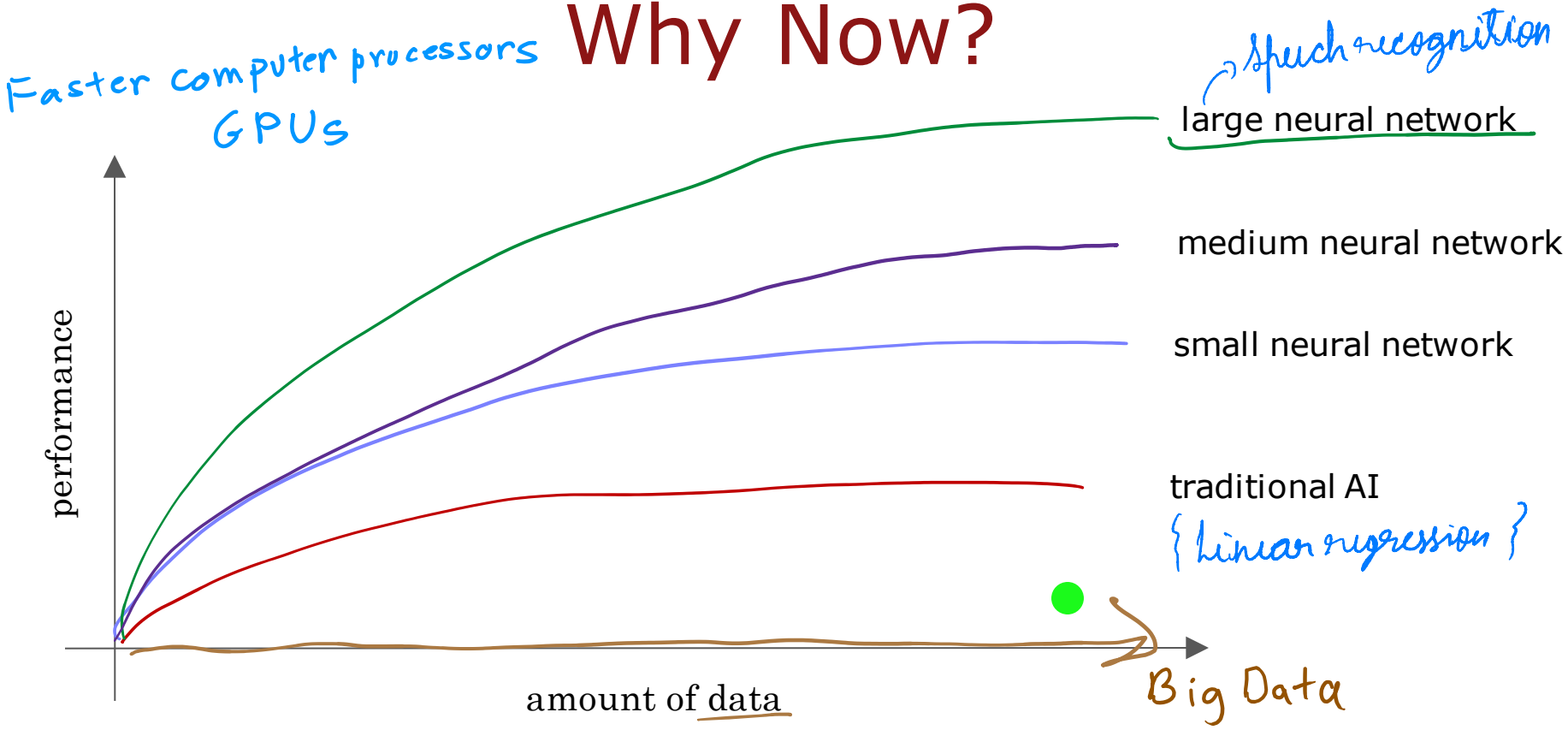


image source: https://biologydictionary.net/sensory-neuron/

Why Now?


Faster computer processors
GPUs
Speech recognition
large neural network
medium neural network
small neural network
traditional AI
{ linear regression }
performance
amount of data
Big Data

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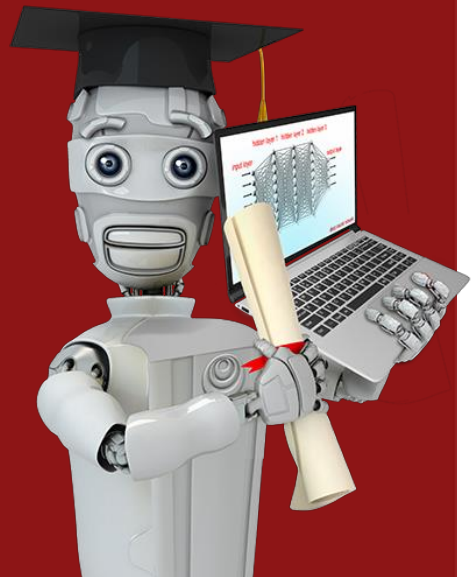
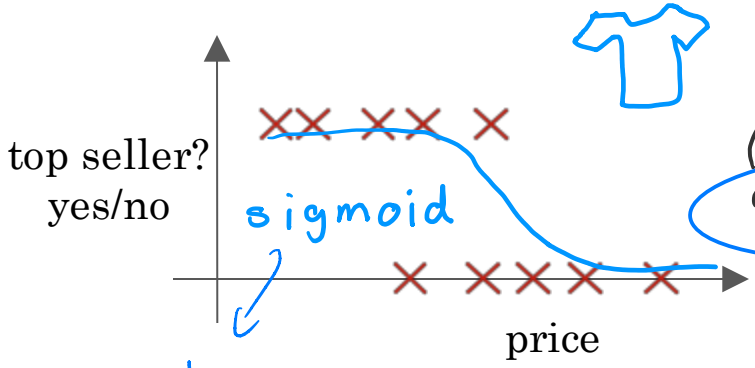


Neural Network Intuition

Demand Prediction

top seller?

Demand Prediction



$g(z) = \frac{1}{1+e^{-z}}$

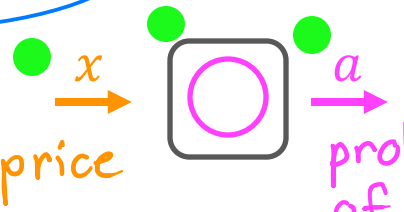
$x = price$

input

$a = f(x) = \frac{1}{1+e^{-(wx+b)}}$

output

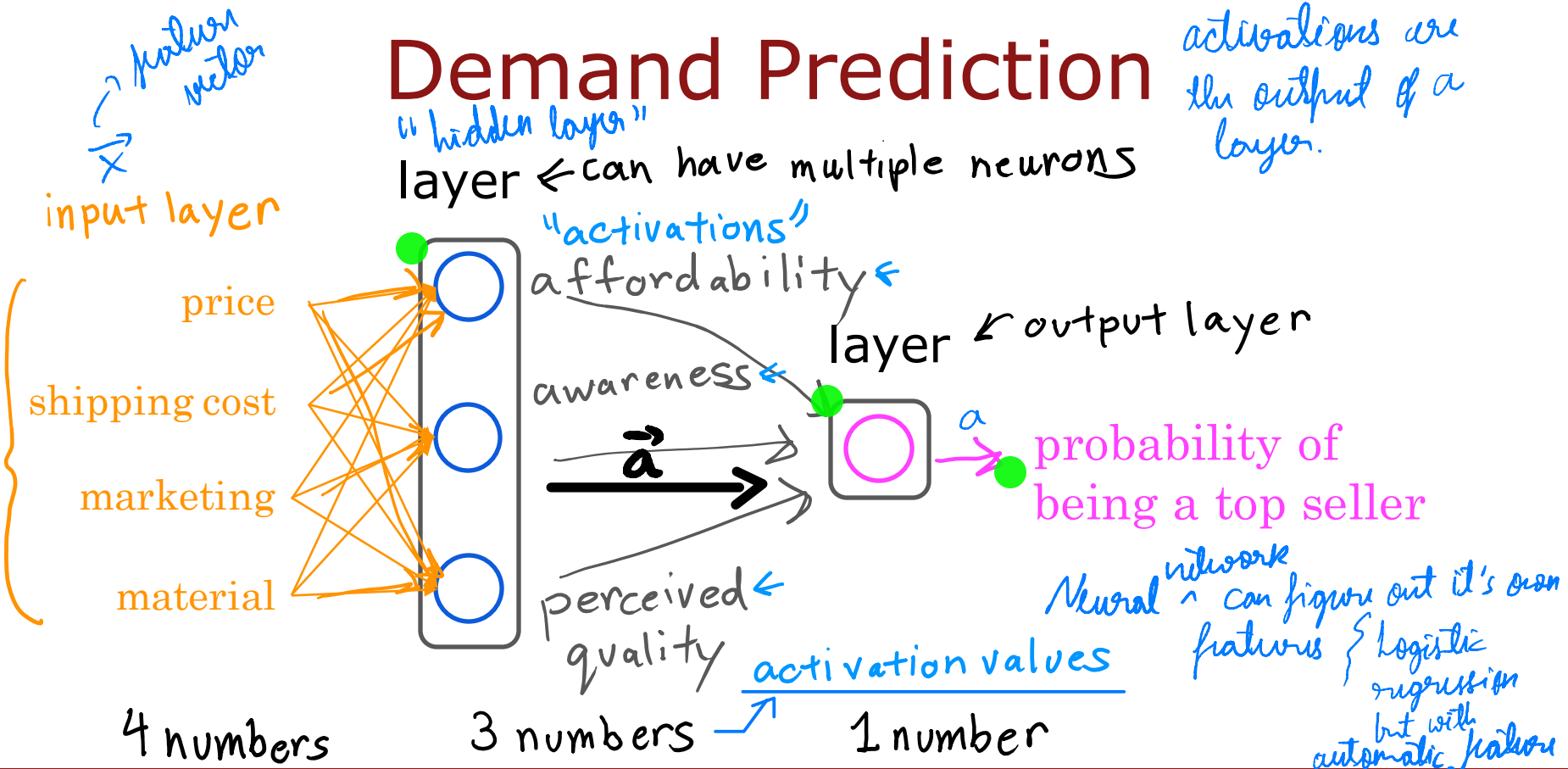
activation



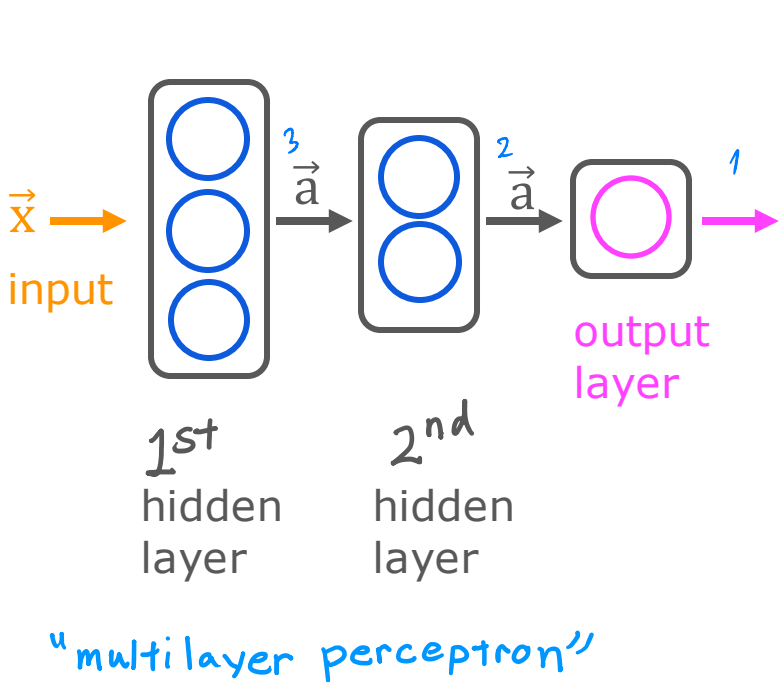
neuron

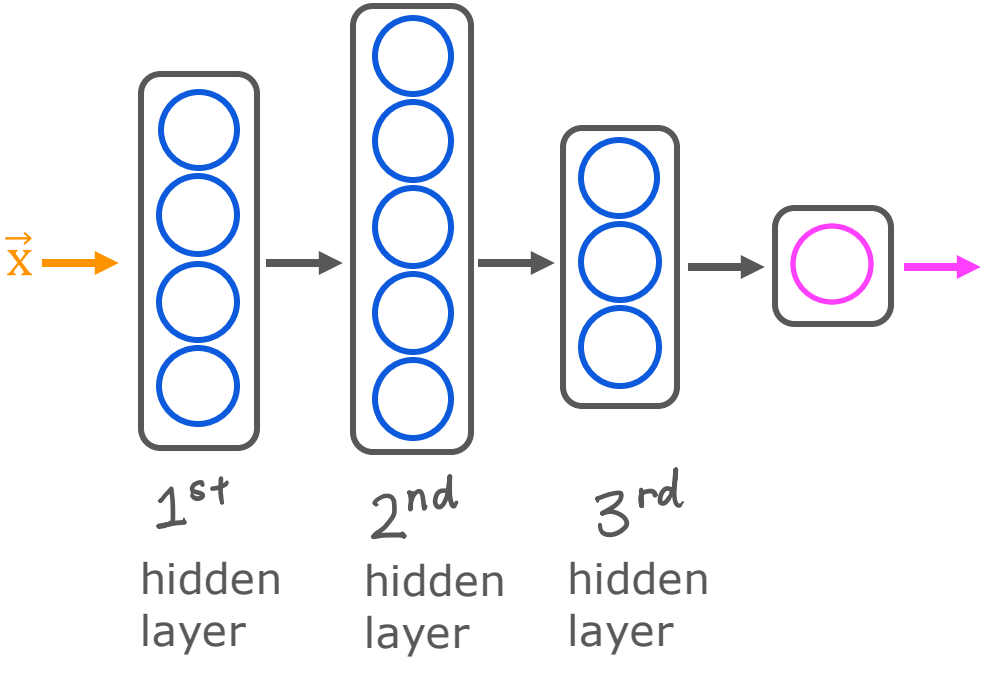
{ tiny computer to work on one or 2 no.s }

Demand Prediction



Multiple hidden layers





neural network architecture { How many hidden layers & neurons }

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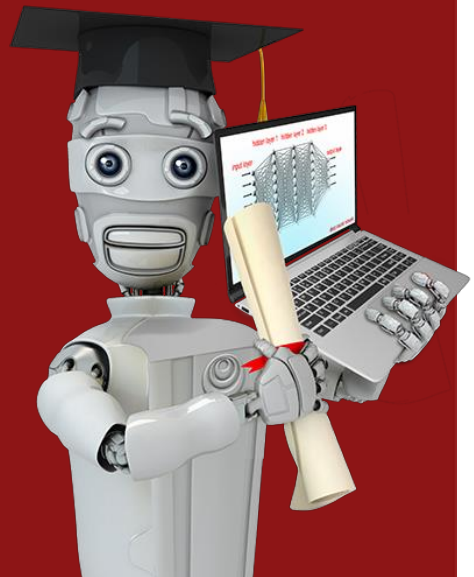


