

COMP5048

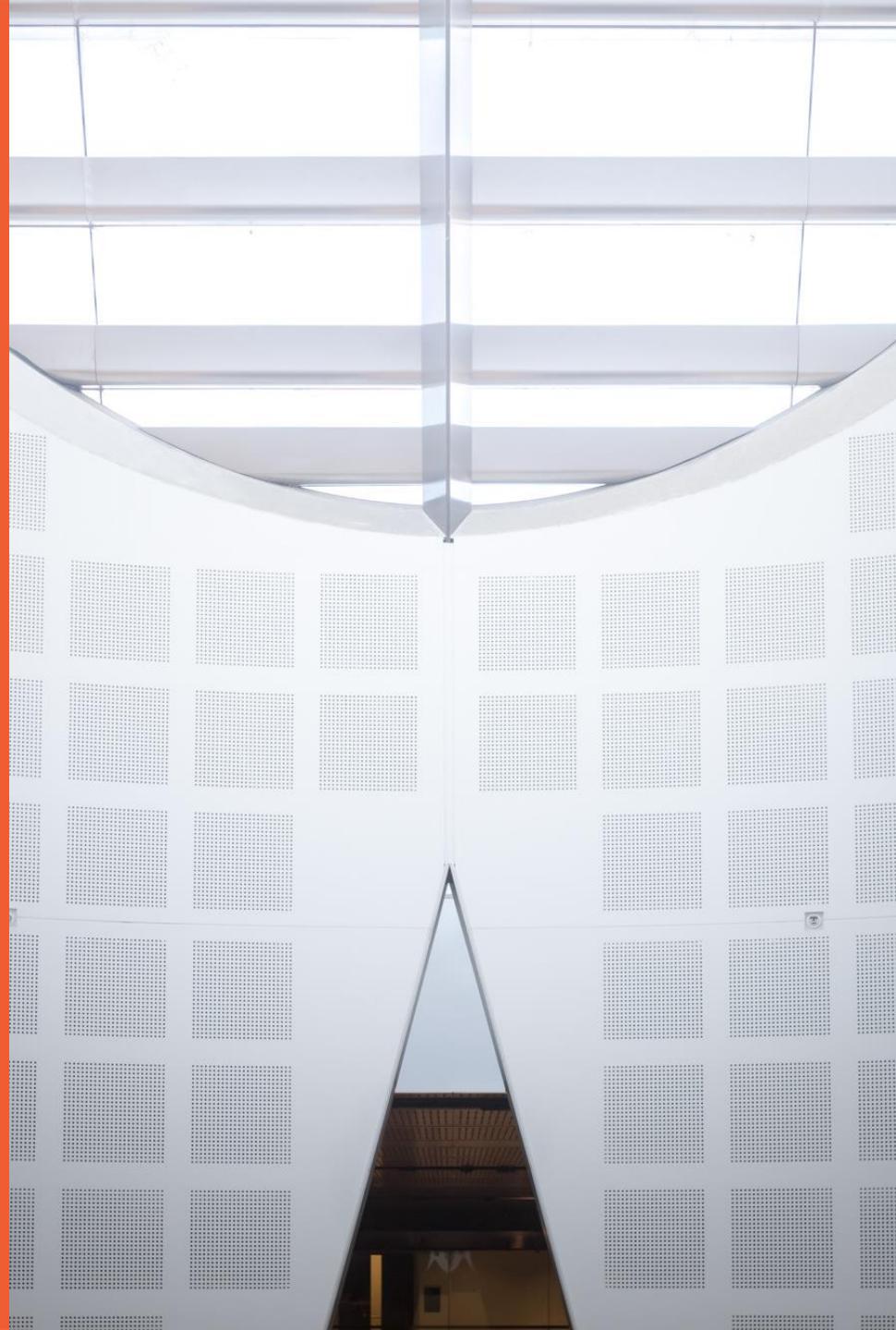
Visual Analytics

Week 2: Complex Data Visualisation

Professor Seokhee Hong
School of Information Technologies



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Data Types

Table data:

Multi-dimensional/Muti-variate/Multi-attribute Data:

Types of Attributes:

-> Nominal value, categorical value, text, image, time stamps, spatial information etc

eg. Student data =(Degree, Year, Academic records, International/local, Address, Phone number, email)

Graph data:

$G=(V, E)$, V: vertex (node) set , E: edge set

Social network, Twitter, facebook, linked-in etc

Complex Data Visualisation

1. Multi-dimensional/Multi-variate Data

- Multiple attributes: ordinal, nominal, categorical, image etc
- (eg) matrix with many columns/rows

2. Spatial Data

- Data with geometry (map, longitude/latitude)

3. Temporal/Dynamic Data

- Data with time stamps: changing over time

4. Relational Data with Constraints (Week 2-4)

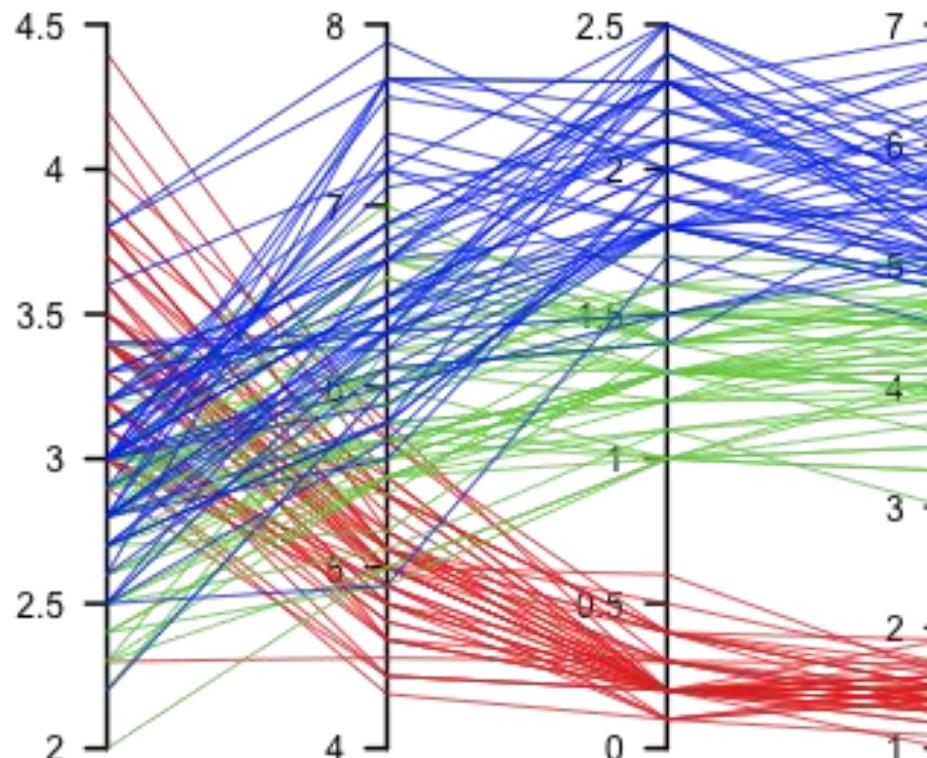
- Relations, Hierarchy, Clusters, Directions
- Tree (Hierarchical Relational Data): Week 2

(1) Multi-dimensional/Multi-variate Data Visualisation

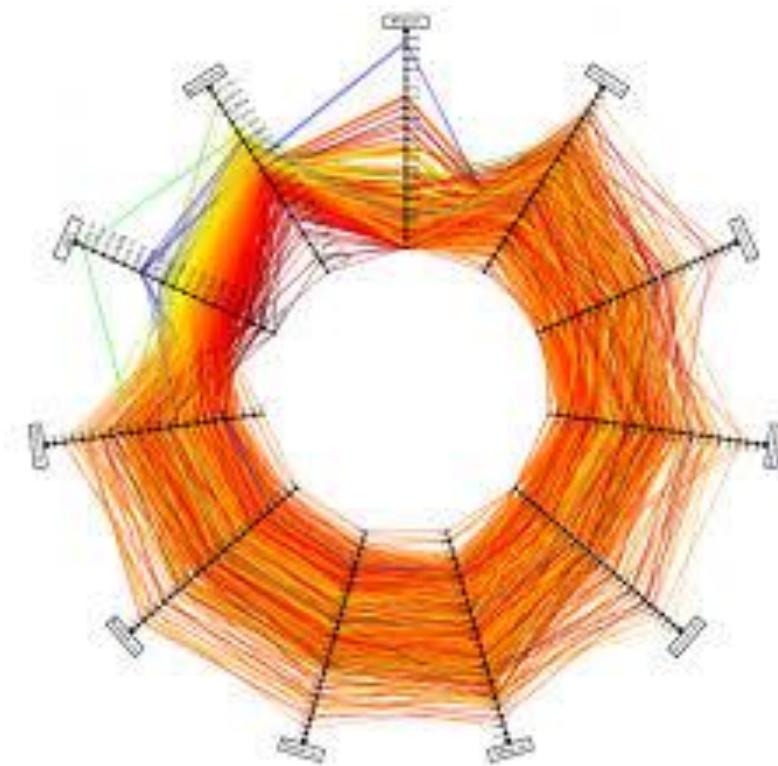
- 1. Parallel Coordinates**
- 2. MDS (Multi-Dimensional Scaling)**
- 3. Glyph (Visual variables)**

