Advanced Model Predictive Control — Fall 2022
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Programming Exercise 2

Linear Robust MPC

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1 Exercise

Linear Robust MPC

Implementation of linear robust MPC in the `RMPC.m` file.

a. **(Graded)** Consider the optimization problem

$$\min_{E,Y,c_{x,j}^2,c_{u,j}^2,\bar{w}^2} \quad \frac{1}{2(1-\rho)}\left((n_x+n_u)\bar{w}^2+\sum_{j=1}^{n_x}c_{x,j}^2+\sum_{j=1}^{n_u}c_{u,j}^2\right) \tag{1a}$$

$$\text{s.t.} \quad E \succeq I, \tag{1b}$$

$$\begin{bmatrix}\rho^2 E & (AE+BY)^\top \\ AE+BY & E\end{bmatrix} \succeq 0, \tag{1c}$$

$$\begin{bmatrix}c_{x,j}^2 & [A_x]_j E \\ E^\top [A_x]_j^\top & E\end{bmatrix} \succeq 0,\ j\in[1,n_x], \tag{1d}$$

$$\begin{bmatrix}c_{u,j}^2 & [A_u]_j Y \\ Y^\top [A_u]_j^\top & E\end{bmatrix} \succeq 0,\ j\in[1,n_u], \tag{1e}$$

$$\begin{bmatrix}\bar{w}^2 & v_w^\top \\ v_w & E\end{bmatrix} \succeq 0,\ \forall v_w \in \mathcal{V}(\mathcal{W}). \tag{1f}$$

Implement (1) in the `compute_tightening` method in the `RMPC.m` file and compute the sublevel δ such that $\mathcal{E}=\{e \mid \|e\|_P \le \delta\}$ is RPI and the corresponding state and input constraint tightenings.

b. **(Graded)** Compute the constraint tightenings for different choices of ρ and observe how the tightenings and the RPI set $\mathcal{E}$ change. Fix ρ for the remainder of the exercise.

c. **(Graded)** Consider the robust MPC problem

$$\min_{V,z_0} \quad \sum_{i=0}^{N-1} z_i^T Q z_i + v_i^T R v_i \tag{2a}$$

$$\text{s.t.} \quad \forall i = 0,\dots,N-1, \tag{2b}$$

$$z_{i+1} = Az_i + Bv_i, \tag{2c}$$

$$[A_x]_j z_i \le [b_x]_j - \tilde{b}_{x,j}, \quad j\in[1,n_x], \tag{2d}$$

$$[A_u]_j v_i \le [b_u]_j - \tilde{b}_{u,j}, \quad j\in[1,n_u], \tag{2e}$$

$$z_N = \begin{bmatrix}0\\0\end{bmatrix}, \tag{2f}$$

$$\|x(k)-z_0\|_P^2 \le \delta^2, \tag{2g}$$

Implement (2) in the provided `RMPC.m` file.

Note: The control parameters, e.g. Q and R, are loaded by the `Controller` class (super class) constructor. Therefore, you can access them with `obj.params.Q`. Additionally, the system object is directly passed to the constructor of the `RMPC` class. This means you can access system properties, like e.g. the state constraints, directly through the `sys` object, i.e., `sys.X`.