Neural Networks Intuition

Example:
Recognizing Images

Face recognition

1000 pixels

1000 pixels

RGB values { 0 to 255 }

1000 columns

1000 rows

197	185	203	...	...
...	57	64	92	...
⋮				
		...	187	214

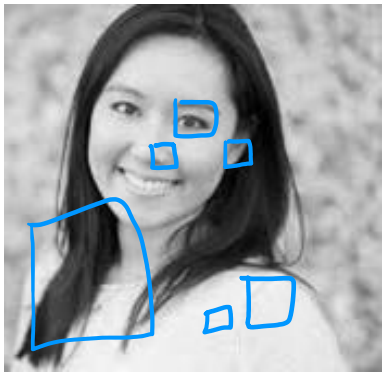
$\vec{x} =$

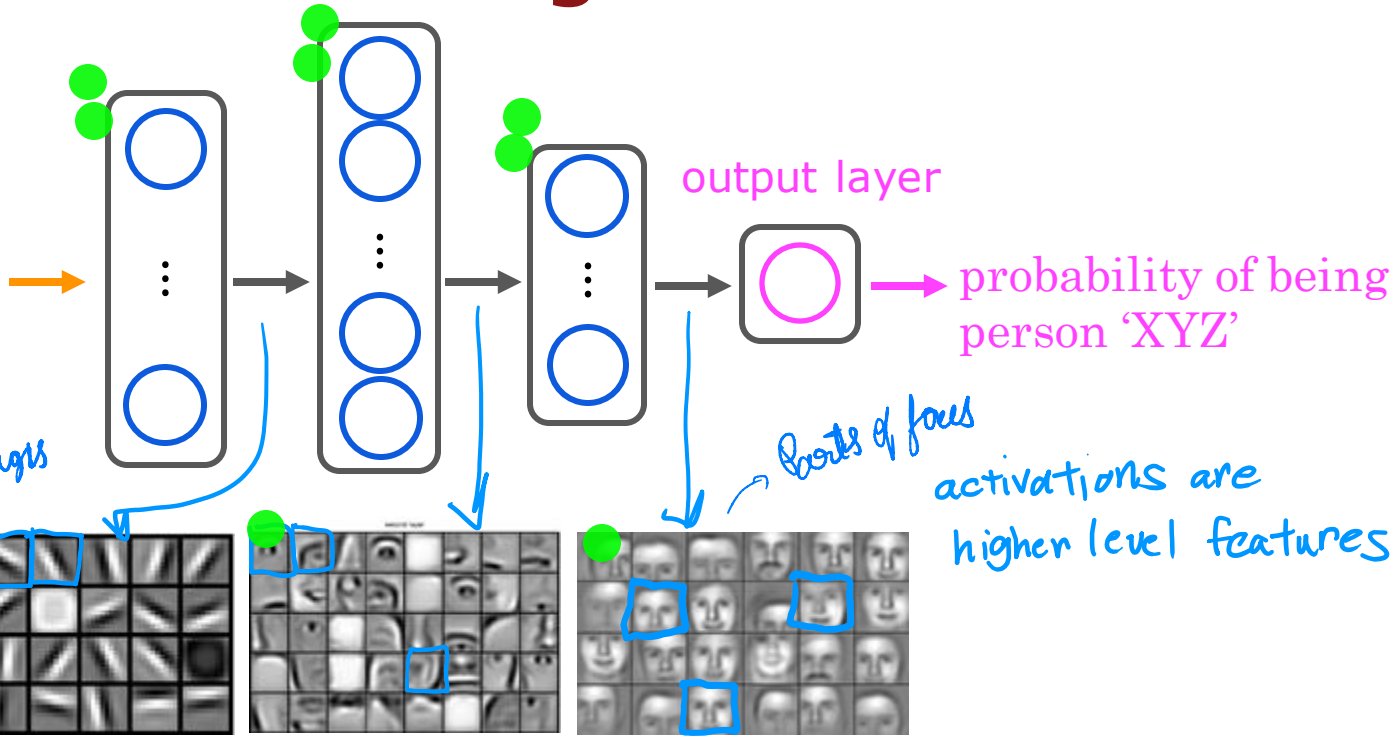
197
185
203
⋮
57
64
92
⋮
187
214

1 million

1000 × 1000

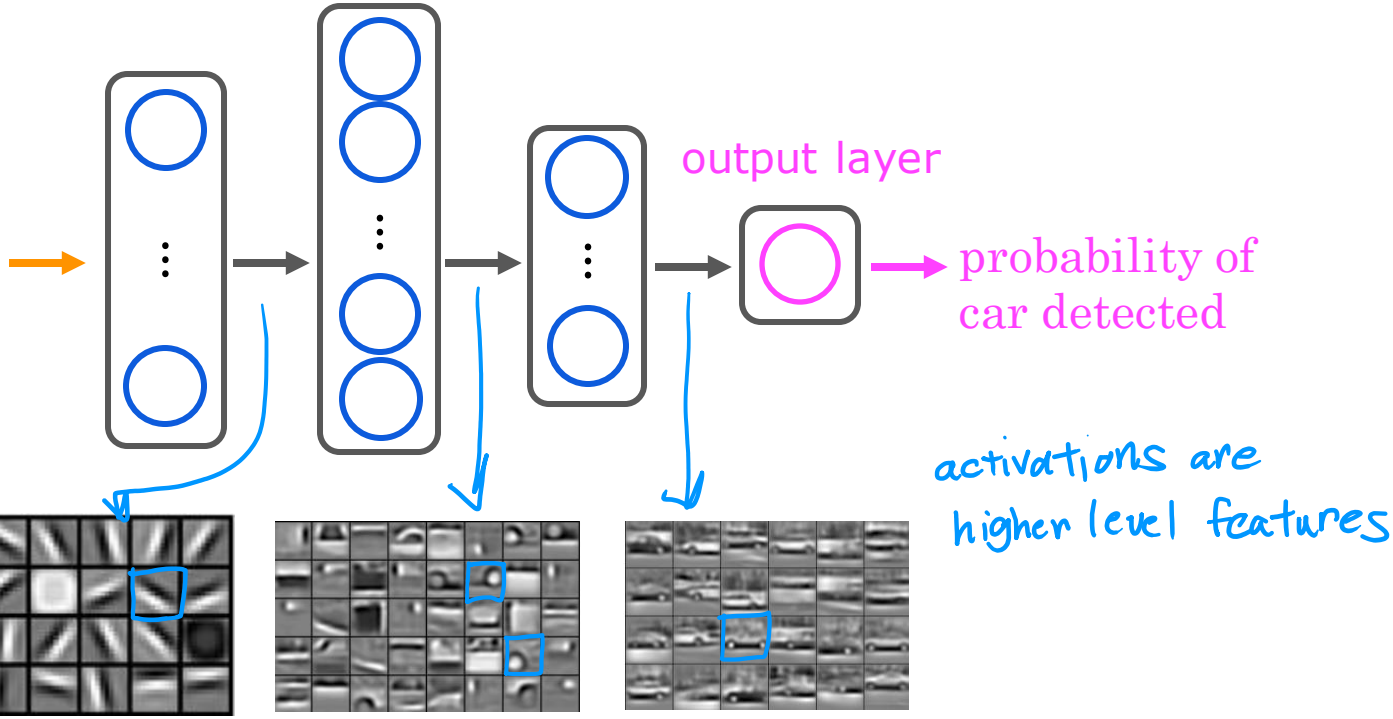
Face recognition





source: Convolutional Deep Belief Networks for Scalable Unsupervised Learning of Hierarchical Representations by Honglak Lee, Roger Grosse, Ranganath Andrew Y. Ng

Car classification



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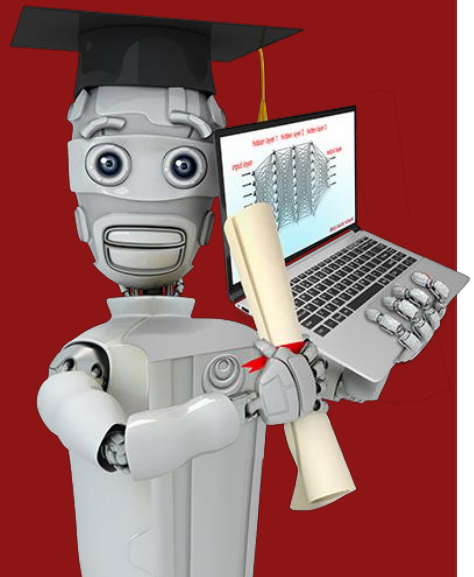
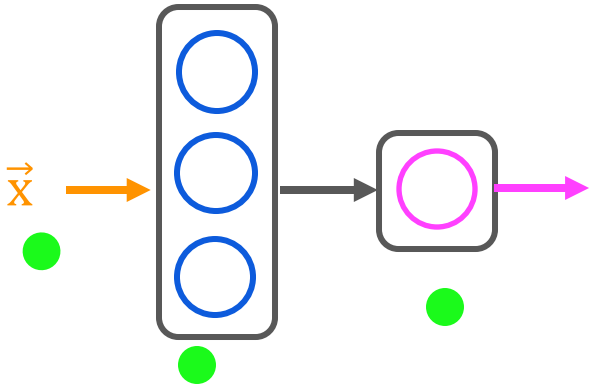