1. Parallel Coordinates [Inselburg]

Parallel coordinate plot, Fisher's Iris data

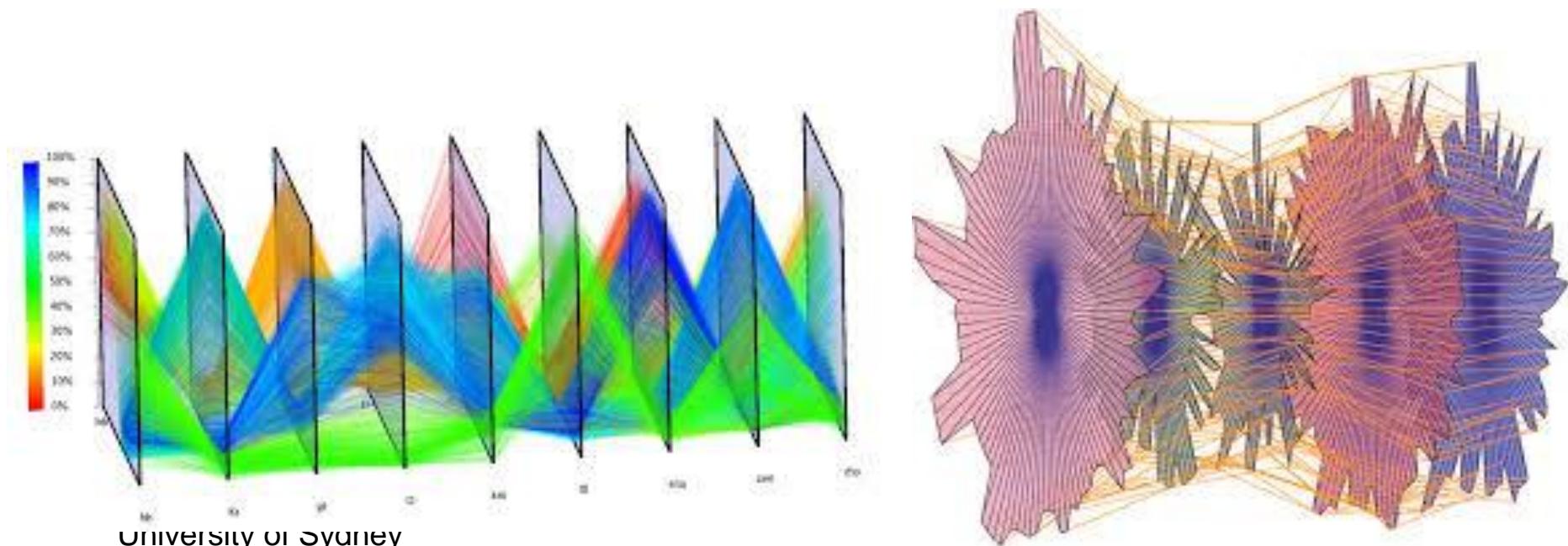
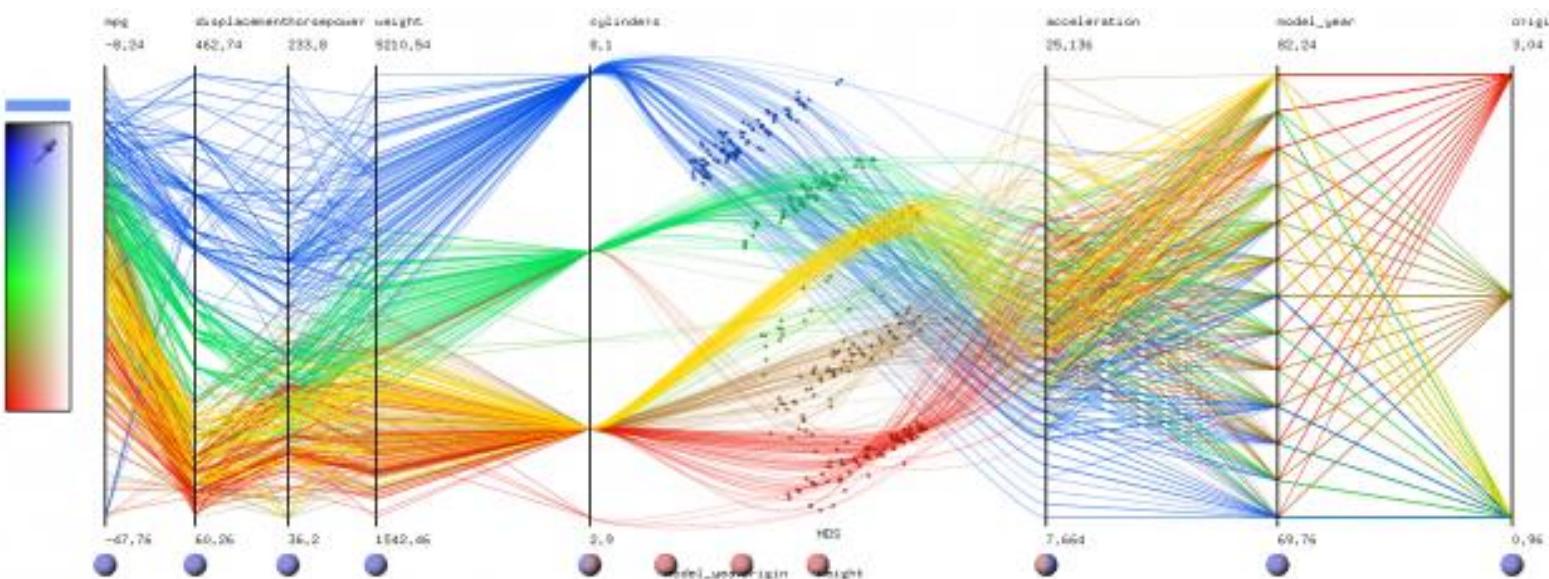


Sepal Width Sepal Length Petal Width
— setosa — versicolor — virginica



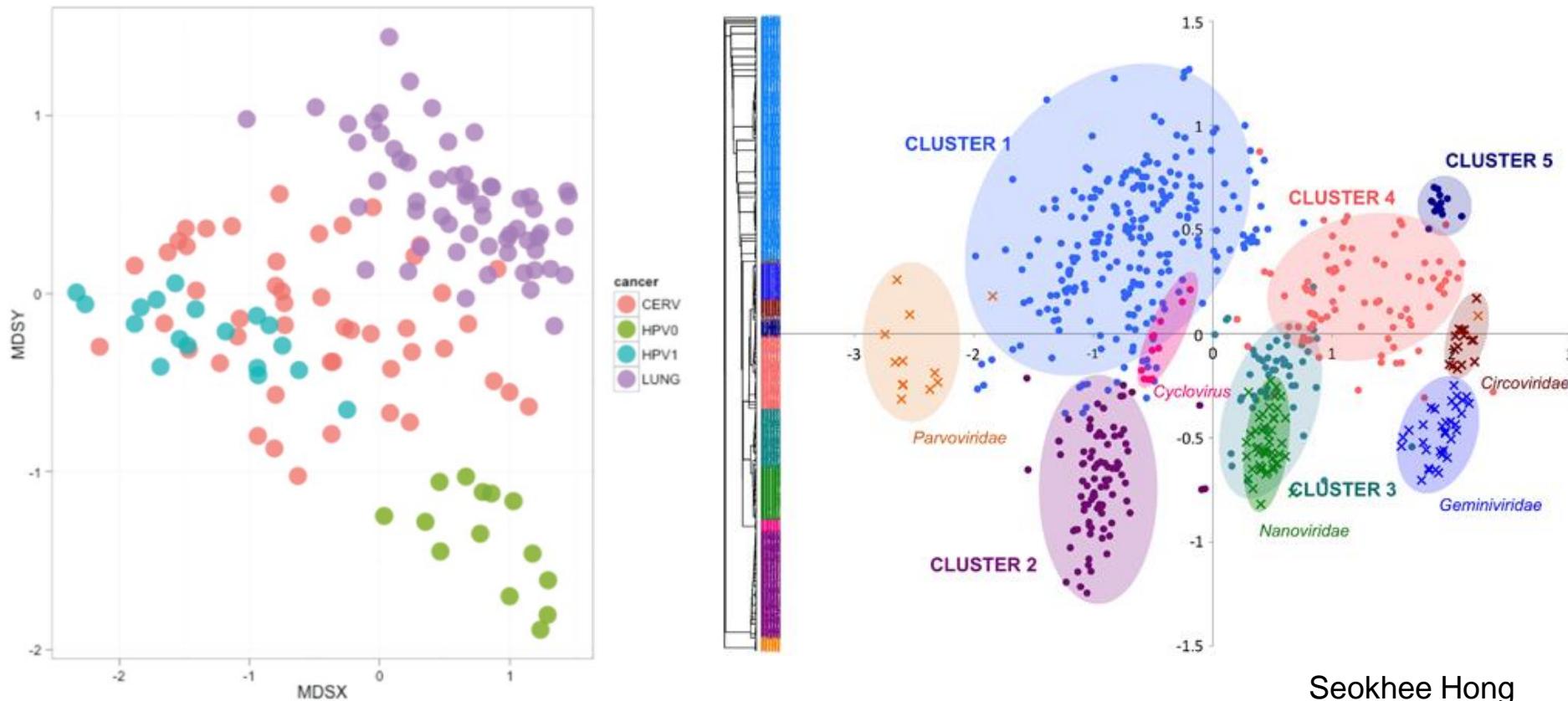
- Nominal values
- Change ordering to see patterns

Variations



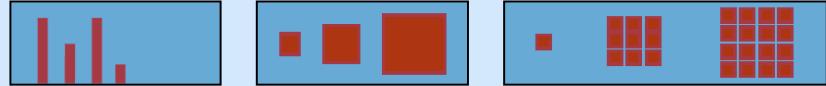
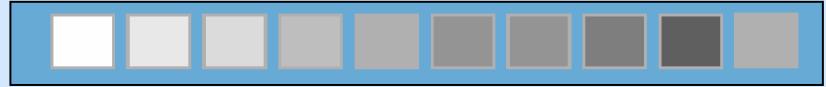
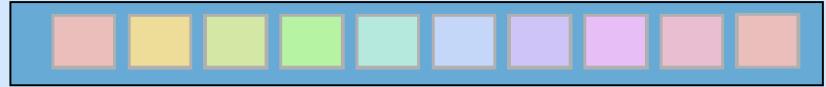
2. MDS (Multi-Dimensional Scaling)

- Compute similarity between the data using distance matrix (n dimension)
- (e.g) PCA (Principal Component Analysis)
 - Eigenspace decomposition using largest eigenvalues
 - Project into 2 or 3 dimensions
- Visualisation: 2D scatter plot



