Advanced Model Predictive Control — Fall 2022
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Exercise

Programming Exercise 1

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1 Exercise

Nominal Nonlinear MPC

1. **(graded)** Consider the nominal nonlinear MPC problem

$$\min_{x,u} \quad l_f(x_N) + \sum_{i=0}^{N-1} l(x_i, u_i) \tag{1a}$$
$$\text{s.t.} \quad \forall i = 0, \dots, N-1, \tag{1b}$$
$$x_{i+1} = f(x_i, u_i), \tag{1c}$$
$$x_i \in \mathcal{X}, \ u_i \in \mathcal{U}, \tag{1d}$$
$$x_N \in \mathcal{X}_f, \ x_0 = x(k). \tag{1e}$$

Implement (1) in the provided `Nonlinear_MPC.m` file, using the following choices of cost function, nonlinear segway dynamics, constraints, and terminal ingredients:

$$\begin{aligned}
l(x,u) &= x^\top Q\, x + u^\top R\, u, \\
f(x,u) &= \begin{bmatrix} x_1 + \delta t \cdot x_2 \\ x_2 + \delta t(-kx_1 - cx_2 + {}^{g}\!/\!_{l} \cdot \sin x_1 + u) \end{bmatrix}, \\
\mathcal{X} &= \{x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \mid A_x x \le b_x\}, \\
\mathcal{U} &= \{u \mid A_u u \le b_u\}, \\
l_f(x) &= 0, \\
\mathcal{X}_f &= \left\{ \begin{bmatrix} 0 \\ 0 \end{bmatrix} \right\}.
\end{aligned}$$

Use the following choices for the parameters

$$Q = \begin{bmatrix} 100 & 0 \\ 0 & 100 \end{bmatrix}, \qquad R = 10, \qquad N = 10,$$
$$k = 4, \qquad c = 1.5, \qquad l = 1.3,$$
$$\begin{bmatrix} -45^\circ \\ -60^\circ \end{bmatrix} \le x \le \begin{bmatrix} 45^\circ \\ 60^\circ \end{bmatrix} \qquad -5 \le u \le 5$$

Hint: The control parameters, e.g. Q and R, are loaded by the `Controller` class (super class) constructor. Therefore, you can access them with `obj.params.Q`. Additionally, the system object is directly passed to the constructor of the `Nonlinear_MPC` class. This means you can access system properties, like e.g. the state constraints, directly through the `sys` object, i.e., `sys.X`.

2. **(optional; not graded)** Consider now the same nonlinear segway system but with additive disturbances.

   a. Run the cell labelled "Exercise 2a" in `main.m` and observe how the initial state and the disturbance affect the feasibilty of the closed-loop trajectories.

   b. Run the cell labelled "Exercise 2b" in `main.m` with different choices of initial states and disturbance sizes. Observe how these two parameters affect the closed-loop trajectories and the cost decrease.