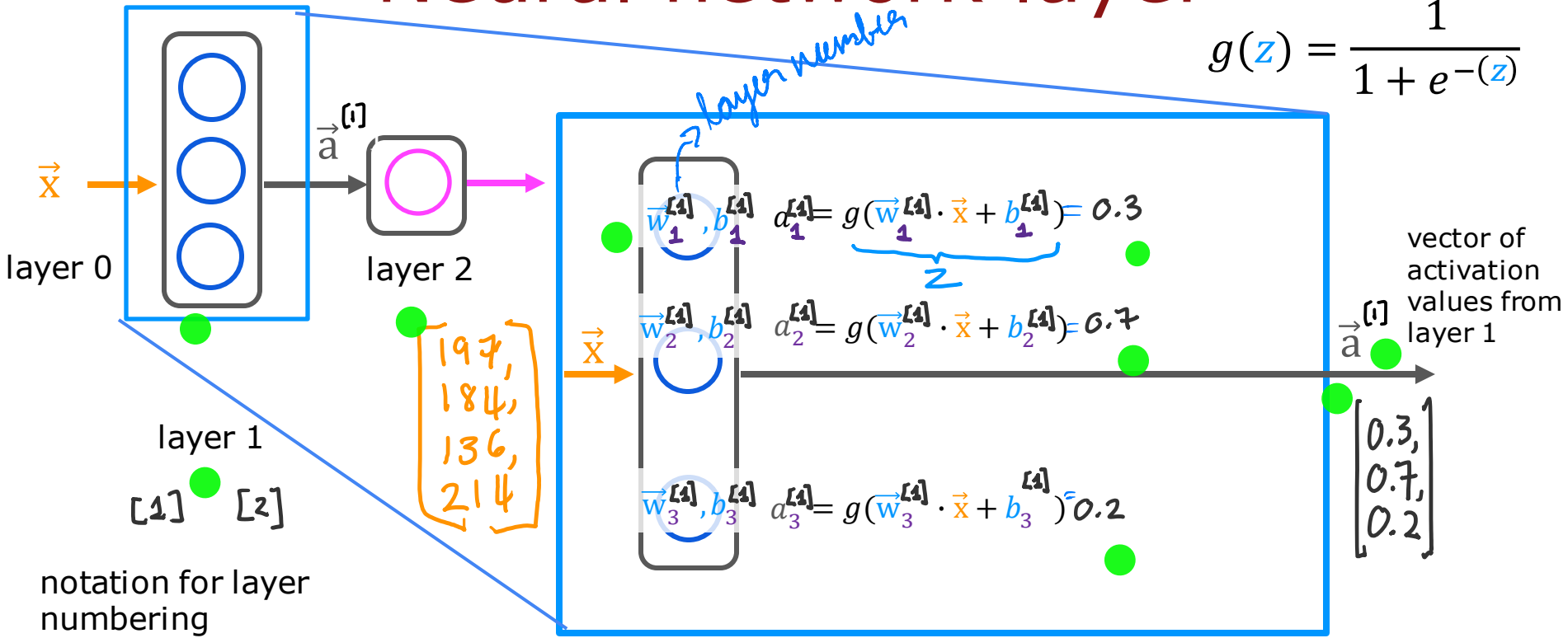
Neural network model

Neural network layer

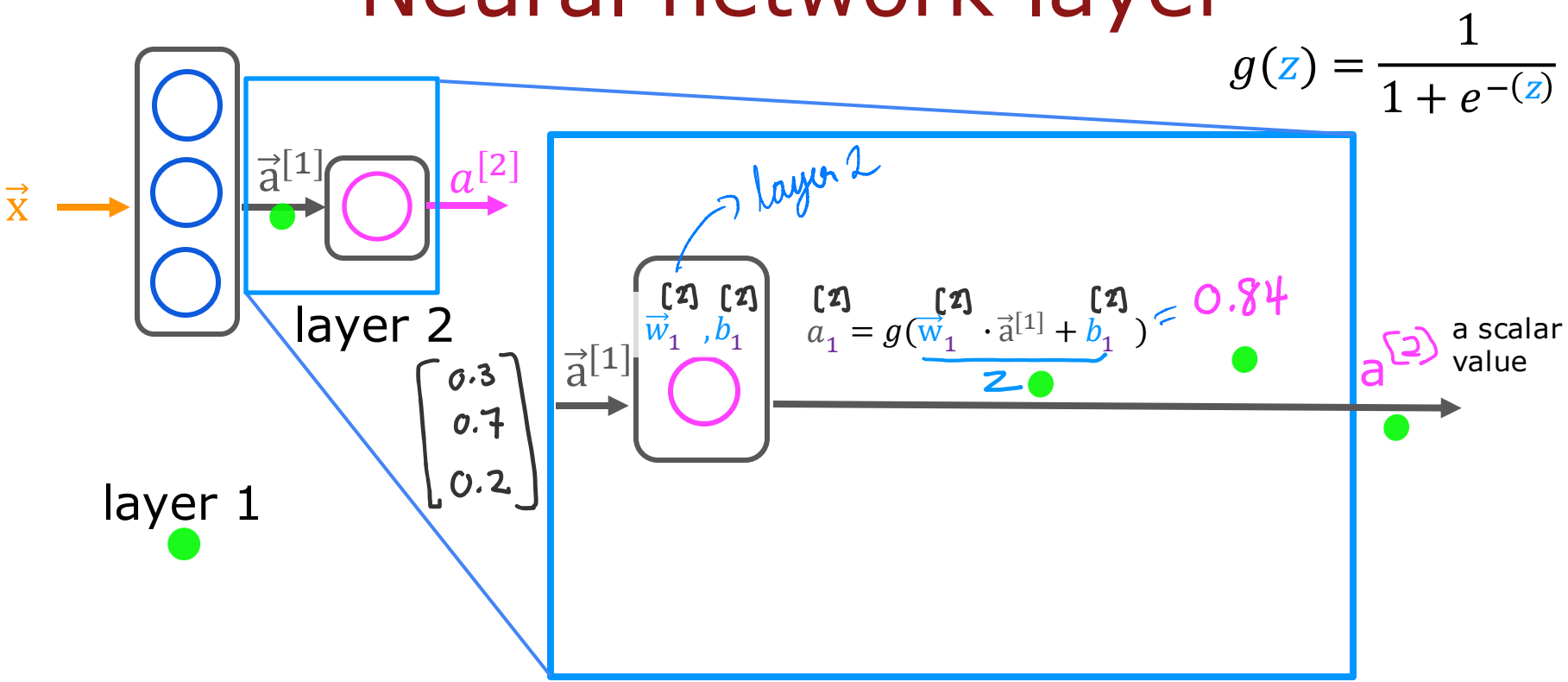
Neural network layer



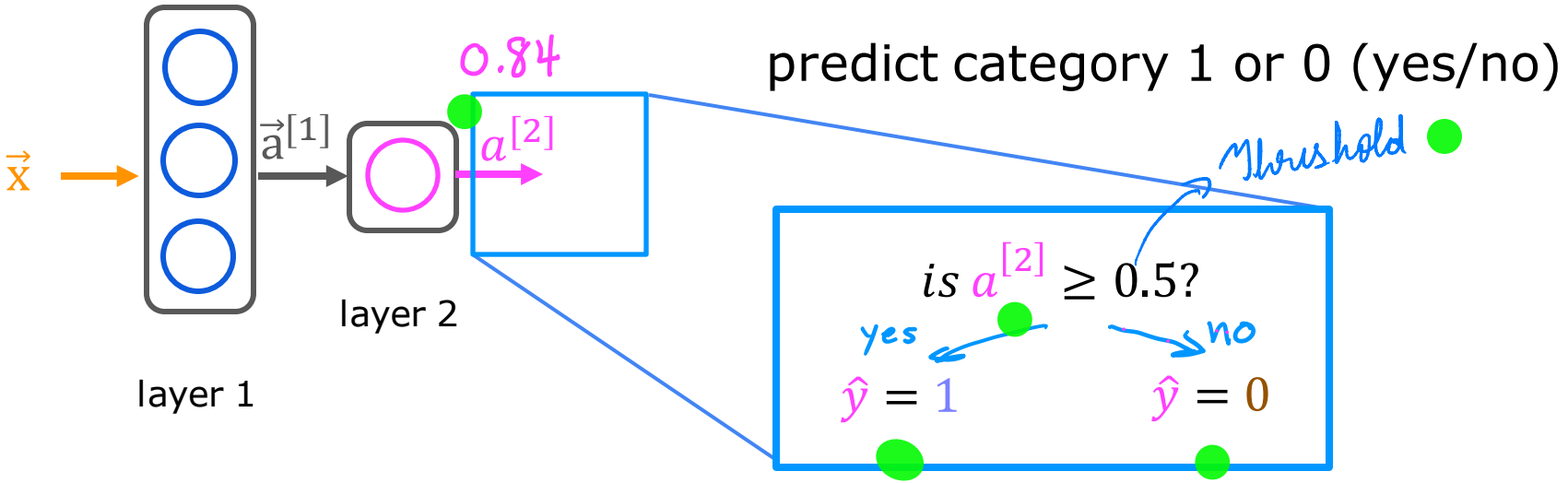
Neural network layer



Neural network layer



Neural network layer



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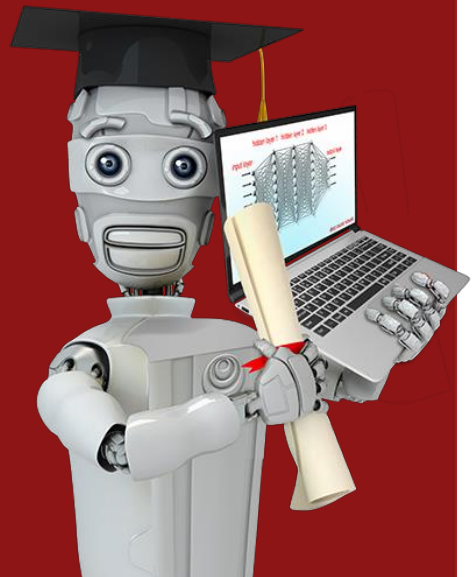
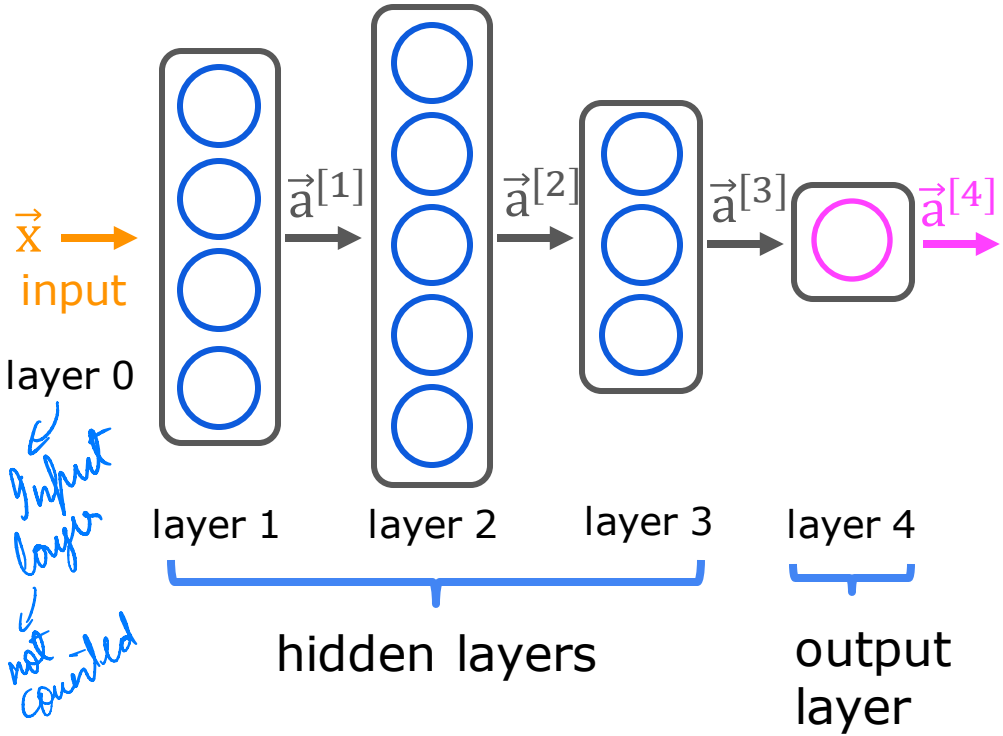


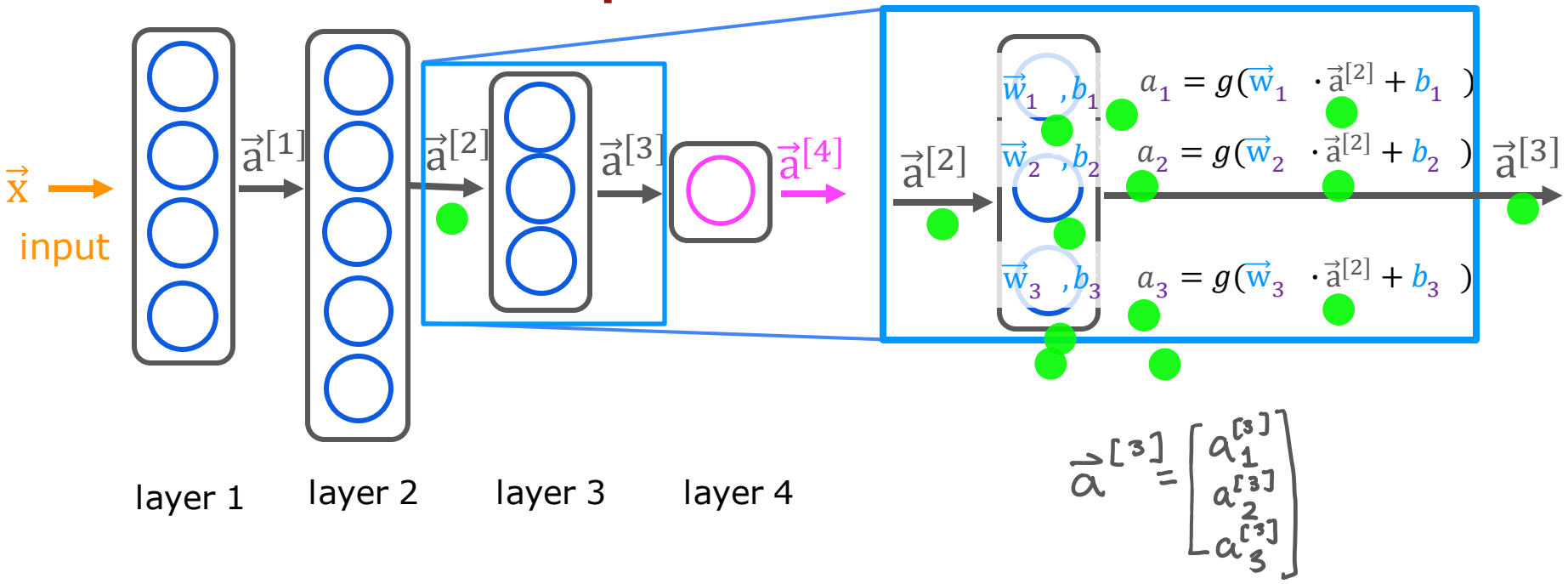
Neural Network Model

More complex neural networks

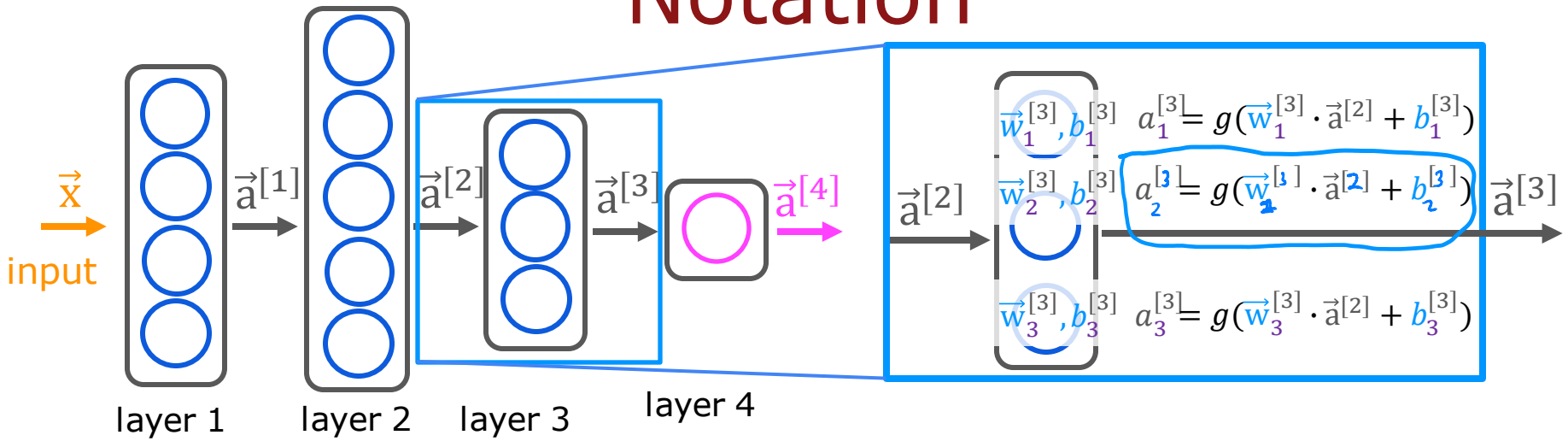
More complex neural network



More complex neural network

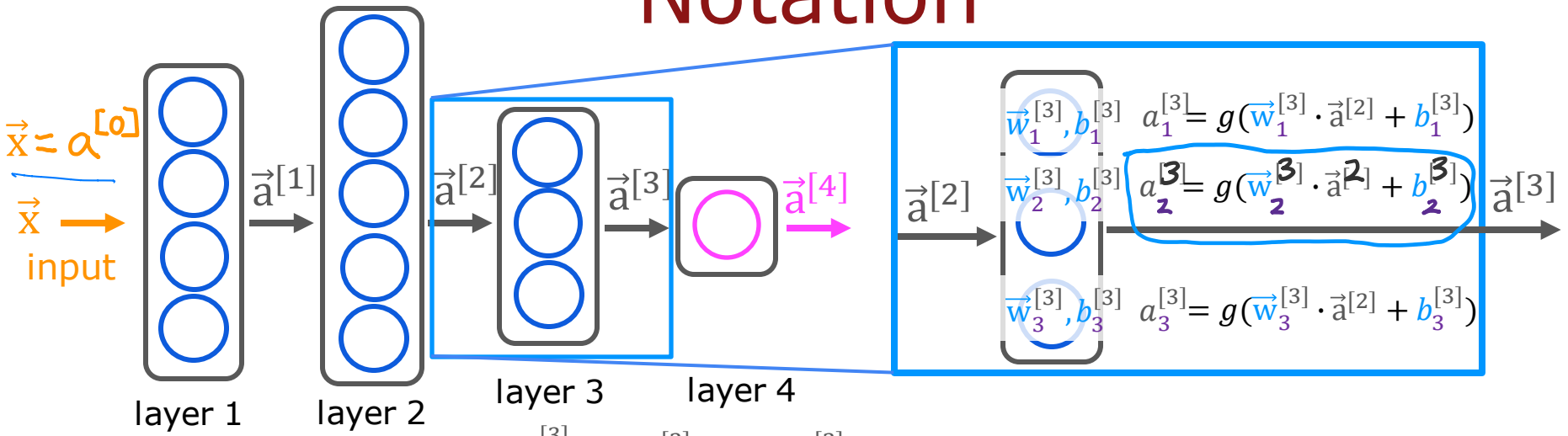


Notation



Question:
Can you fill in the superscripts and subscripts for the second neuron?