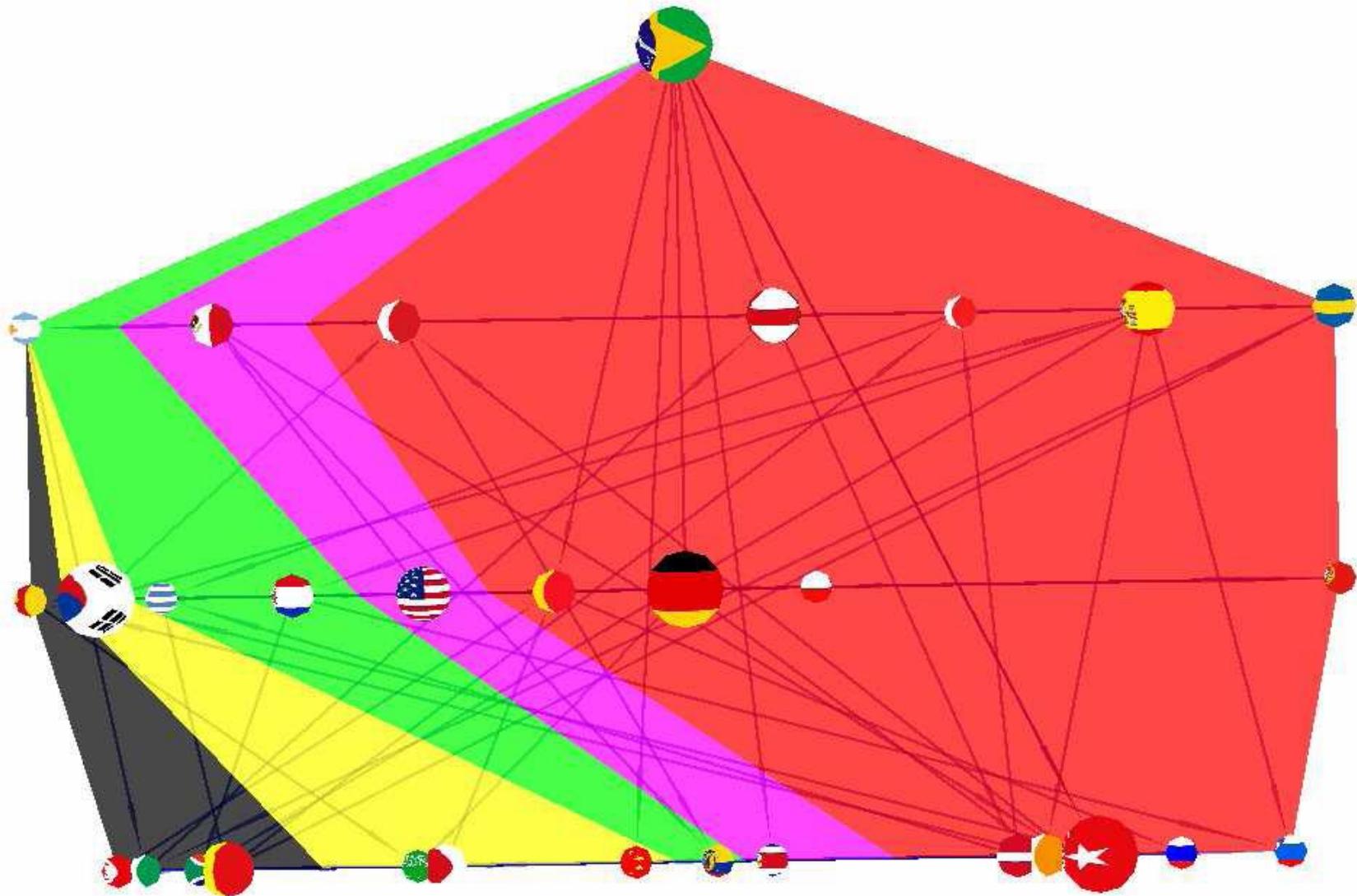
3. Glyph

- ✓ **Data-Ink-Ratio [Tufte]**: minimise ink to represent information
- ✓ **Marks (signs)**: line, point, area
- ✓ **Visual Variables** [Bertin]

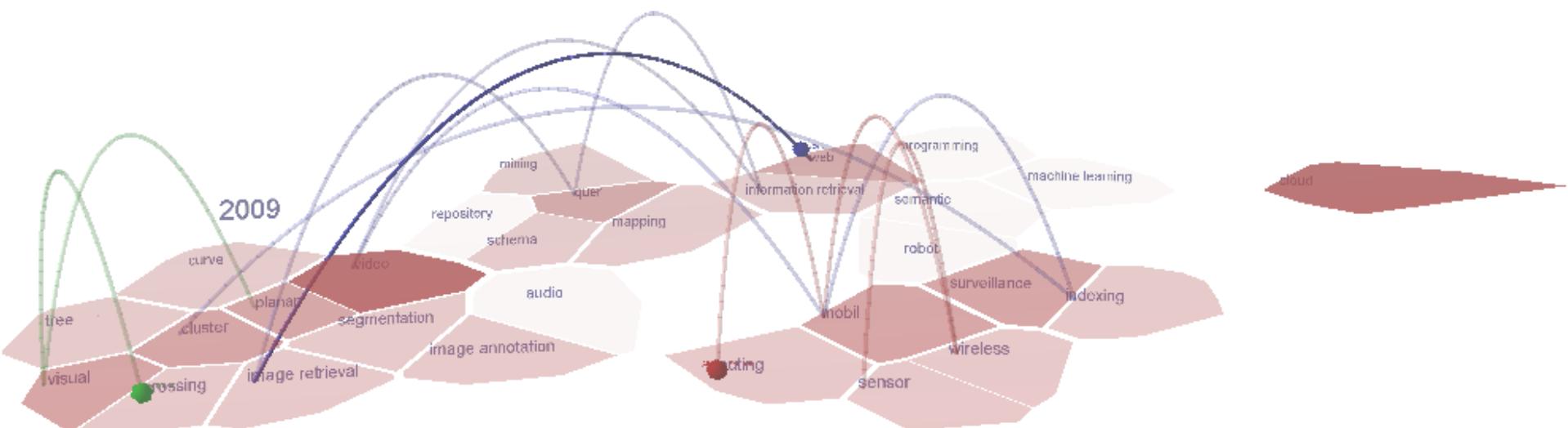
- **position**
 - changes in the x, y, (z) location
- **size**
 - change in length, area or repetition
- **shape**
 - infinite number of shapes
- **value**
 - changes from light to dark
- **orientation**
 - changes in alignment
- **colour**
 - changes in hue at a given value
- **texture**
 - variation in pattern
- **motion**

History of World Cup

2002



Comparison of Trajectories



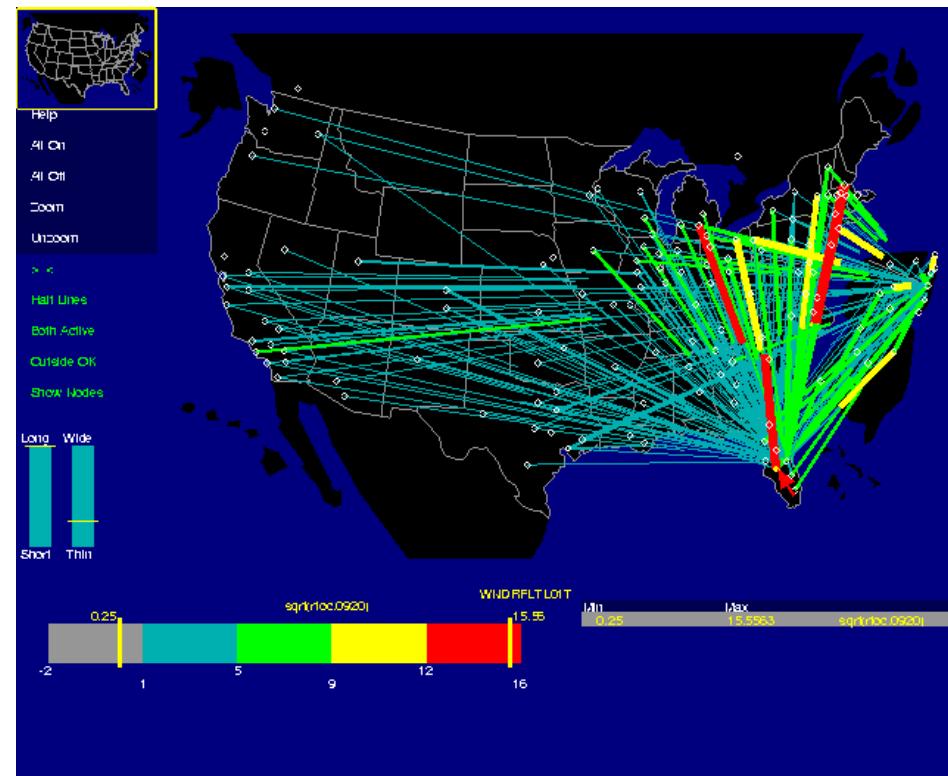
Glyph: research area

George G. Robertson

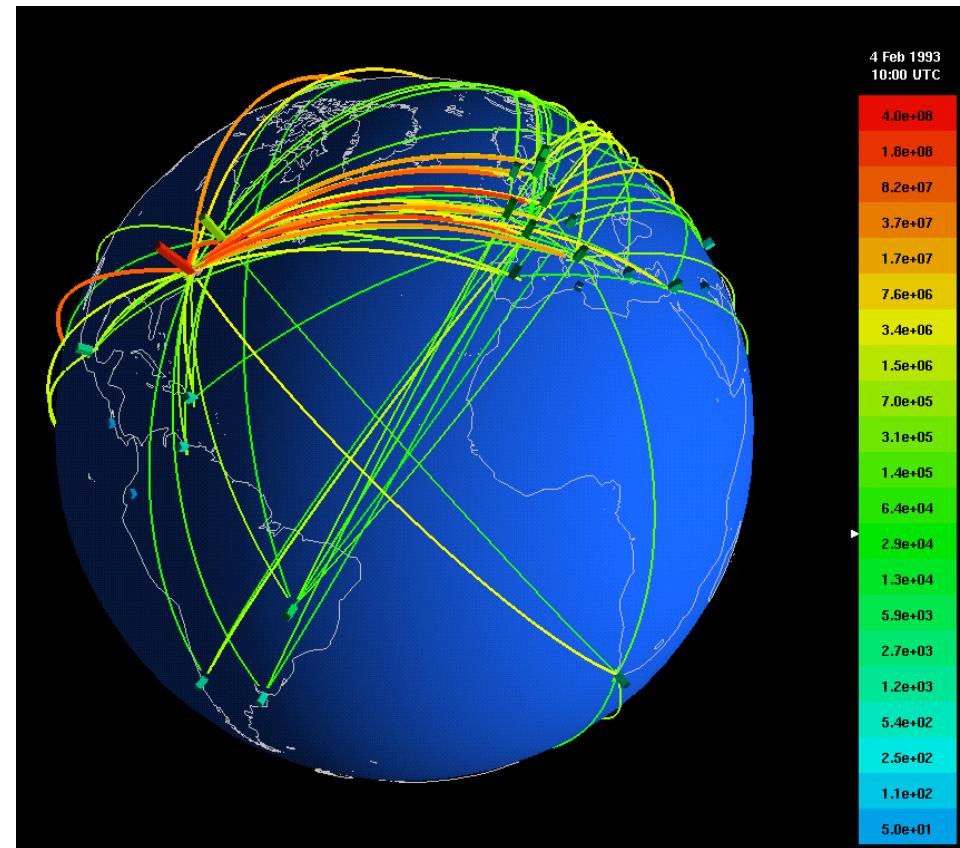
(2) Spatial Data Visualisation

**Geo Visualisation (map)
Trajectory visualisation**

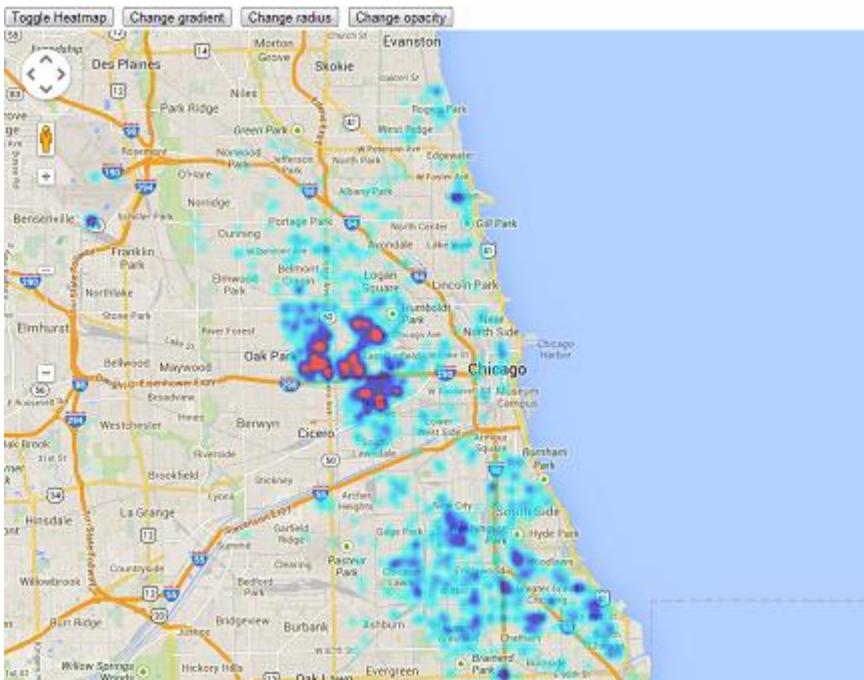
Geographic/Spatial Data



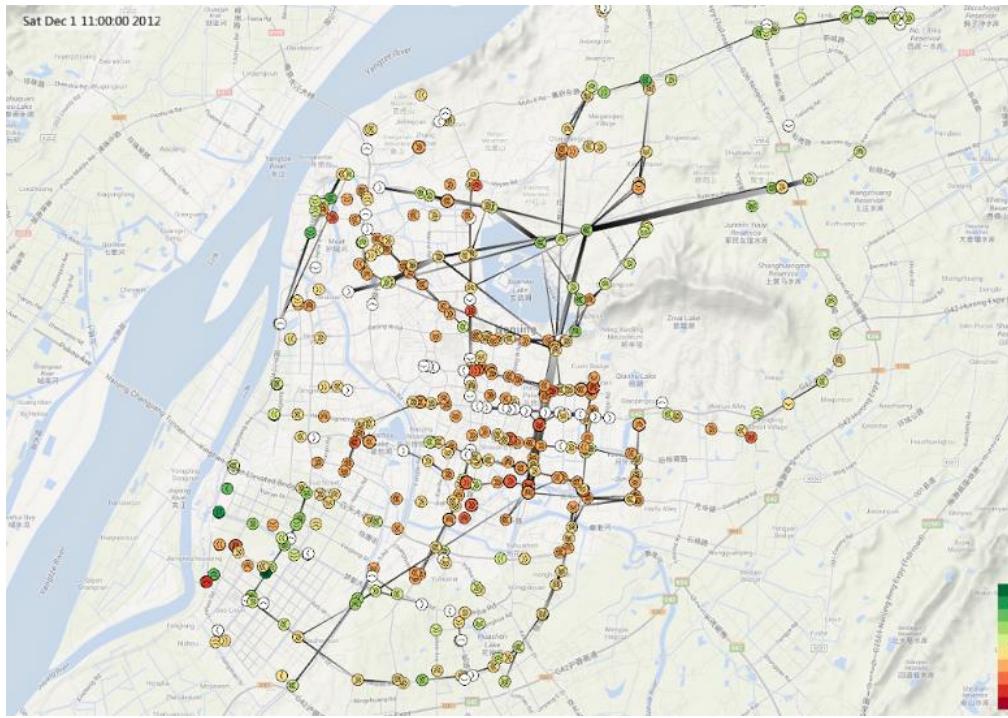
- [Becker, Eick, Wilks 95]
- SeeNet



[Cox, Eick 95] 3D Displays of
Network Traffic
SeeNet3D



Crime Data Visualisation



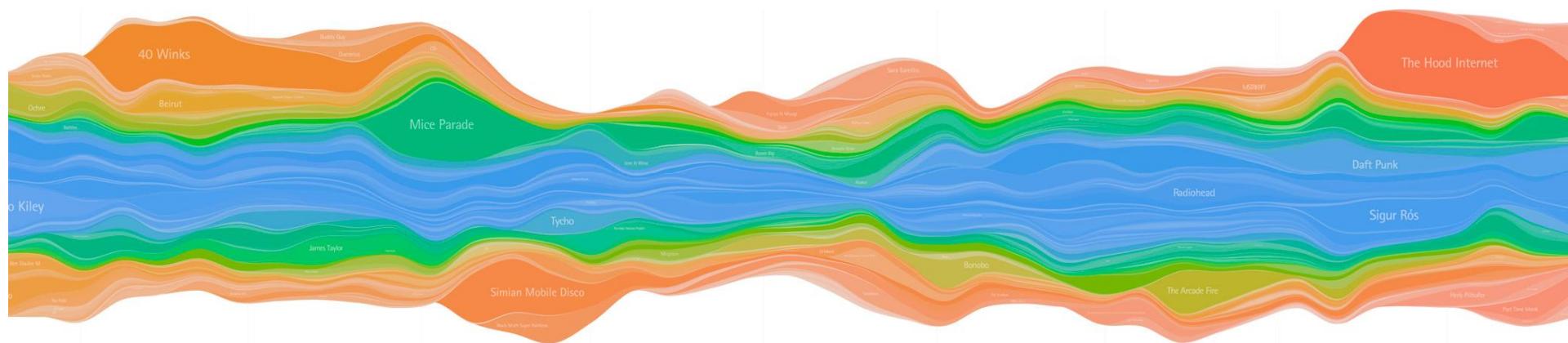
Traffic analysis using Trajectory

Seokhee Hong

(3) Temporal/Dynamic Data Visualisation

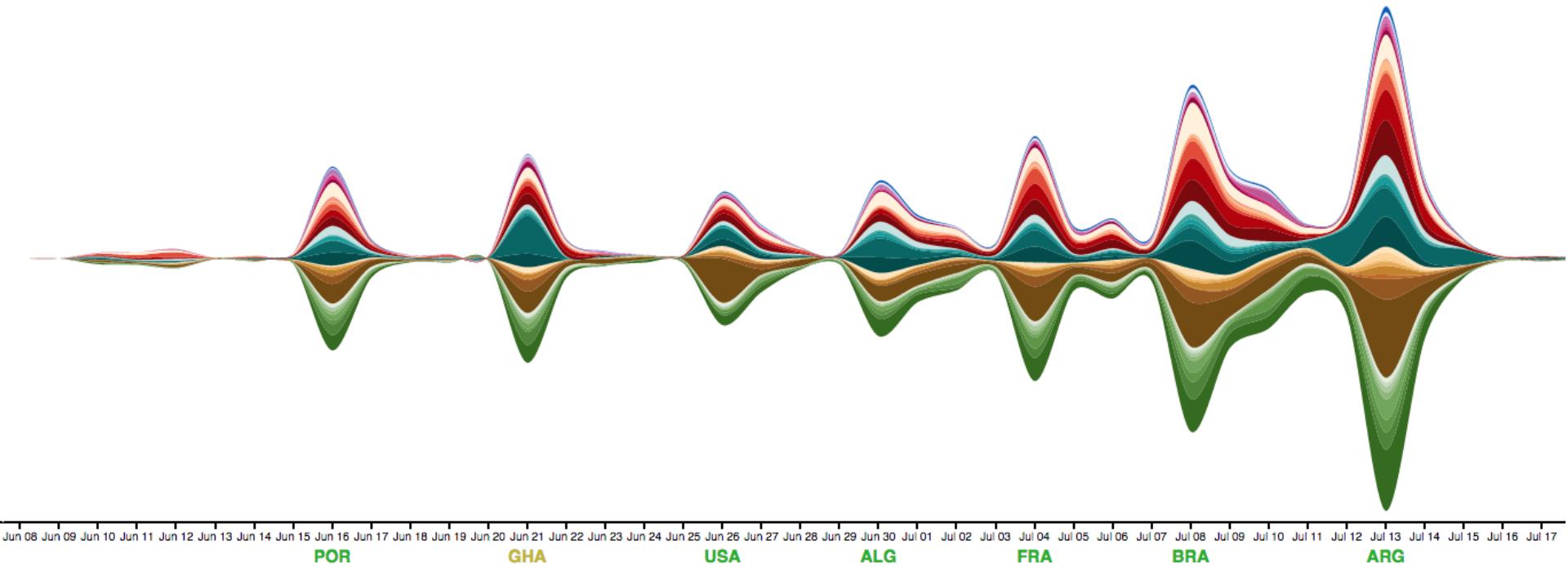
- 1. Stream Graph**
- 2. Theme River**
- 3. Storyline**