Notation



$$a_2^{[3]} = g(\vec{w}_2^{[3]} \cdot \vec{a}^{[2]} + b_2^{[3]})$$

Activation value of layer l, unit(neuron) j

output of layer $l-1$ (previous layer)

$$a_j^{[l]} = g(\vec{w}_j^{[l]} \cdot \vec{a}^{[l-1]} + b_j^{[l]})$$

sigmoid "activation function"

Parameters w & b of layer l, unit j

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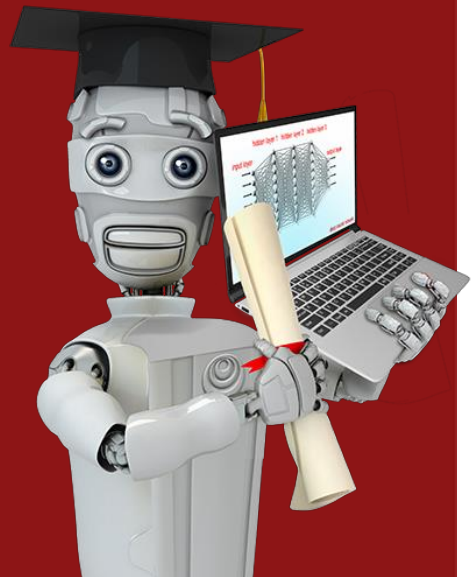
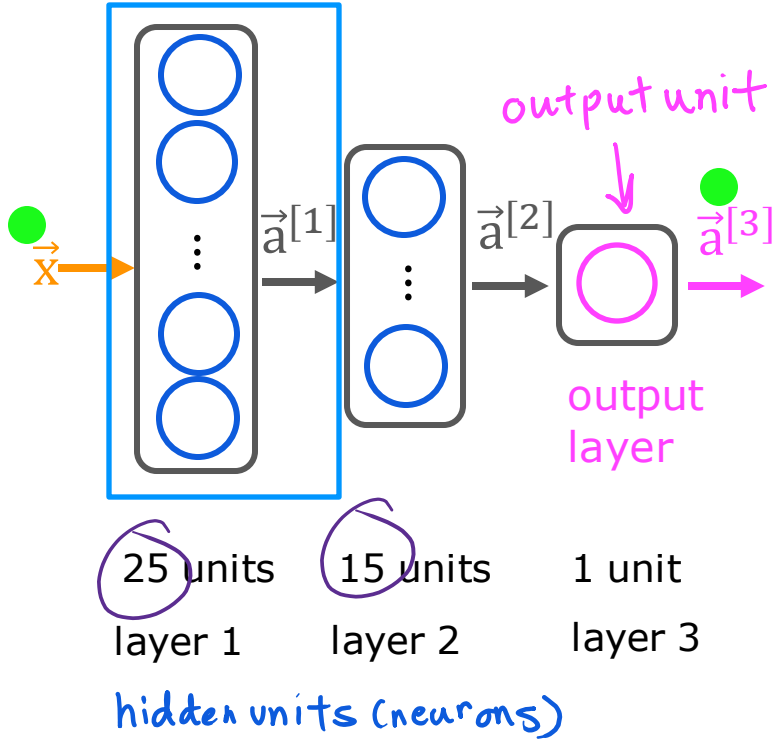
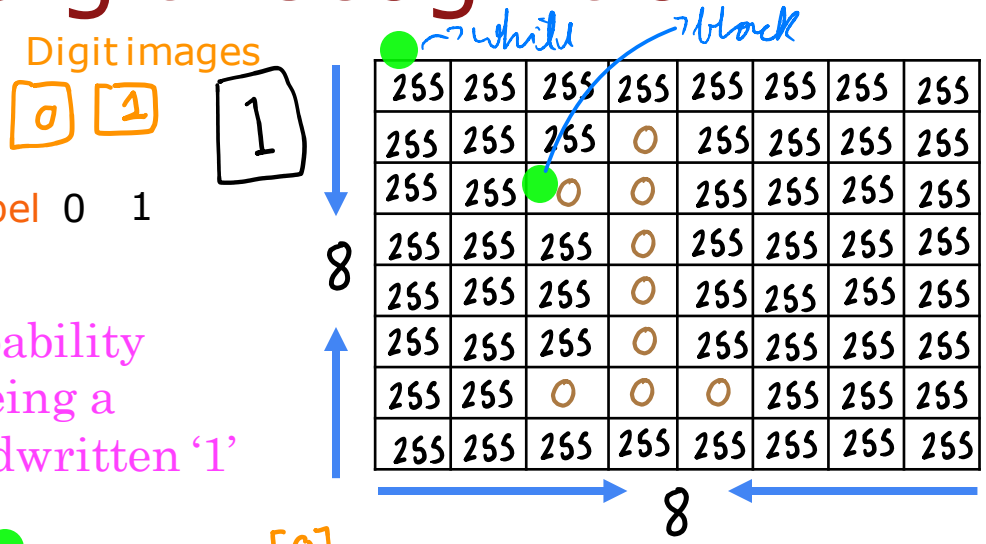


Neural Network Model

Inference: making predictions (forward propagation)

Handwritten digit recognition

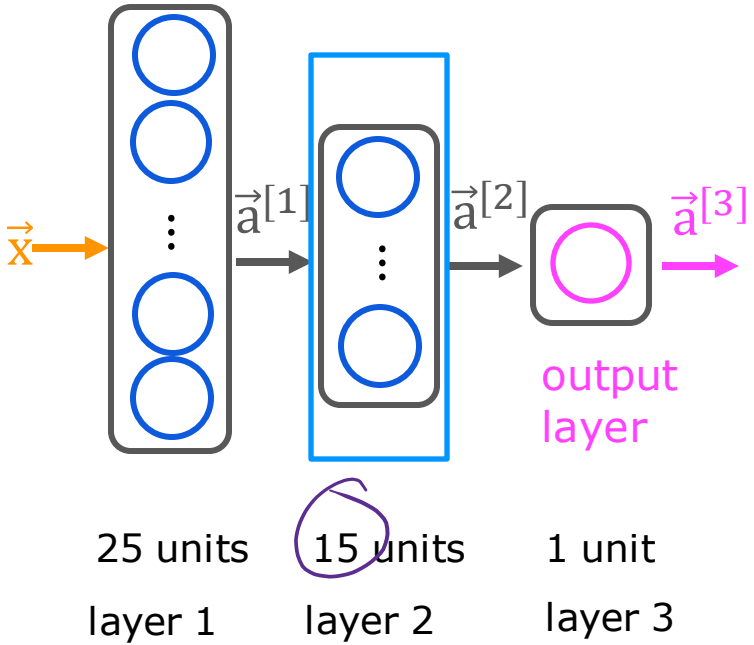


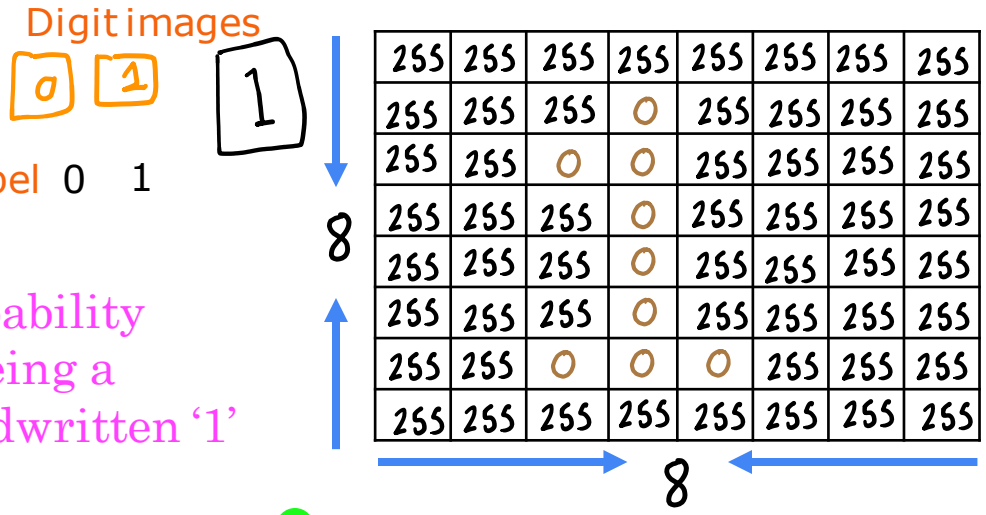


$$\vec{a}^{[1]} = \begin{bmatrix} g(\vec{w}_1^{[1]} \cdot \vec{x} + b_1^{[1]}) \\ \vdots \\ g(\vec{w}_{25}^{[1]} \cdot \vec{x} + b_{25}^{[1]}) \end{bmatrix}$$

$a^{[0]}$

Handwritten digit recognition

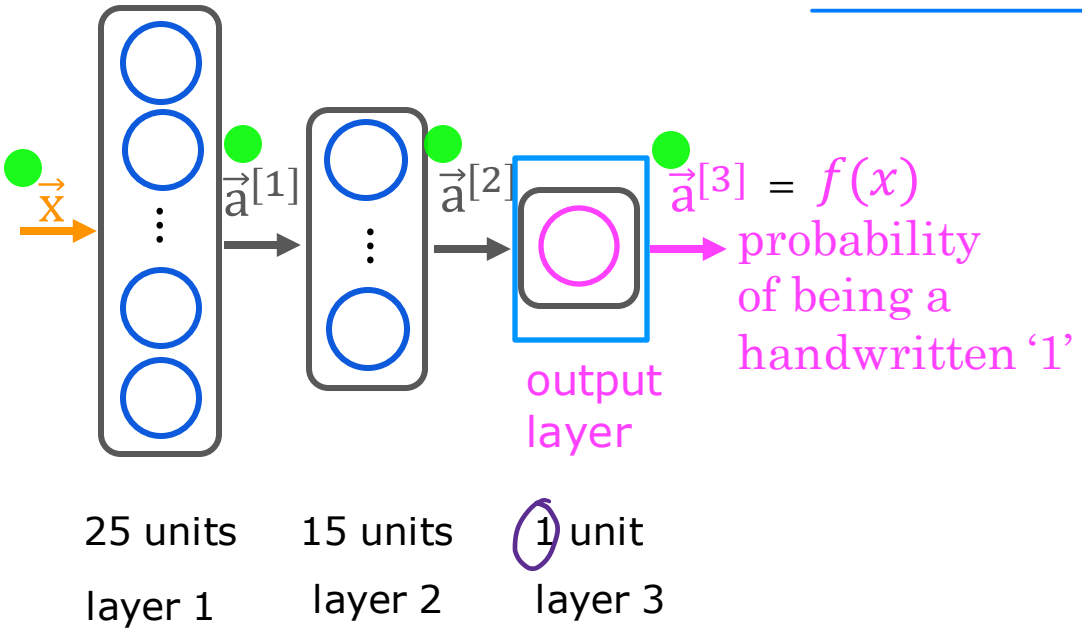




$$\vec{a}^{[2]} = \begin{bmatrix} g(\vec{w}_1^{[2]} \cdot \vec{a}^{[1]} + b_1^{[2]}) \\ \vdots \\ g(\vec{w}_{15}^{[2]} \cdot \vec{a}^{[1]} + b_{15}^{[2]}) \end{bmatrix}$$

Handwritten digit recognition

forward propagation → algo name



$$\vec{a}^{[3]} = \left[g\left(\vec{w}_1^{[3]} \cdot \vec{a}^{[2]} + b_1^{[3]} \right) \right]$$

is $a_1^{[3]} \geq 0.5$?

yes → $\hat{y} = 1$ image is digit 1

no → $\hat{y} = 0$ image isn't digit 1

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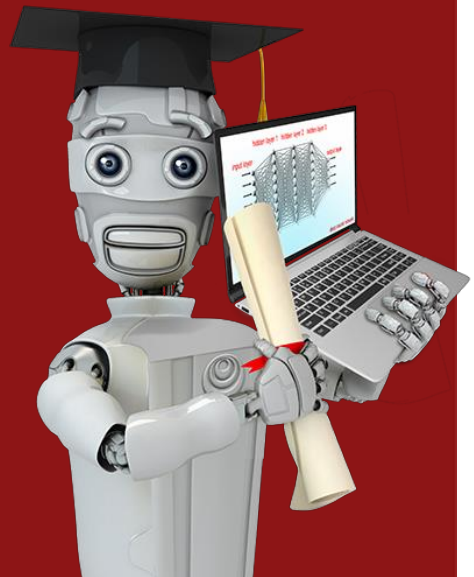
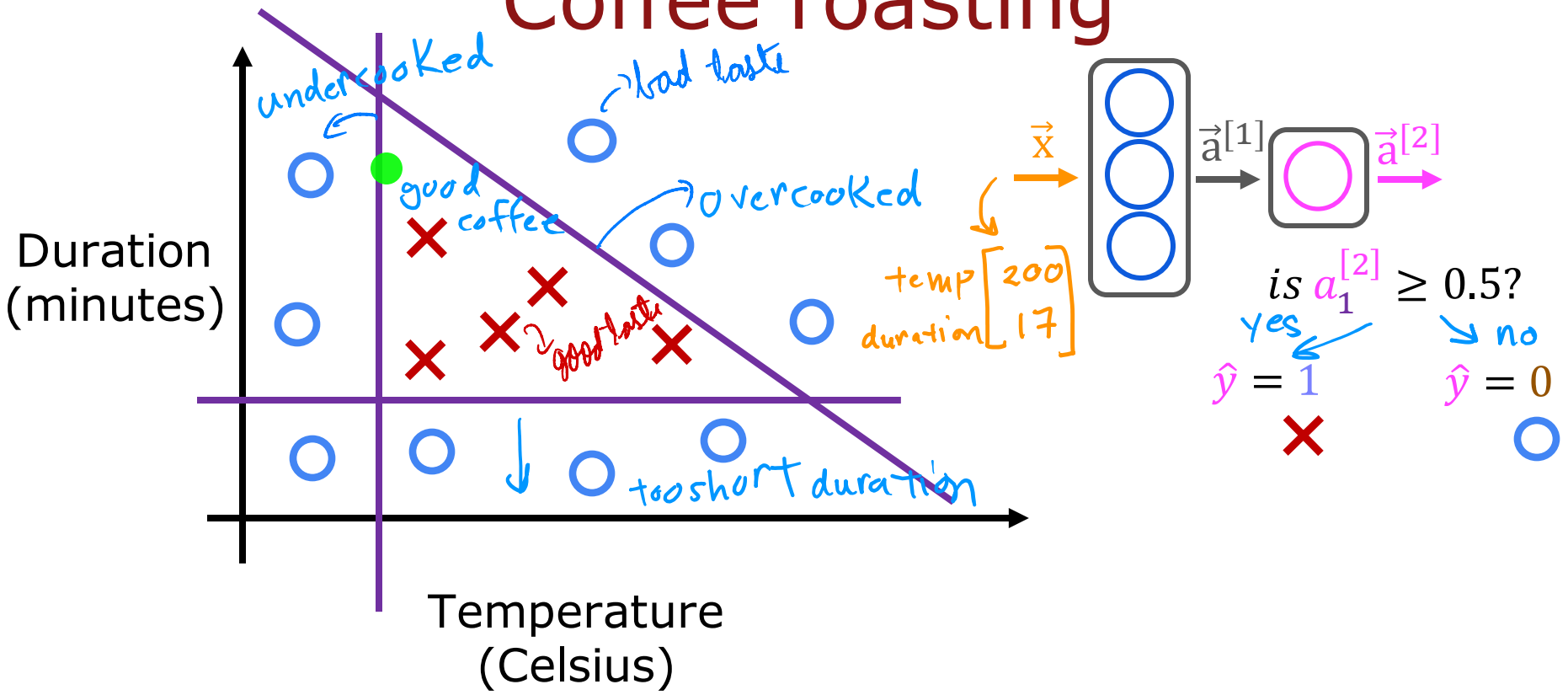
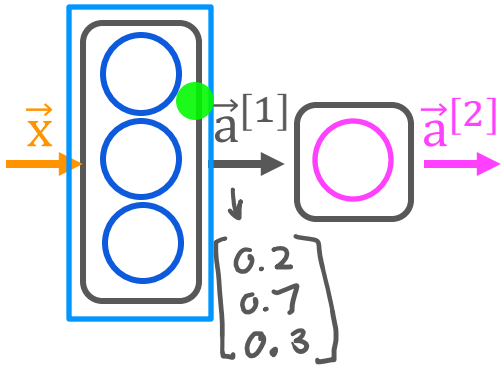