1. Stream Graph



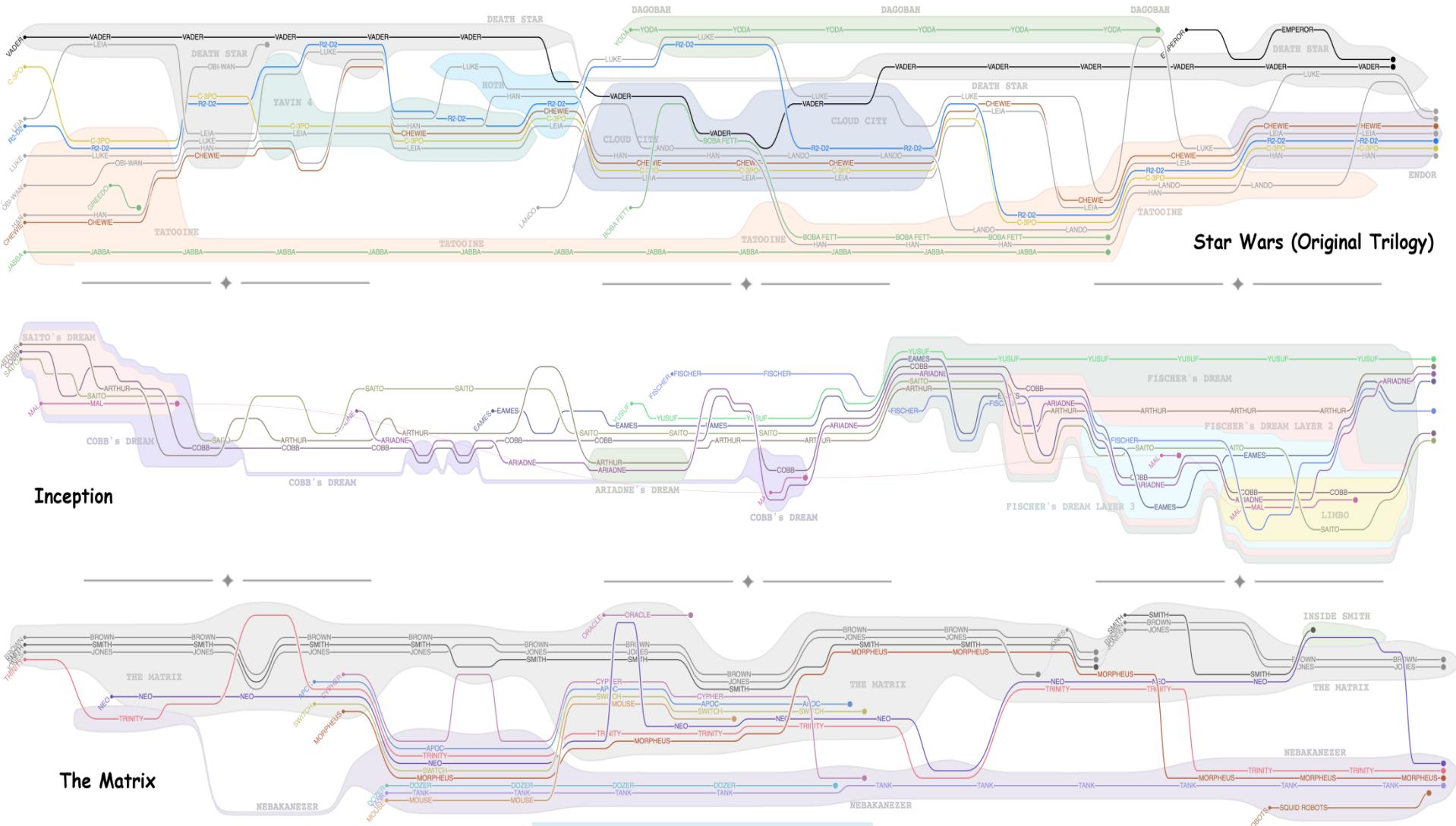
Shows overview
Trend analysis

2. Theme River



Shows overview
Trend analysis

3. Storyline



Shows overview
Dynamic clustering

(4) Relational Data Visualisation:

Graph Drawing (Network Visualisation)

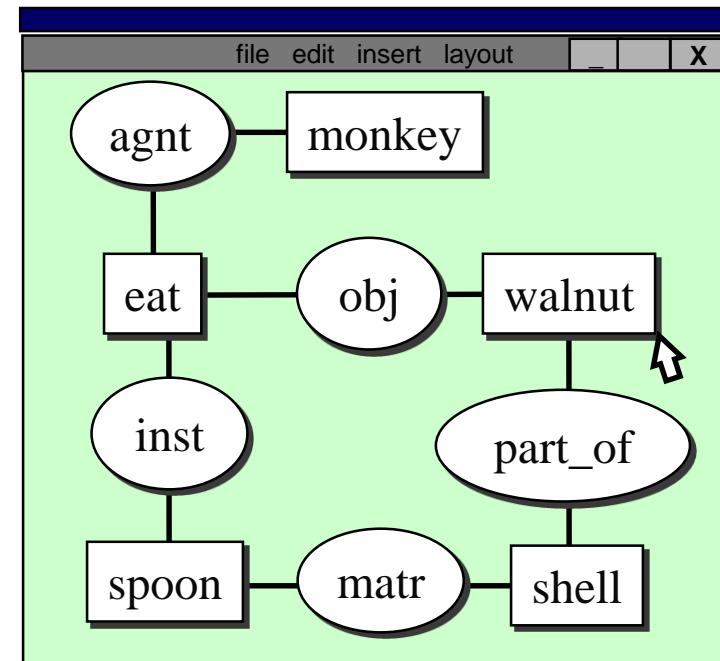
(4A) Graph Drawing

The input graph is usually some relational description of a software system.

```
agt(monkey, eat).  
inst(eat, spoon).  
obj(eat, walnut).  
part_of(walnut, shell).  
matr(spoon, shell).
```



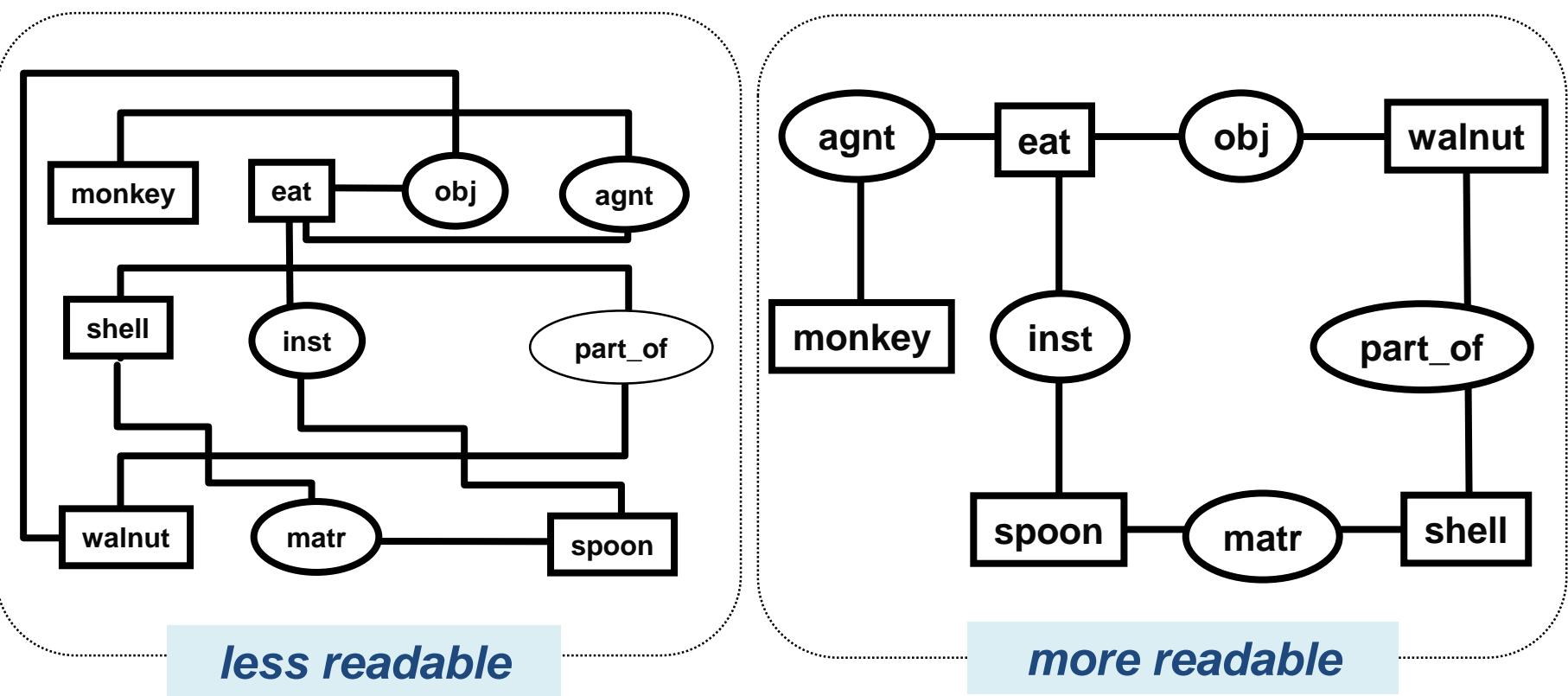
The output picture is used in a system design/analysis tool.



The graph drawing problem is to design methods to give good drawings of graphs.

Aesthetics

readability: the drawing should be easy to read, easy to understand, easy to remember, beautiful.

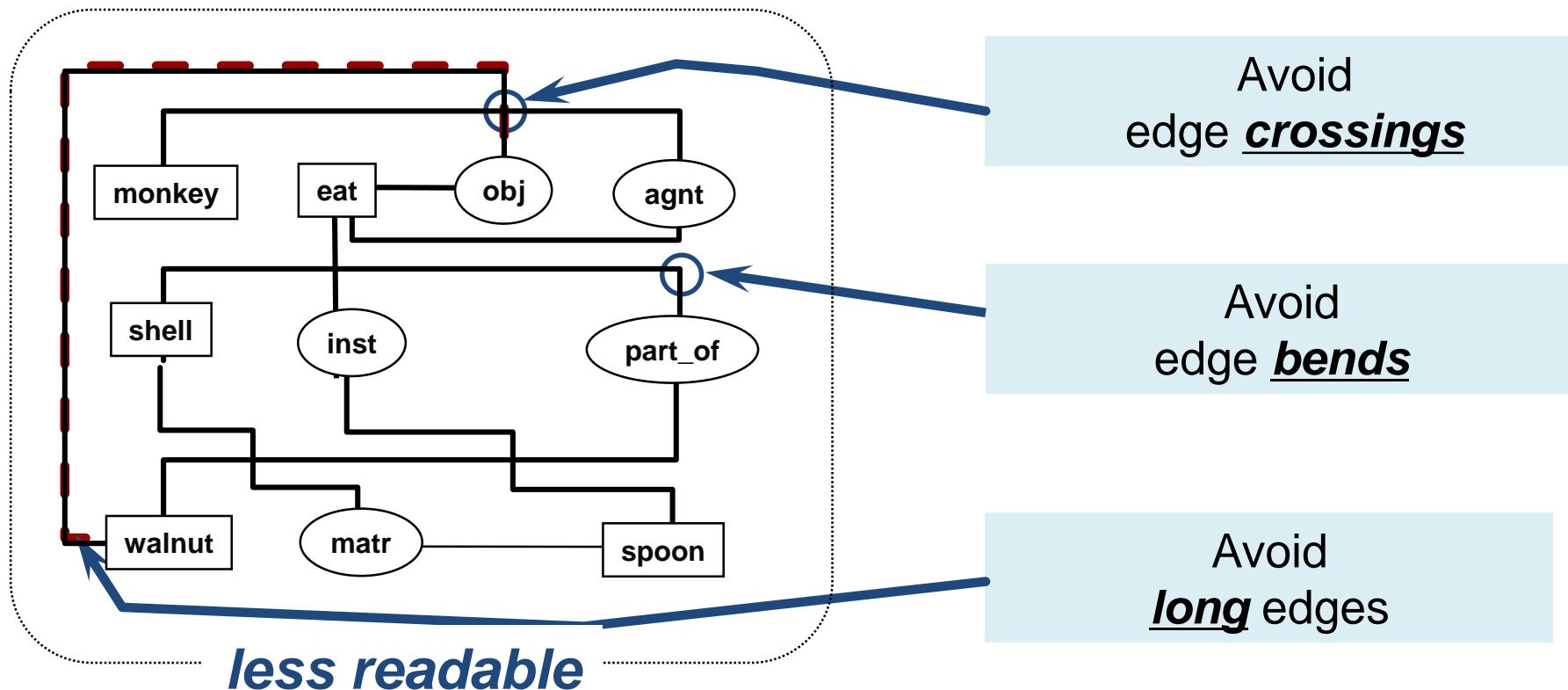


Aesthetics (quality metrics)

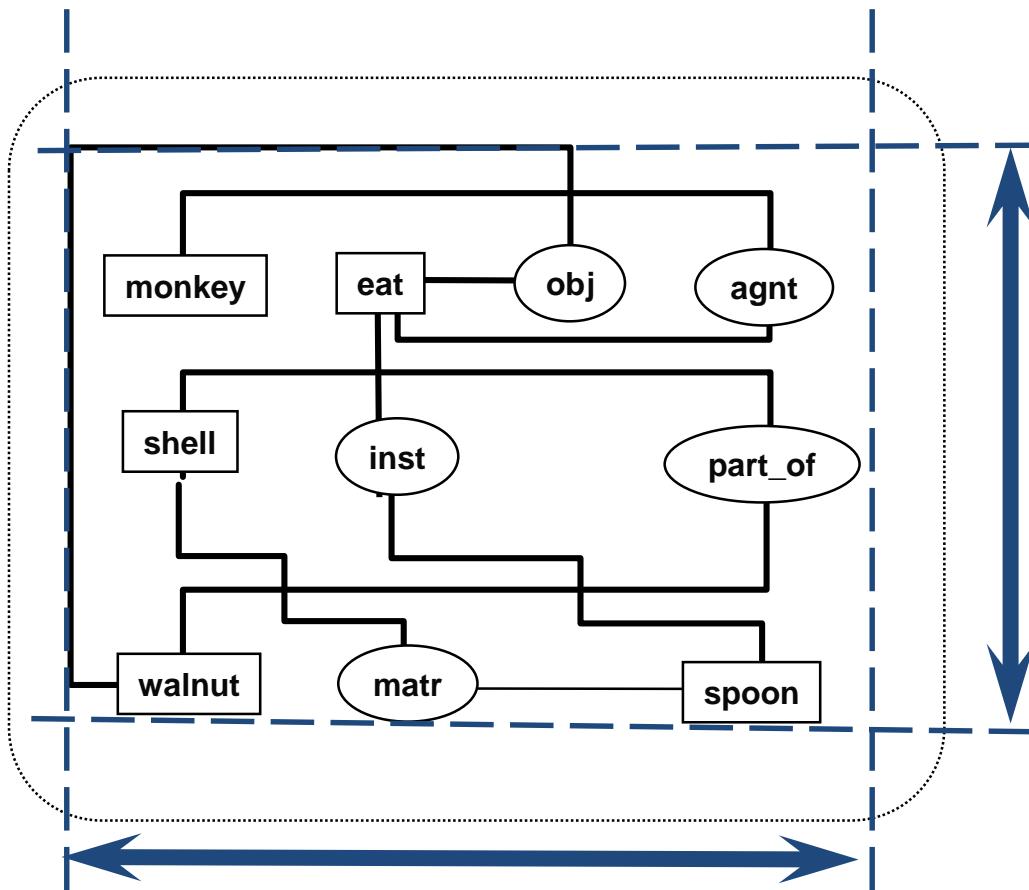
Readability is sometimes measured by aesthetic criteria

- crossings
- area
- symmetry
- edge length
 - total edge length, maximum edge length, uniform edge length
- bends
 - total bends, maximum bends, uniform bends
- angular resolution
- aspect ratio

Crossings and bends



Area and resolution



One should spread the nodes evenly over the page. This can be measured:

- minimise **area** (for fixed size nodes)

or equivalently

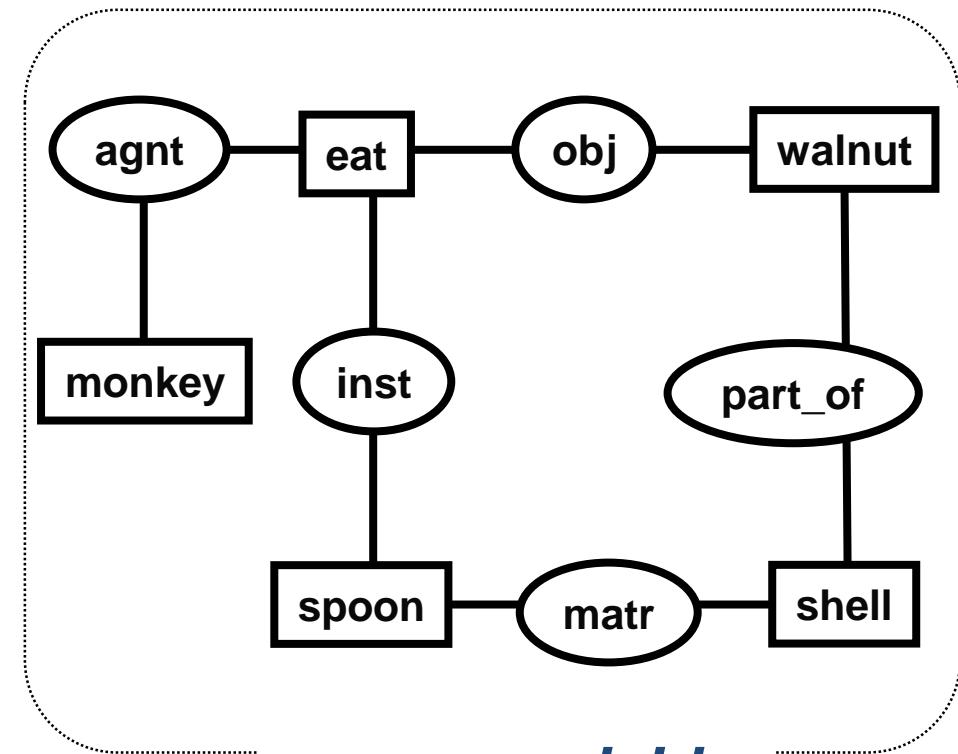
- maximise **resolution** (for a fixed size screen).

Aesthetics (Quality Metrics)

There are many aesthetic criteria for good diagrams:

- minimum **edge crossings**,
- minimum **bends**,
- minimum **edge lengths**,
- maximum **resolution**,

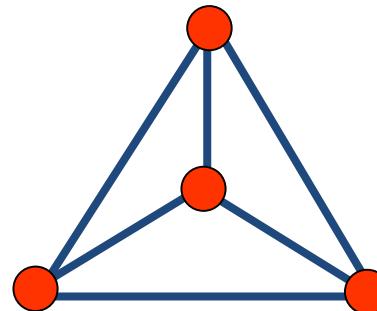
and **many more**.



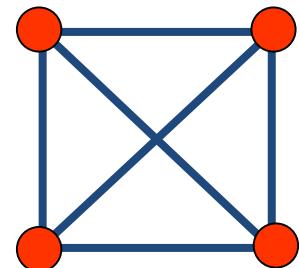
Optimisation Problem

- minimize crossings
 - minimize area
 - maximize symmetry
 - minimize total edge length
 - minimize number of bends
 - maximize angular resolution
- > **NP-hard** problem