TensorFlow implementation

Inference in Code

Coffee roasting

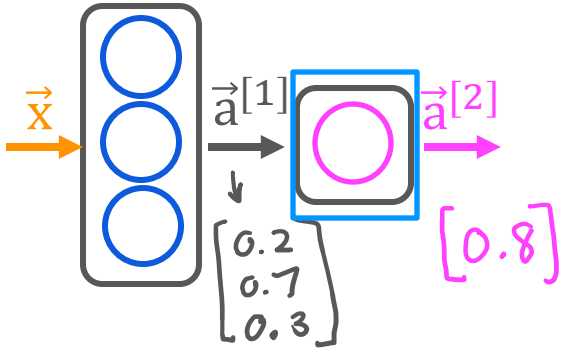





```
x = np.array([[200.0, 17.0]])
layer_1 = Dense(units=3, activation='sigmoid')
a1 = layer_1(x)
```

layer type

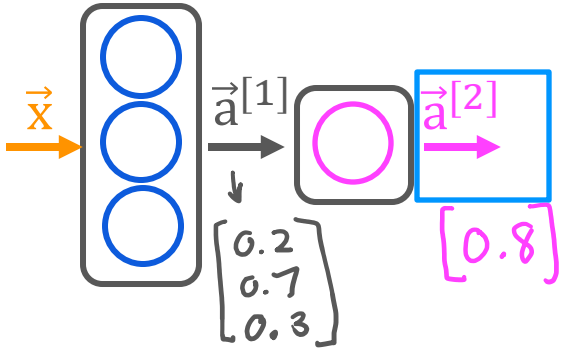
Build the model using TensorFlow

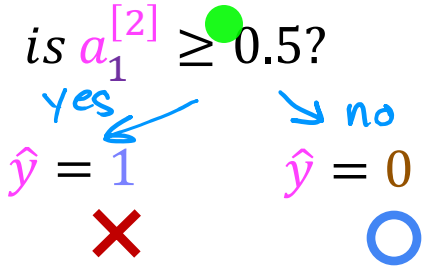



```
x = np.array([[200.0, 17.0]])
layer_1 = Dense(units=3, activation='sigmoid')
a1 = layer_1(x)
```

```
layer_2 = Dense(units=1, activation='sigmoid')
a2 = layer_2(a1)
```

Build the model using TensorFlow



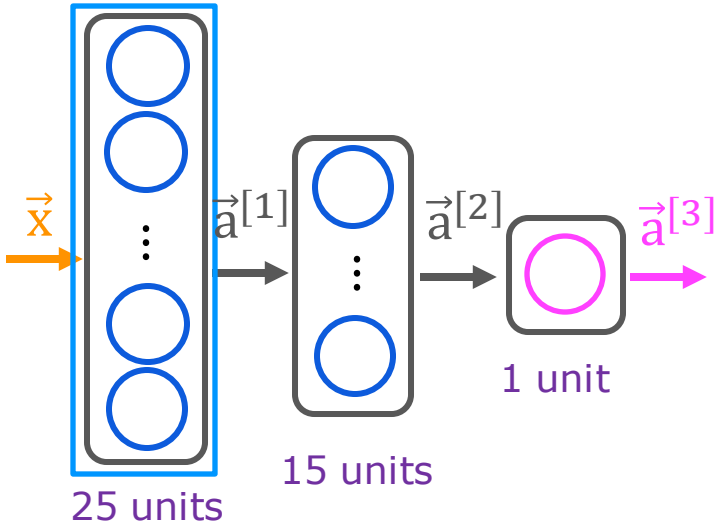



```
x = np.array([[200.0, 17.0]])
layer_1 = Dense(units=3, activation='sigmoid')
a1 = layer_1(x)
```

```
layer_2 = Dense(units=1, activation='sigmoid')
a2 = layer_2(a1)
```

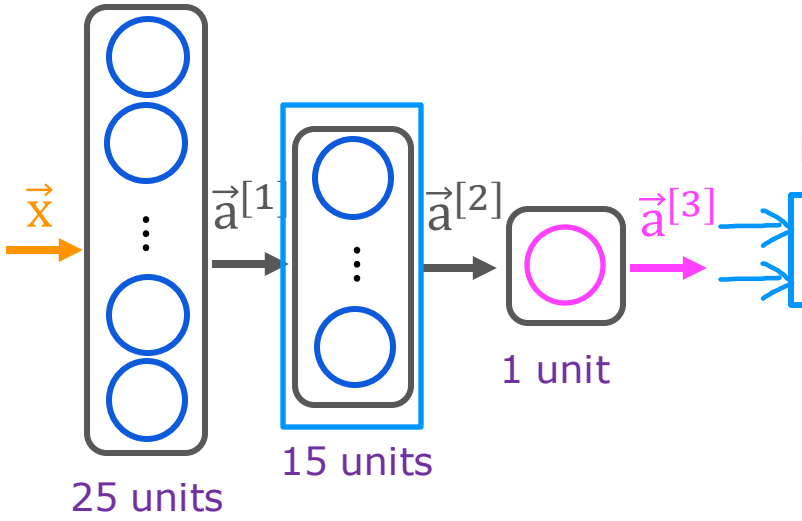
```
if a2 >= 0.5:
  yhat = 1
else:
  yhat = 0
```

Model for digit classification




```
x = np.array([[0.0,...245,...240...0]])
layer_1 = Dense(units=25, activation='sigmoid')
a1 = layer_1(x)
```

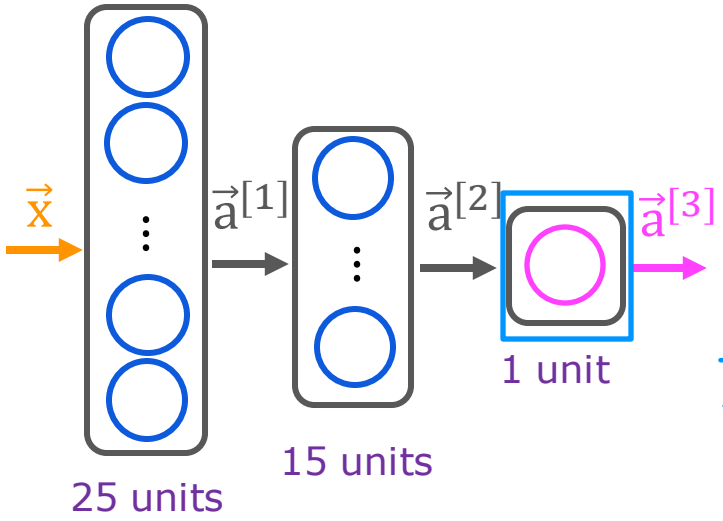
Model for digit classification




```
x = np.array([[0.0,...245,...240...0]])
layer_1 = Dense(units=25, activation='sigmoid')
a1 = layer_1(x)
```

```
layer_2 = Dense(units=15, activation='sigmoid')
a2 = layer_2(a1)
```

Model for digit classification

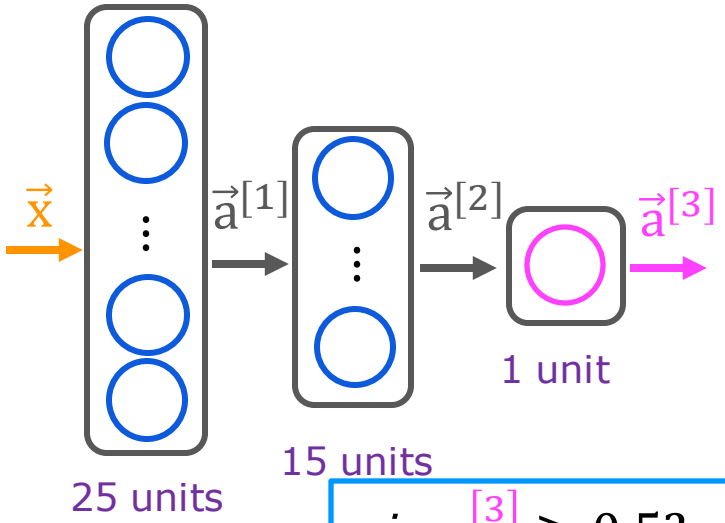



```
x = np.array([[0.0,...245,...240...0]])
layer_1 = Dense(units=25, activation='sigmoid')
a1 = layer_1(x)

layer_2 = Dense(units=15, activation='sigmoid')
a2 = layer_2(a1)

layer_3 = Dense(units=1, activation='sigmoid')
a3 = layer_3(a2)
```

Model for digit classification




```
x = np.array([[0.0,...245,...240...0]])
layer_1 = Dense(units=25, activation='sigmoid')
a1 = layer_1(x)

layer_2 = Dense(units=15, activation='sigmoid')
a2 = layer_2(a1)

layer_3 = Dense(units=1, activation='sigmoid')
a3 = layer_3(a2)
```

$is\ a_1^{[3]} \geq 0.5?$

$\hat{y} = 1$ ✗ $\hat{y} = 0$ ○

```
if a3 >= 0.5:
  yhat = 1
else:
  yhat = 0
```

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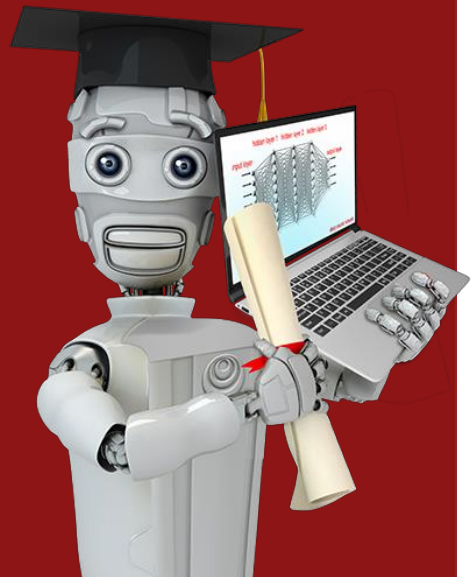


TensorFlow implementation

Data in TensorFlow

Feature vectors

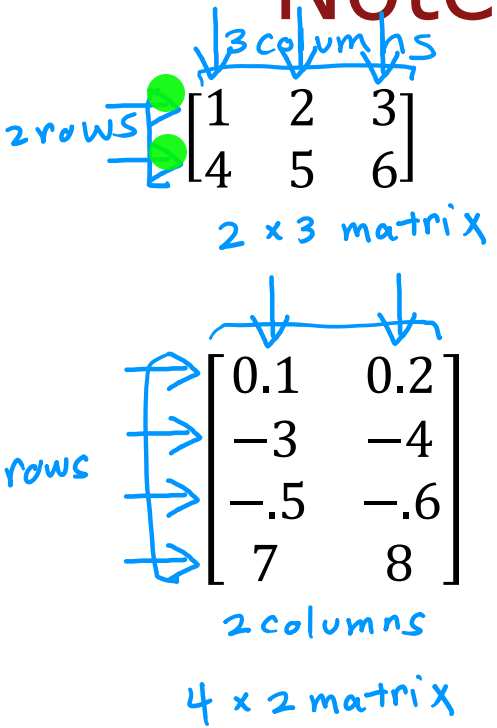
temperature (Celsius)	duration (minutes)	Good coffee? (1/0)
200.0	17.0	1
425.0	18.5	0
…	…	…

```
x = np.array([[200.0, 17.0]])
```

```
[[200.0, 17.0]]
```

Why? {Double brackets}

Note about numpy arrays




```
x = np.array([[1, 2, 3],
              [4, 5, 6]])
```

```
[[1, 2, 3],
 [4, 5, 6]]
```

```
x = np.array([[0.1, 0.2],
              [-3.0, -4.0,],
              [-0.5, -0.6,],
              [7.0, 8.0,]])
```

```
[[0.1, 0.2],
 [-3.0, -4.0,],
 [-0.5, -0.6,],
 [7.0, 8.0,]]
```

rows × columns

2D array

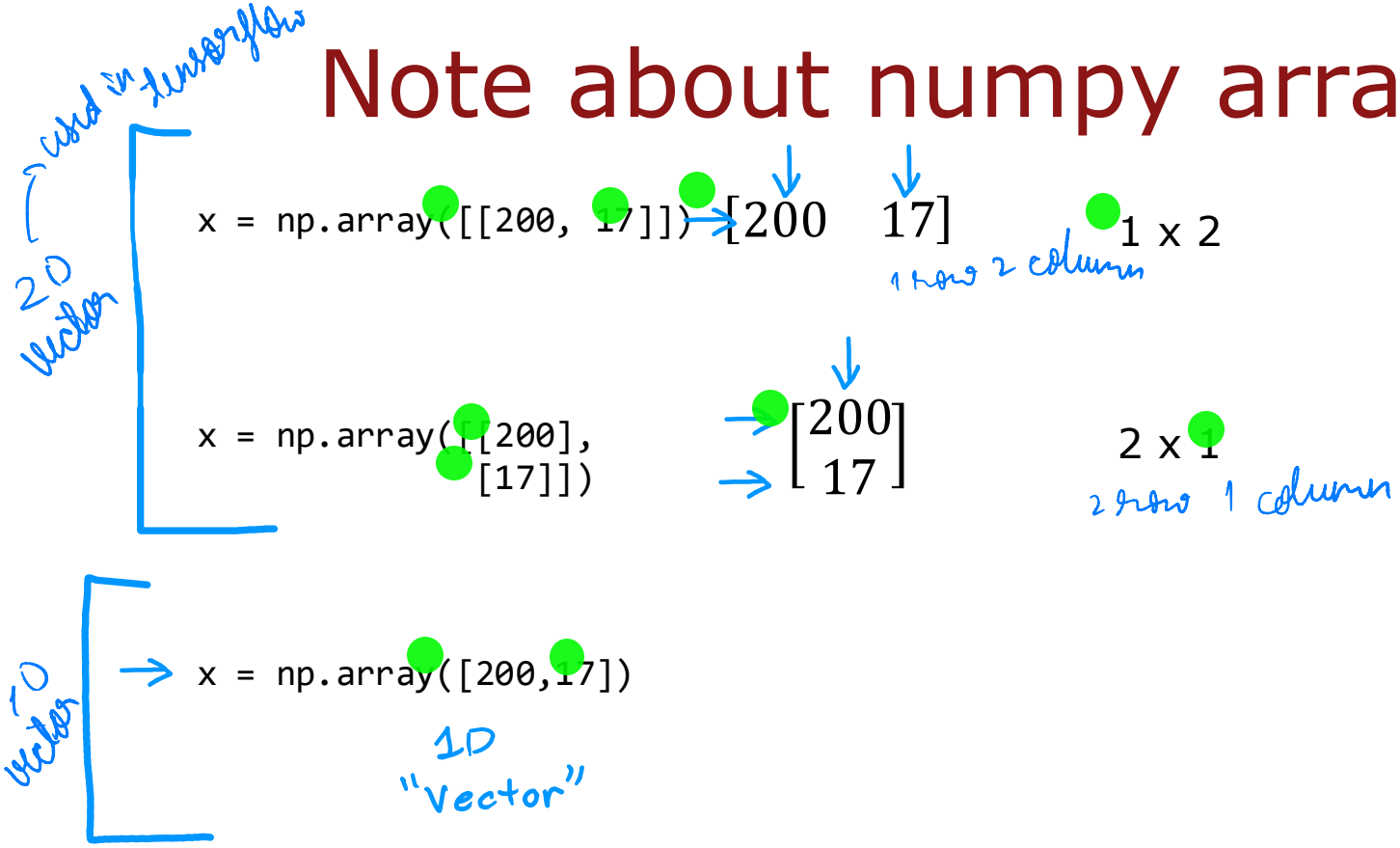
2 x 3

4 x 2

1 x 2

2 x 1

Note about numpy arrays



Feature vectors

temperature (Celsius)	duration (minutes)	Good coffee? (1/0)
200.0	17.0	1
425.0	18.5	0
…	…	…