Conflicts

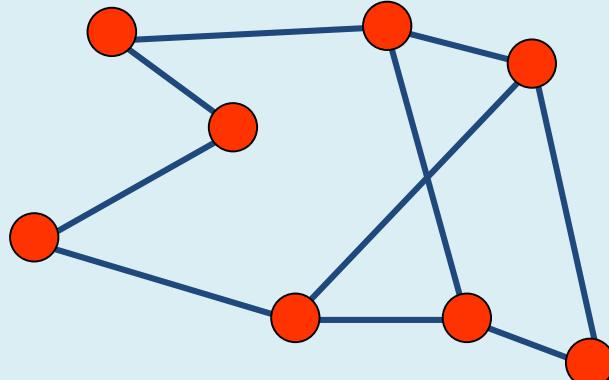


Minimize
edge
crossings

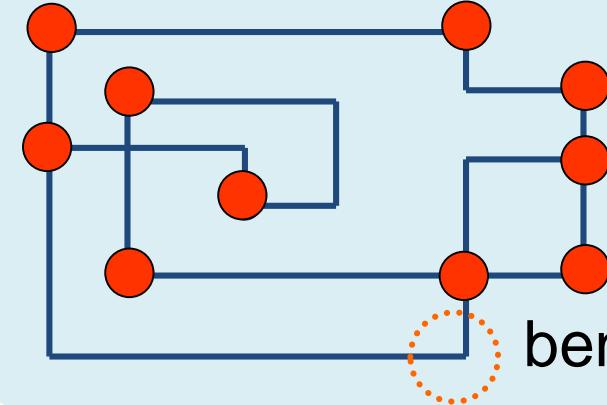


Maximize
symmetry

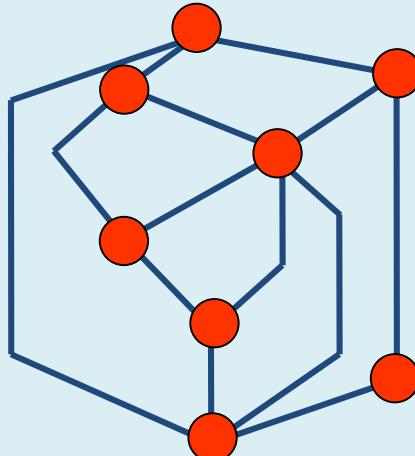
Drawing conventions



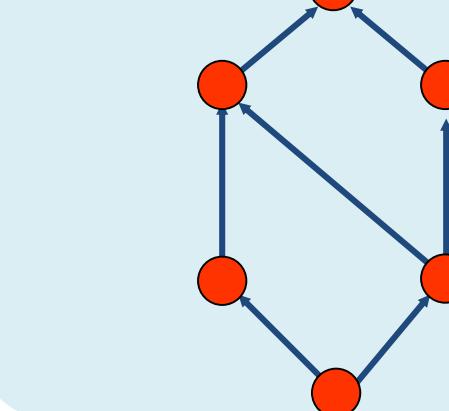
Straight-line drawing



Orthogonal drawing



Polyline drawing



Upward drawing

Graph Classes

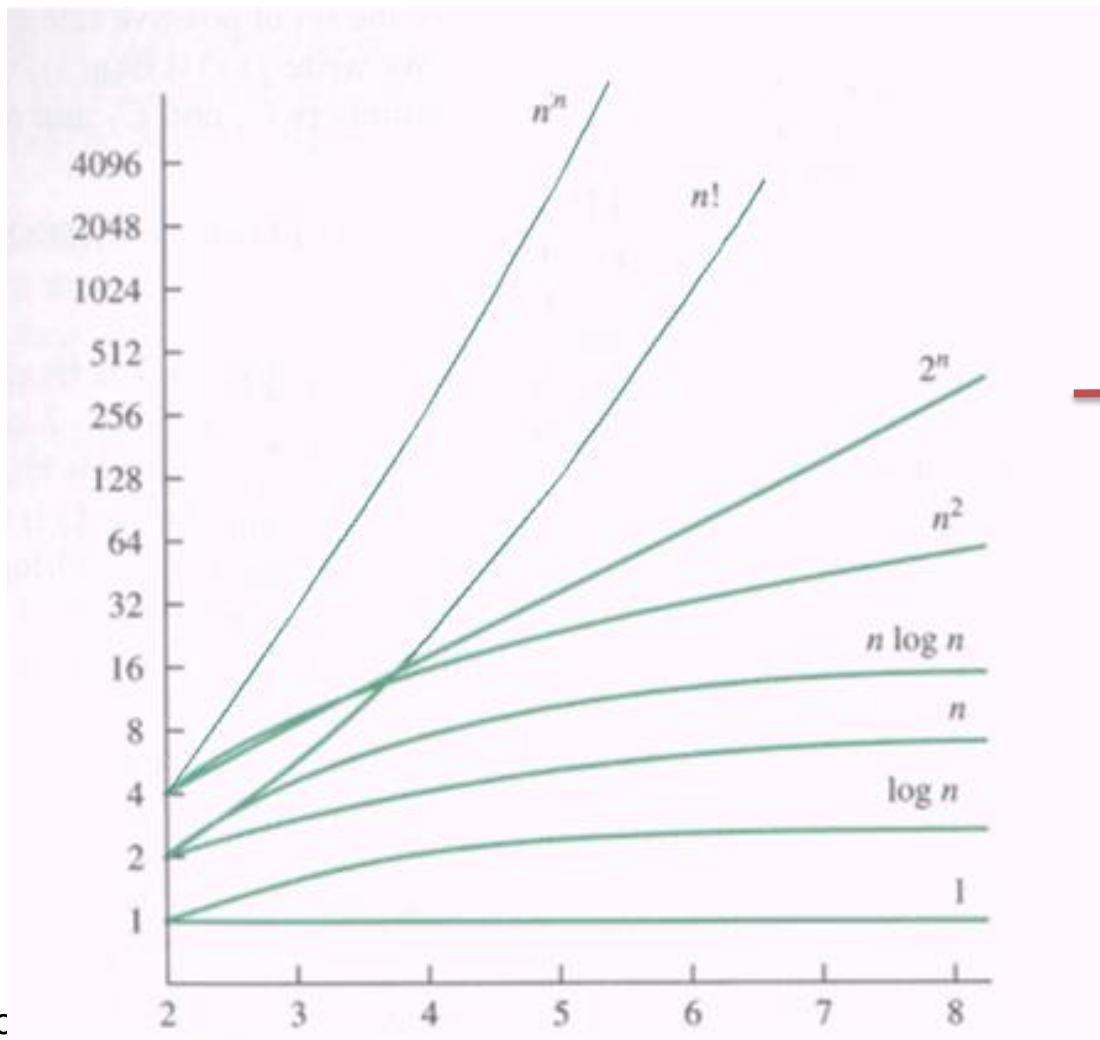
- **tree (Week 2)**
 - free tree
 - binary tree
 - rooted tree
- planar graphs
- **general graphs (Week 3)**
- **directed/hierarchical graphs (Week 4)**
- **clustered graphs (Week 5)**
 - hyper graphs/ higraphs

(4B) Tree Drawing Algorithm

- 1. Terminology**
- 2. Layered Drawing**
- 3. Radial Drawing**
- 4. HV-Drawing**
- 5. Inclusion Drawing**

Big O Notation

- Measure Time complexity (Efficiency) of algorithm: run-time
- Asymptotic: eg. $5n+10 \Rightarrow O(n)$, $10000n \Rightarrow O(n)$, n : size of input data



Exponential time

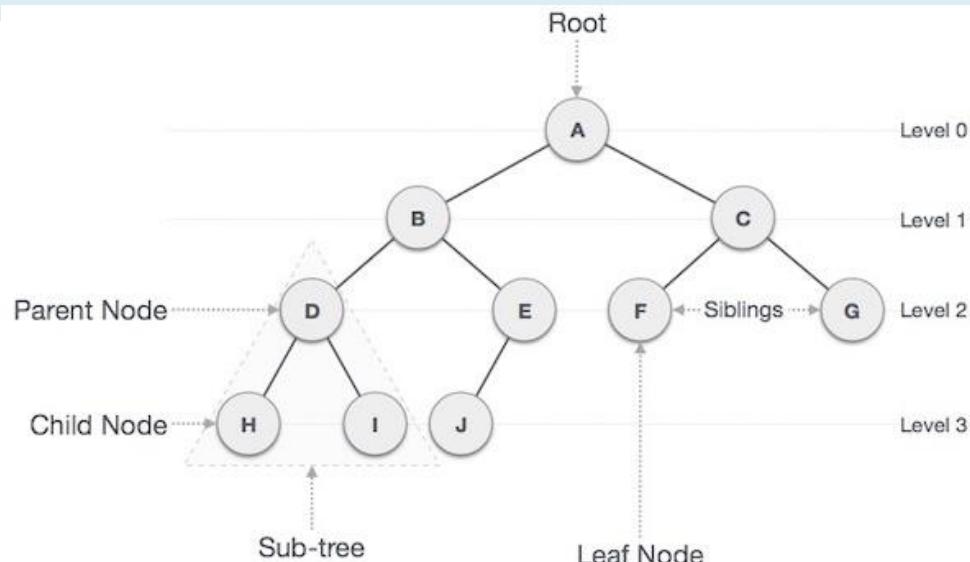
Polynomial time

Algorithm: Terminology

- **Divide and Conquer algorithm**
 - Divide into smaller subproblems/instances recursively
 - Solve solutions for subproblems
 - Merge solutions to obtain a solution to the original problem
- **Dynamic Programming algorithm**
 - breaking it down into a collection of simpler subproblems
 - solve each subproblem, and store the solution in a table.
 - look up previously solved subproblems and use the solutions to compute the solution for the original problem.
- **Recursion:**
 - a method where the solution to a problem depends on solutions to smaller instances of the same problem

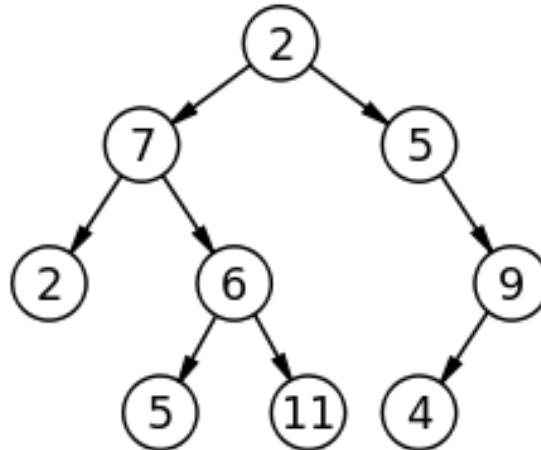
1. Tree: terminology

- Data structure to represent hierarchical information (no cycle)
 - **Rooted** tree
 - **Root**: a distinguished vertex in a tree
 - directed edge $u \rightarrow v$ (u : parent of v , v : child of u)
 - Leaf node: no child
 - **Subtree** rooted at v : subgraph induced by all “descendants” of v
 - **Depth** (level) of a vertex v : number of edges from v to the root
 - **Height** of a tree T : maximum depth
 - Ordered tree: rooted tree with a fixed ordering for children of each vertex



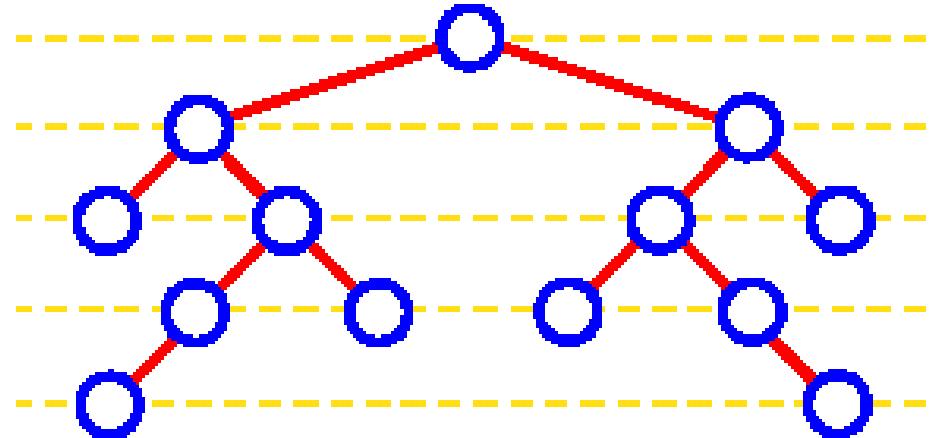
Binary Tree

- **Binary** tree: rooted tree with every node has at most two children
 - Children: Left child and Right child
 - Subtrees: Left subtree and Right subtree
- **Binary Tree Traversal**
 - Inorder Traversal: Left subtree-Root-Right subtree
 - Preorder Traversal: Root-Left subtree-Right subtree
 - Postorder Traversal: Left subtree-Right-Root subtree



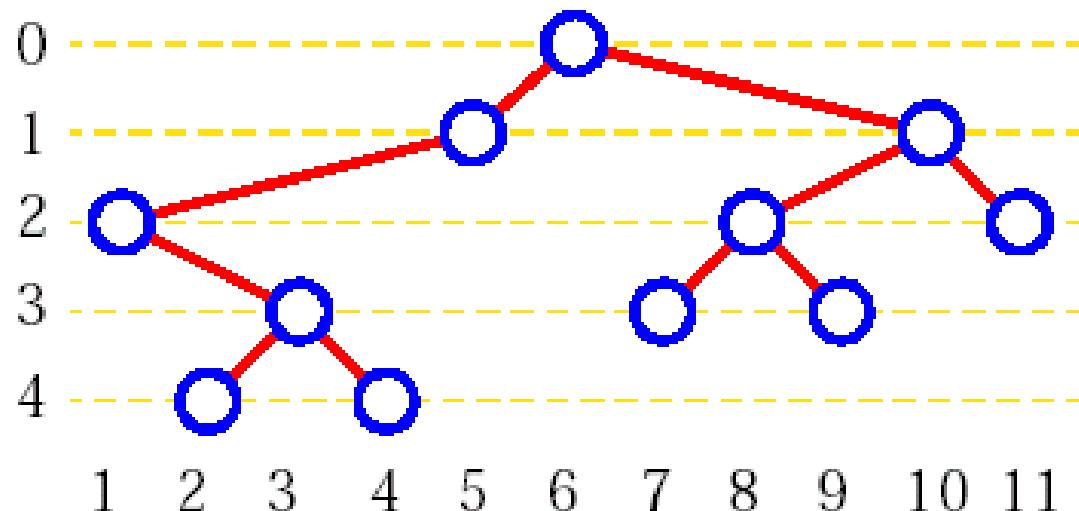
2. Layered Drawing

- rooted (binary) tree T
- assign layer according to the depth
-> y-coordinates: $y(v) = \text{depth of } v$
- how to compute x-coordinates?