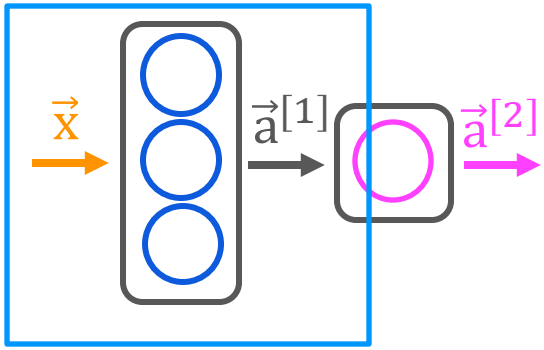
```
x = np.array([[200.0, 17.0]])
```

```
[[200.0, 17.0]]
```

1 x 2

$$\begin{bmatrix} 200.0 & 17.0 \end{bmatrix}$$

Activation vector




```
x = np.array([[200.0, 17.0]])
layer_1 = Dense(units=3, activation='sigmoid')
a1 = layer_1(x)
```

[[0.2, 0.7, 0.3]] 1 x 3 matrix

```
tf.Tensor([[0.2 0.7 0.3]], shape=(1, 3), dtype=float32)
```

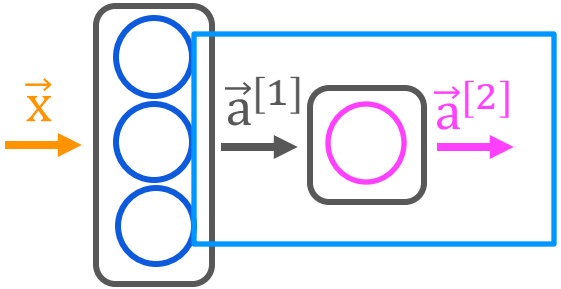
Matrix

```
a1.numpy()
```

converts to numpy array

```
array([[1.4661001, 1.125196 , 3.2159438]], dtype=float32)
```

Activation vector




```
layer_2 = Dense(units=1, activation='sigmoid')
a2 = layer_2(a1)
```

[[0.8]]

1 x 1

```
tf.Tensor([[0.8]], shape=(1, 1), dtype=float32)
```

1 x 1 matrix {Tensor}

```
a2.numpy()
```

```
array([[0.8]], dtype=float32)
```

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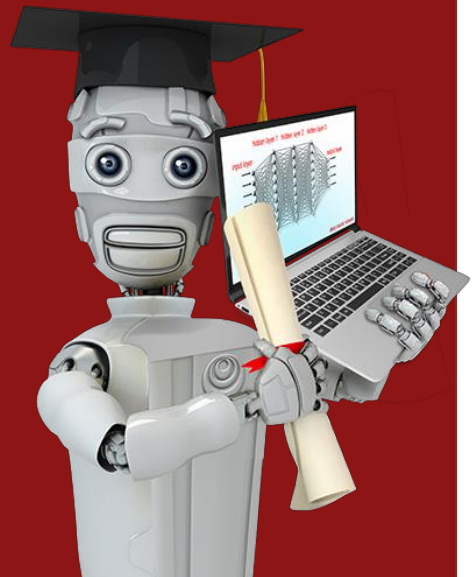
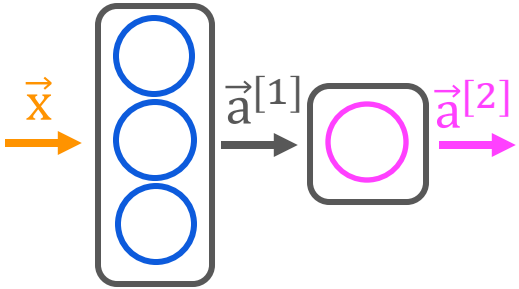


TensorFlow implementation

Building a neural network

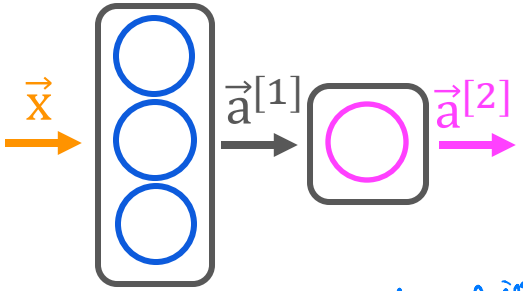
What you saw earlier




```
x = np.array([[200.0, 17.0]])
layer_1 = Dense(units=3, activation="sigmoid")
a1 = layer_1(x)

layer_2 = Dense(units=1, activation="sigmoid")
a2 = layer_2(a1)
```

Building a neural network architecture




```
layer_1 = Dense(units=3, activation="sigmoid")
layer_2 = Dense(units=1, activation="sigmoid")
model = Sequential([layer_1, layer_2])
```

layer1 layer2

left to right

		y
200	17	1
120	5	0
425	20	0
212	18	1

```
x = np.array([[200.0, 17.0],
              [120.0, 5.0],
              [425.0, 20.0],
              [212.0, 18.0]])
```

4 x 2

targets

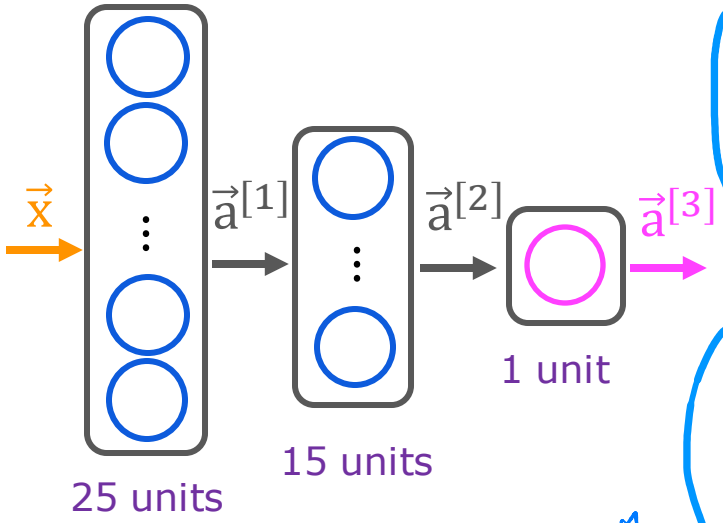
```
y = np.array([1,0,0,1])

model.compile(...)
model.fit(x,y)

model.predict(x_new)
```

more about this next week!

Digit classification model



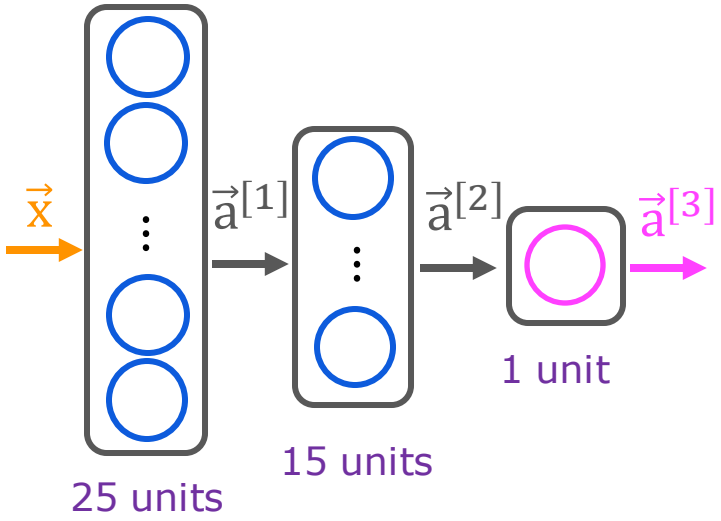

```
layer_1 = Dense(units=25, activation="sigmoid")
layer_2 = Dense(units=15, activation="sigmoid")
layer_3 = Dense(units=1, activation="sigmoid")
model = Sequential([layer_1, layer_2, layer_3])
model.compile(...)
x = np.array([[0..., 245, ..., 17],
              [0..., 200, ..., 184]])
y = np.array([1,0])
model.fit(x,y)
model.predict(x_new)
```

Defines loss func

← more about this next week!

runs gradient descent

Digit classification model




```
model = Sequential([
    Dense(units=25, activation="sigmoid"),
    Dense(units=15, activation="sigmoid"),
    Dense(units=1, activation="sigmoid")])

model.compile(...)

x = np.array([[0..., 245, ..., 17],
              [0..., 200, ..., 184]])
y = np.array([1,0])

model.fit(x,y)

model.predict(x_new)
```

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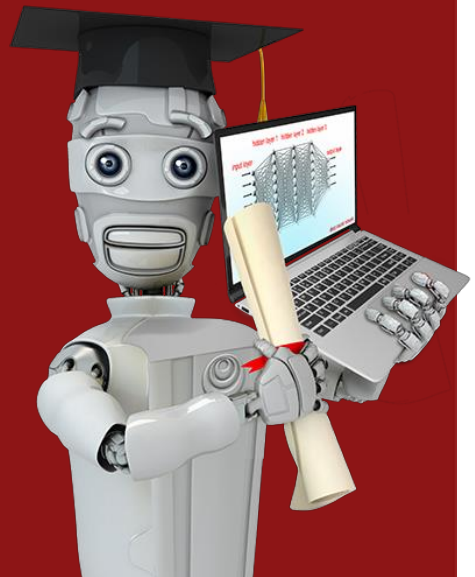
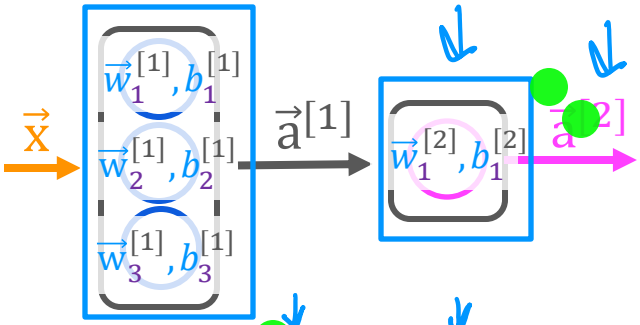


Neural network implementation in Python

Forward prop in a single layer

forward prop (coffee roasting model)



$a_1^{[2]} = g(\vec{w}_1^{[2]} \cdot \vec{a}^{[1]} + b_1^{[2]})$

```
w2_1 = np.array([-7, 8])
b2_1 = np.array([3])
z2_1 = np.dot(w2_1,a1)+b2_1
a2_1 = sigmoid(z1_1)
```

$w_1^{[2]}$ w2_1

```
x = np.array([200, 17])
```

1D arrays

$a_1^{[1]} = g(\vec{w}_1^{[1]} \cdot \vec{x} + b_1^{[1]})$ $a_2^{[1]} = g(\vec{w}_2^{[1]} \cdot \vec{x} + b_2^{[1]})$ $a_3^{[1]} = g(\vec{w}_3^{[1]} \cdot \vec{x} + b_3^{[1]})$

```
w1_1 = np.array([1, 2])
b1_1 = np.array([-1])
z1_1 = np.dot(w1_1,x)+ b
a1_1 = sigmoid(z1_1)
```

```
w1_2 = np.array([-3, 4])
b1_2 = np.array([1])
z1_2 = np.dot(w1_2,x)+b
a1_2 = sigmoid(z1_2)
```

```
w1_3 = np.array([5, -6])
b1_3 = np.array([2])
z1_3 = np.dot(w1_3,x)+b
a1_3 = sigmoid(z1_3)
```

```
a1 = np.array([a1_1, a1_2, a1_3])
```

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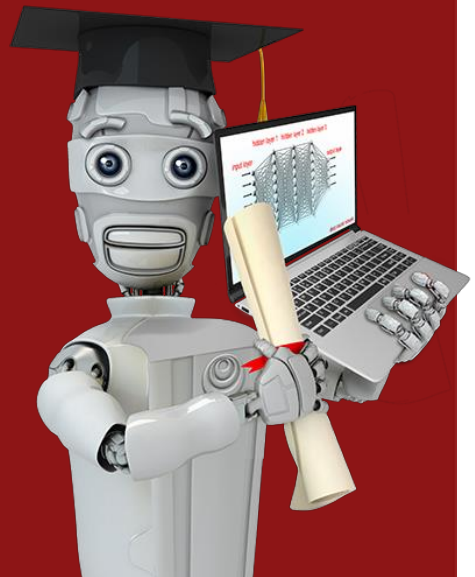
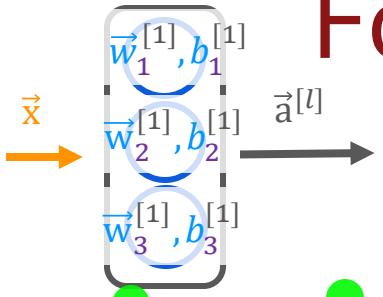


Neural network implemenation in Python

General implementation of forward propagation

Forward prop in NumPy



$\vec{w}_1^{[1]} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ $\vec{w}_2^{[1]} = \begin{bmatrix} -3 \\ 4 \end{bmatrix}$ $\vec{w}_3^{[1]} = \begin{bmatrix} 5 \\ -6 \end{bmatrix}$

```
W = np.array([
    [1, -3, 5]
    [2, 4, -6]])
```
2 by 3

$b_1^{[l]} = -1 \quad b_2^{[l]} = 1 \quad b_3^{[l]} = 2$

```
b = np.array([-1, 1, 2])
```

$\vec{a}^{[0]} = \vec{x}$

```
a_in = np.array([-2, 4])
```

func" to get a single layer

```
def dense(a_in,W,b, g):
    units = W.shape[1]
    a_out = np.zeros(units)
    for j in range(units):
        w = W[:,j]
        z = np.dot(w,a_in) + b[j]
        a_out[j] = g(z)
    return a_out
```

3 [0,0,0] 0,1,2 j-th column

$a^{[1]}$ $a^{[1]}$

```
def sequential(x):
    a1 = dense(x,W1,b1,g)
    a2 = dense(a1,W2,b2,g)
    a3 = dense(a2,W3,b3,g)
    a4 = dense(a3,W4,b4,g)
    f_x = a4
    return f_x
```

capital W refers to a matrix

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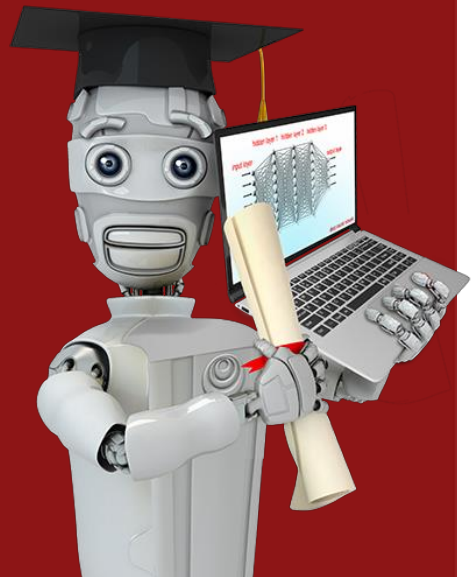
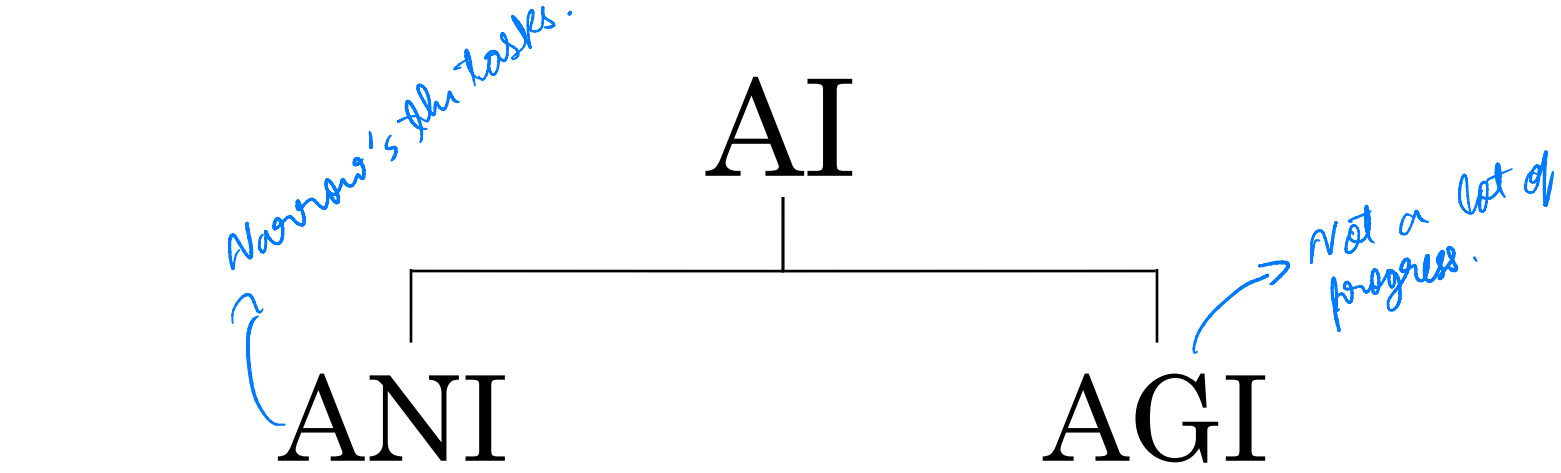


Speculations on artificial general intelligence (AGI)

Is there a path to AGI?



(artificial narrow intelligence)

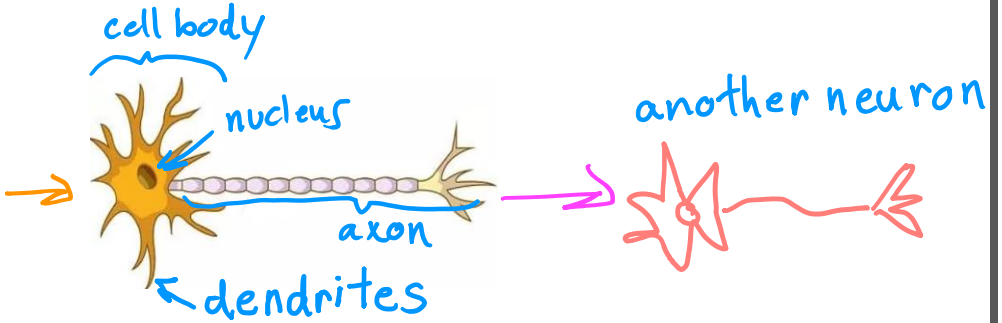
E.g., smart speaker, self-driving car, web search, AI in farming and factories

(artificial general intelligence)

Do anything a human can do

Biological neuron

inputs outputs



Simplified mathematical model of a neuron

inputs outputs

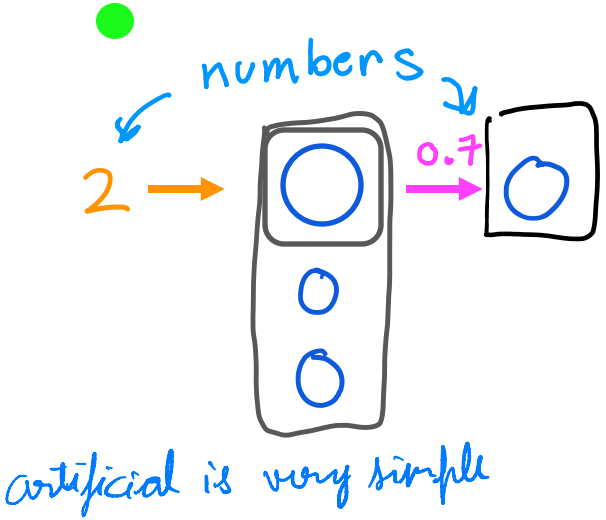
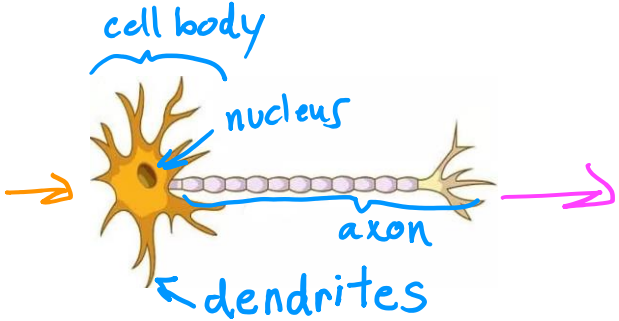


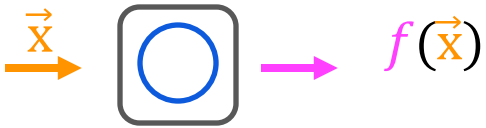
image source: https://biologydictionary.net/sensory-neuron/

Neural network and the brain

Can we mimic the human brain?

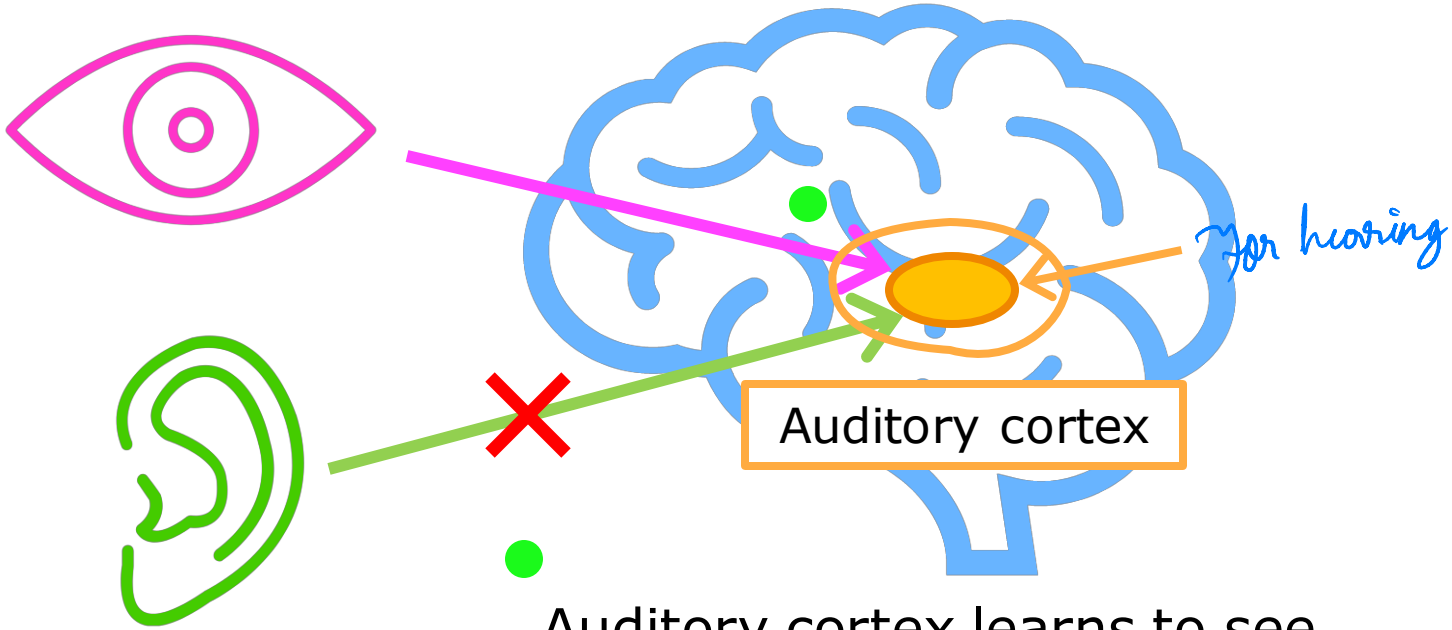


vs



We have (almost) no idea how the brain works

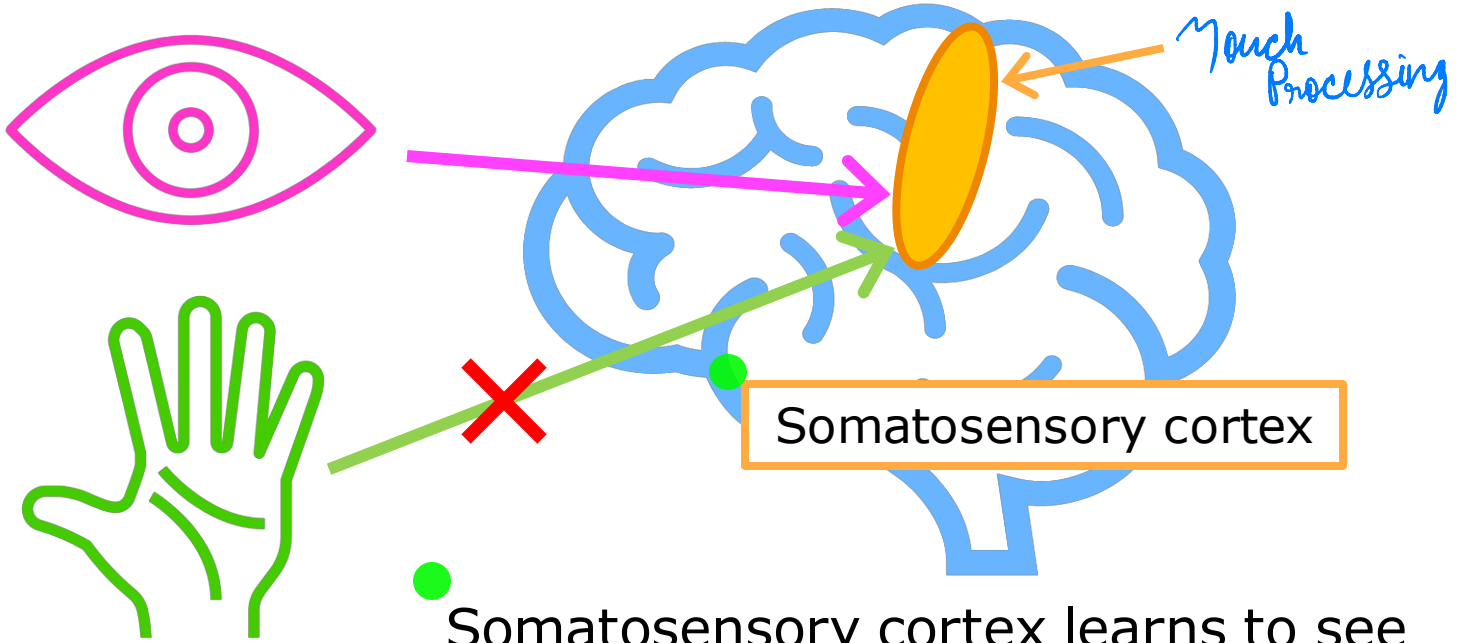
The ”one learning algorithm” hypothesis



Auditory cortex learns to see

[Roe et al., 1992]

The ”one learning algorithm” hypothesis



Somatosensory cortex learns to see

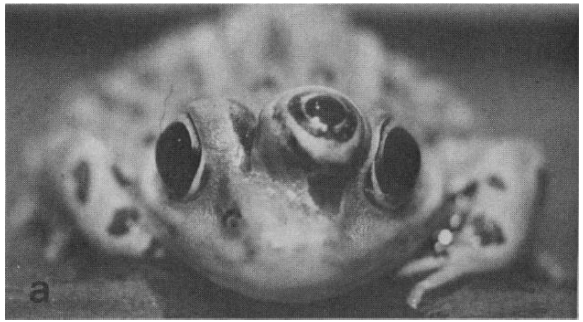
[Metin & Frost, 1989]

Sensor representations in the brain

Seeing with your tongue

Human echolocation (sonar)

Haptic belt: Direction sense



Implanting a 3rd eye

Brains learns to deal with input.

[BrainPort; Welsh & Blasch, 1997; Nagel et al., 2005; Constantine-Paton & Law, 2009]

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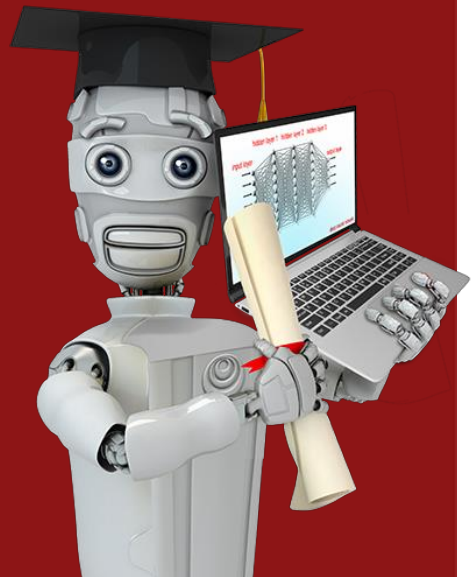

np.dot($\vec{x}$, $\vec{w}$)

Vectorization (optional)

How neural networks are implemented efficiently

For loops vs. vectorization

```
x = np.array([200, 17])

W = np.array([[1, -3, 5],
              [-2, 4, -6]])

b = np.array([-1, 1, 2])

def dense(a_in,W,b):
    a_out = np.zeros(units)
    for j in range(units):
      w = W[:,j]
      z = np.dot(w,x) + b[j]
      a[j] = g(z)
  return a
```

[1,0,1]

Vectorized

```
X = np.array([[200, 17]])
W = np.array([[1, -3, 5],
              [-2, 4, -6]])
B = np.array([[-1, 1, 2]])

def dense(A_in,W,B):
    Z = np.matmul(A_in,W) + B
    A_out = g(Z)
    return A_out
```

[[1,0,1]]

2D array

same

1x3 2D array

all 2D arrays

matrix multiplication {numpy}

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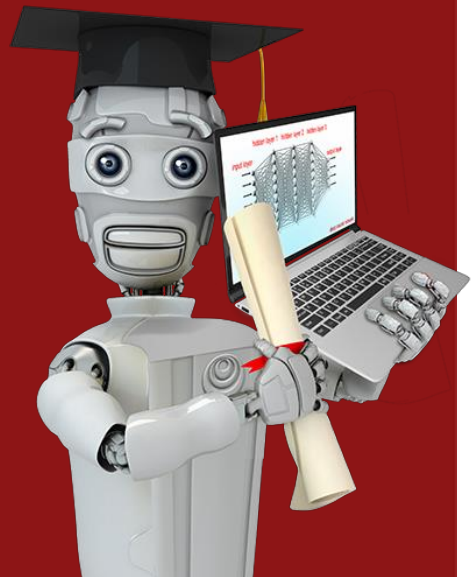


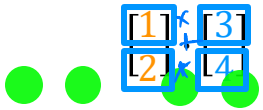
Vectorization (optional)

matmul

Matrix multiplication

Dot products

example



$z = (1 \times 3) + (2 \times 4)$

$3 + 8$

11

in general

$z = \vec{a} \cdot \vec{w}$

transpose

$\vec{a} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ $(2,1)$

$\vec{a}^T = [1 \quad 2]$ $(1,2)$

vector vector multiplication

$[\leftarrow \ \vec{a}^T \ \rightarrow] \begin{bmatrix} \uparrow \\ \vec{w} \\ \downarrow \end{bmatrix}$ 1×2 2×1

$z = \vec{a}^T \vec{w}$

equivalent

useful for understanding matrix multiplication

Vector matrix multiplication

$$\vec{a} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$\vec{a}^T = \begin{bmatrix} 1 & 2 \end{bmatrix} \qquad W = \begin{bmatrix} 3 & 5 \\ 4 & 6 \end{bmatrix}$$

$$Z = \vec{a}^T W \quad \begin{bmatrix} \leftarrow & \vec{a}^T & \rightarrow \end{bmatrix} \begin{bmatrix} \uparrow & \uparrow \\ \vec{w}_1 & \vec{w}_2 \\ \downarrow & \downarrow \end{bmatrix}$$

1 by 2

$$Z = \begin{bmatrix} \vec{a}^T \vec{w}_1 & a^T \vec{w}_2 \end{bmatrix}$$

$(1 * 3) + (2 * 4)$ $\qquad$ $(1 * 5) + (2 * 6)$

$3 + 8$ $\qquad$ $5 + 12$

11 $\qquad$ 17

$$Z = [11 \quad 17]$$

matrix matrix multiplication

$$A = \begin{bmatrix} 1 & -1 \\ 2 & -2 \end{bmatrix} \quad A^T = \begin{bmatrix} 1 & 2 \\ -1 & -2 \end{bmatrix} \quad W = \begin{bmatrix} 3 & 5 \\ 4 & 6 \end{bmatrix}$$

rows

columns

$$Z = A^T W = \begin{bmatrix} \leftarrow & \vec{a}_1^T & \rightarrow \\ \leftarrow & \vec{a}_2^T & \rightarrow \end{bmatrix} \begin{bmatrix} \uparrow & \uparrow \\ \vec{w}_1 & \vec{w}_2 \\ \downarrow & \downarrow \end{bmatrix}$$

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \quad A^T = \begin{bmatrix} a_{11} & a_{21} \\ a_{12} & a_{22} \end{bmatrix}$$

$$\begin{bmatrix} \vec{a}_1^T \vec{w}_1 & \vec{a}_1^T \vec{w}_2 \\ \vec{a}_2^T \vec{w}_1 & \vec{a}_2^T \vec{w}_2 \end{bmatrix}$$

row 1 col 1: $(-1 \times 3) + (-2 \times 4) = -3 + -8 = -11$

row 1 col 2: $(-1 \times 5) + (-2 \times 6) = -5 + -12 = -17$

$$= \begin{bmatrix} 11 & 17 \\ -11 & -17 \end{bmatrix}$$

general rules for matrix multiplication
↳ next video!

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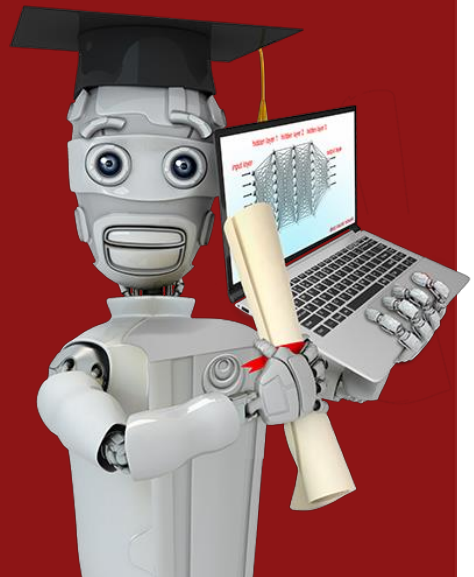

Vectorization (optional)

Matrix multiplication rules

Matrix multiplication rules

$$\mathbf{A} = \begin{bmatrix} 1 & -1 & 0.1 \\ 2 & -2 & 0.2 \end{bmatrix} \quad \mathbf{A}^T = \begin{bmatrix} 1 & 2 \\ -1 & -2 \\ 0.1 & 0.2 \end{bmatrix} \quad \mathbf{W} = \begin{bmatrix} 3 & 5 & 7 & 9 \\ 4 & 6 & 8 & 0 \end{bmatrix}$$

$\vec{a}_1^T$ $\vec{a}_2^T$ $\vec{a}_3^T$

$\vec{w}_1$ $\vec{w}_2$ $\vec{w}_3$ $\vec{w}_4$

(3, 2) (2, 4) (3, 4)

$$\mathbf{Z} = \mathbf{A}^T\mathbf{W} = \begin{bmatrix} & & & \\ & & & \\ & & & \end{bmatrix}$$

Matrix multiplication rules

$$A = \begin{bmatrix} 1 & -1 & 0.1 \\ 2 & -2 & 0.2 \end{bmatrix} \quad A^T = \begin{bmatrix} 1 & 2 \\ -1 & -2 \\ 0.1 & 0.2 \end{bmatrix} \quad W = \begin{bmatrix} 3 & 5 & 7 & 9 \\ 4 & 6 & 8 & 0 \end{bmatrix} \quad Z = A^T W = \begin{bmatrix} & & & \\ & & & \\ & & & \end{bmatrix}$$

3 by 4 matrix

$$\vec{a}_1^T \vec{w}_1 = (1 \times 3) + (2 \times 4) = 11$$

row 3 column 2

$$\vec{a}_3^T \vec{w}_2 = (0.1 \times 5) + (0.2 \times 6) = 1.7$$

0.5 + 1.2

row 2 column 3?

$$\vec{a}_2^T \vec{w}_3 = (-1 \times 7) + (-2 + 8) = -23$$

-7 + -16

Matrix multiplication rules

$$A = \begin{bmatrix} 1 & -1 & 0.1 \\ 2 & -2 & 0.2 \end{bmatrix} \quad A^T = \begin{bmatrix} 1 & 2 \\ -1 & -2 \\ 0.1 & 0.2 \end{bmatrix} \quad W = \begin{bmatrix} 3 & 5 & 7 & 9 \\ 4 & 6 & 8 & 0 \end{bmatrix} \quad Z = A^T W = \begin{bmatrix} 11 & 17 & 23 & 9 \\ -11 & -17 & -23 & -9 \\ 1.1 & 1.7 & 2.3 & 0.9 \end{bmatrix}$$

3 by 4 matrix

$\vec{a}_1^T \vec{w}_1 = (1 \times 3) + (2 \times 4) = 11$

row 3 column 2

$\vec{a}_3^T \vec{w}_2 = (0.1 \times 5) + (0.2 \times 6) = 1.7$

0.5 + 1.2

row 2 column 3?

$\vec{a}_2^T \vec{w}_3 = (-1 \times 7) + (-2 + 8) = -23$

-7 + -16

Matrix multiplication rules

$$\mathbf{A} = \begin{bmatrix} 1 & -1 & 0.1 \\ 2 & -2 & 0.2 \end{bmatrix} \quad \mathbf{A}^T = \begin{bmatrix} 1 & 2 \\ -1 & -2 \\ 0.1 & 0.2 \end{bmatrix} \quad \mathbf{W} = \begin{bmatrix} 3 & 5 & 7 & 9 \\ 4 & 6 & 8 & 0 \end{bmatrix} \quad \mathbf{Z} = \mathbf{A}^T\mathbf{W} = \begin{bmatrix} 11 & 17 & 23 & 9 \\ -11 & -17 & -23 & -9 \\ 1.1 & 1.7 & 2.3 & 0.9 \end{bmatrix}$$

3 x 2 2 x 4

can only take dot products of vectors that are same length

[0.1 0.2] length 2

$\begin{bmatrix} 5 \\ 6 \end{bmatrix}$ length 2

3 by 4 matrix

↳ same # rows as A^T

same # columns as W

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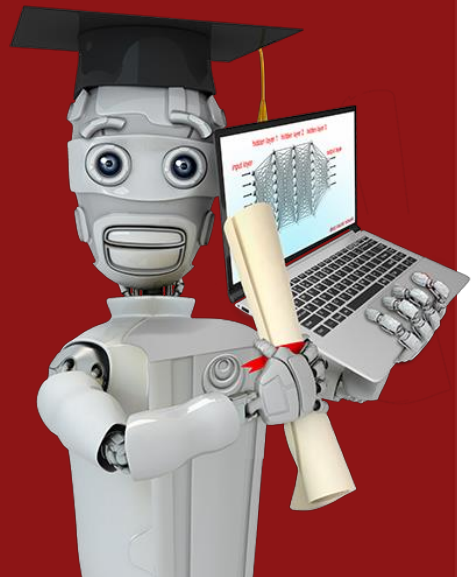


Vectorization (optional)

Matrix multiplication code

Matrix multiplication in NumPy

$$A = \begin{bmatrix} 1 & -1 & 0.1 \\ 2 & -2 & 0.2 \end{bmatrix} \quad A^T = \begin{bmatrix} 1 & 2 \\ -1 & -2 \\ 0.1 & 0.2 \end{bmatrix} \quad W = \begin{bmatrix} 3 & 5 & 7 & 9 \\ 4 & 6 & 8 & 0 \end{bmatrix} \quad Z = A^T W = \begin{bmatrix} 11 & 17 & 23 & 9 \\ -11 & -17 & -23 & -9 \\ 1.1 & 1.7 & 2.3 & 0.9 \end{bmatrix}$$

```
A=np.array([1,-1,0.1],
           [2,-2,0.2]])
```

```
W=np.array([3,5,7,9],
           [4,6,8,0])
```

```
Z = np.matmul(AT,W)
```

or

```
Z = AT @ W
```

```
AT=np.array([1,2],
            [-1,-2],
            [0.1,0.2])
```

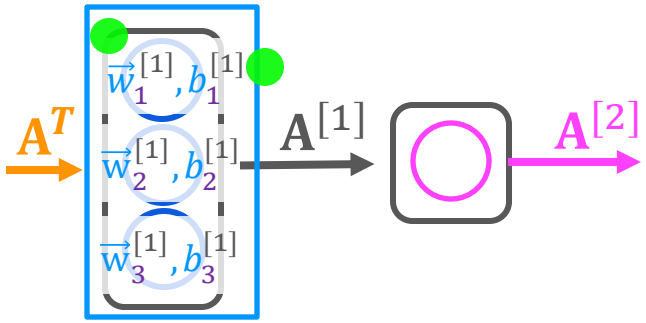
```
AT=A.T
```

transpose

result

```
[[11,17,23,9],
 [-11,-17,-23,-9],
 [1.1,1.7,2.3,0.9]
]
```

Dense layer vectorized



$$\mathbf{A}^T = [200 \quad 17] \qquad 1 \times 2$$

$$\mathbf{W} = \begin{bmatrix} 1 & -3 & 5 \\ -2 & 4 & -6 \end{bmatrix} \qquad 2 \times 3$$

$$\vec{\mathbf{b}} = [-1 \quad 1 \quad 2] \qquad 1 \times 3$$

$$\mathbf{Z} = \mathbf{A}^T\mathbf{W} + \mathbf{B}$$

$$[165 \quad -531 \quad 900]$$

$$z_1^{[1]} \quad z_2^{[1]} \quad z_3^{[1]}$$

$$\mathbf{A} = g(\mathbf{Z})$$

$$[1 \quad 0 \quad 1]$$

A

```
AT = np.array([[200, 17]])
W = np.array([[1, -3, 5],
              [-2, 4, -6]])
b = np.array([[-1, 1, 2]])
```

a_in

```
def dense(AT,W,b,g):
  z = np.matmul(AT,W) + b
  a_out = g(z)
  return a_out
```

a_in

```
[[1,0,1]]
```