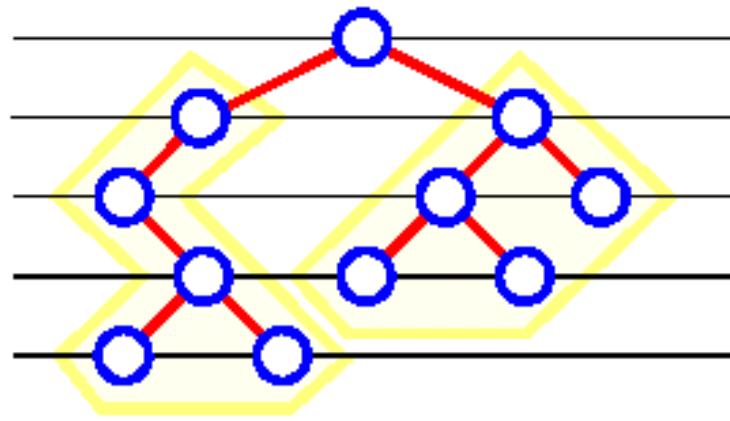
Simple Method

- Inorder Tree Traversal algorithm
 - layered grid drawing
 - two drawback:
 - too wide: width $n-1$
 - parent vertex is not centered with respect to the children



[Reingold–Tilford 81] Tidier Drawing Algorithm

- **Divide**: recursively apply the algorithm to draw the left and right subtrees of T .
- **Conquer**
 - move the drawings of subtrees until their horizontal distance equals 2.
 - place the root r vertically one level above and horizontally half way between its children.
 - If there is only one child, place the root at horizontal distance 1 from the child.



Tidier Drawing Algorithm

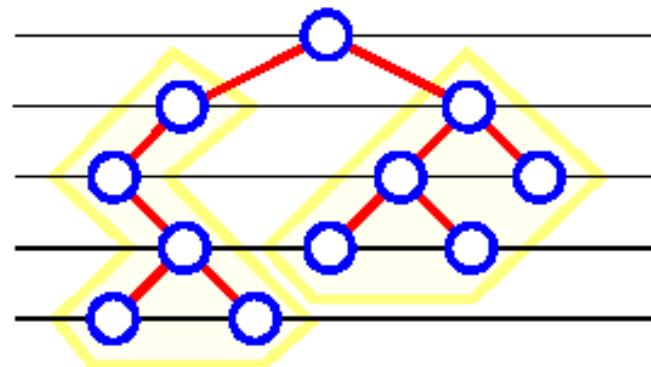
- Two traversals:

- **Step 1. Postorder traversal**

For each vertex v , recursively computes the horizontal displacement of the left & right children of v with respect to v .

- **Step 2. Preorder traversal**

Computes x-coordinates of the vertices by accumulating the displacements on the path from each vertex to the root.



Left/Right Contours for Postorder Traversal

计算v的轮廓:

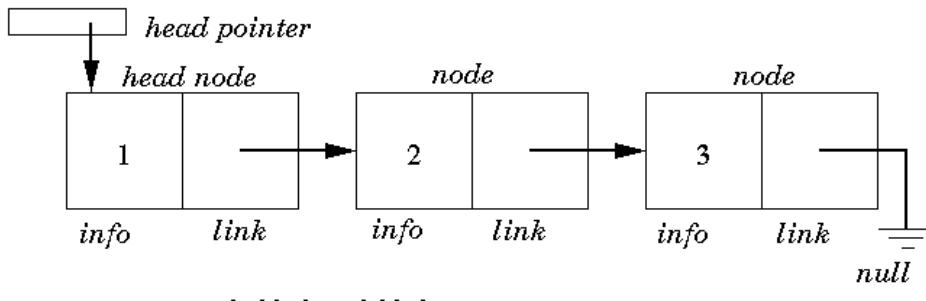
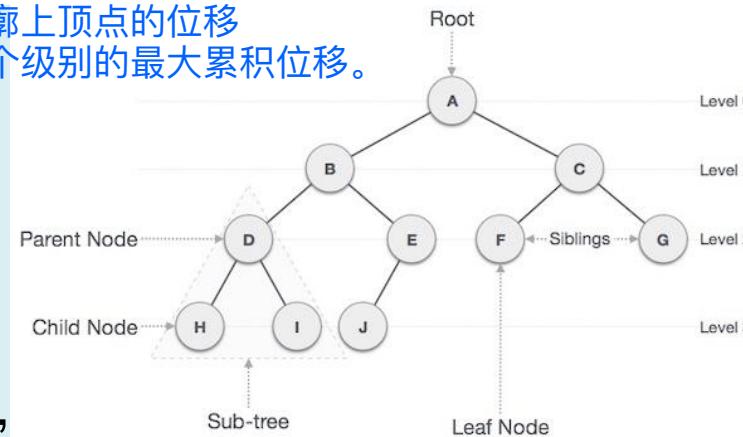
- 扫描左子树的右轮廓和右子树的左轮廓（按链接列表）
- 累积轮廓上顶点的位移
- 跟踪每个级别的最大累积极位移。

Left (Right) Contour:

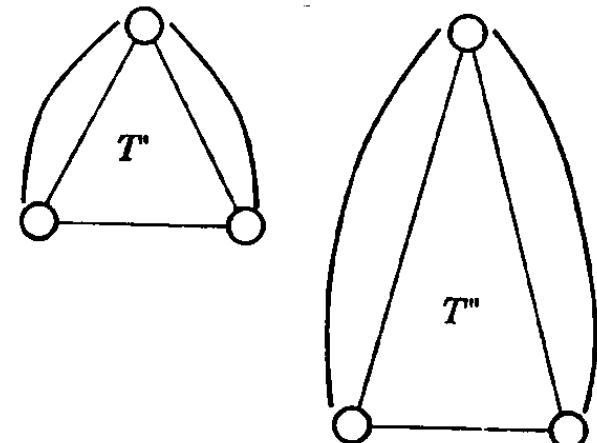
- sequence of vertices v_i such that v_i is the Leftmost (Rightmost) vertex of T at level i.
- store in linked list for each v.

Compute contours of v:

- scan the Right Contour of the Left Subtree, and the Left Contour of the Right Subtree (follow the linked list).
- accumulate displacements of vertices on the contours
- keep track of the maximum cumulative displacement at each level.



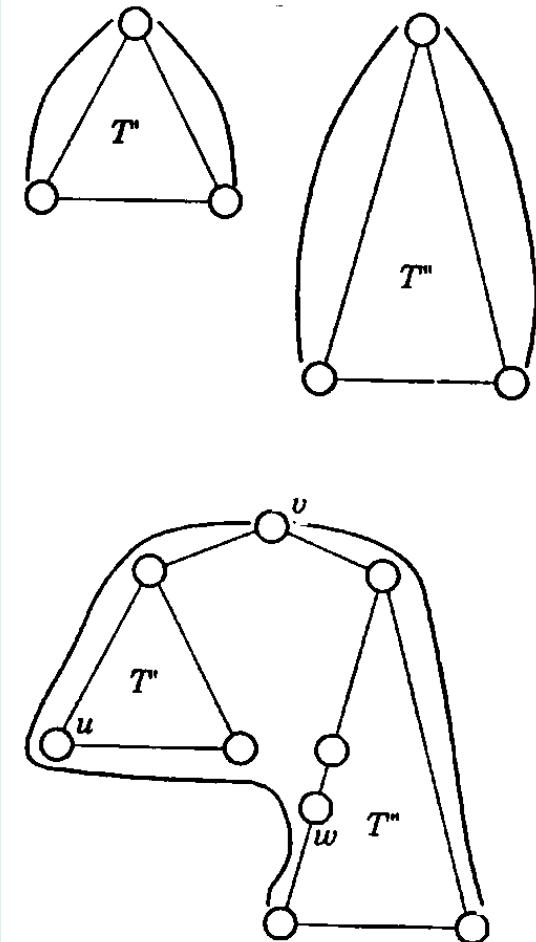
A Linked List



- T : subtree rooted at v
- T' (T''): left (right) subtree of T
- $L(T)$ ($R(T)$): left (right) contours of T

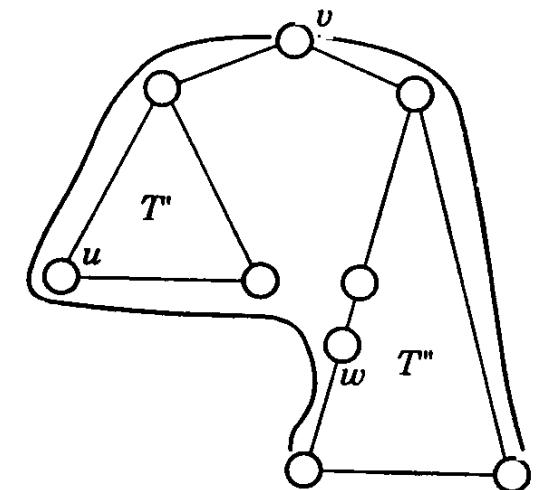
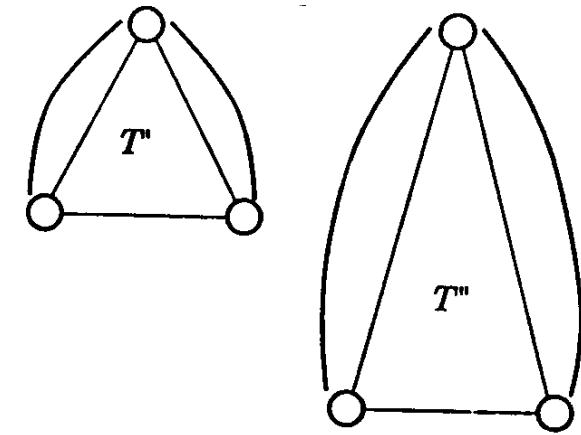
How to Construct $L(T)$ ($R(T)$) ?

- Case 1: $\text{height}(T') = \text{height}(T'')$
 - $L(T) = v + L(T')$
 - $R(T) = v + R(T'')$
- Case 2: $\text{height}(T') < \text{height}(T'')$
 - $R(T) = v + R(T'')$
 - $L(T) = v + L(T') + \{\text{part of } L(T'') \text{ starting from } w\}$
 - h' : height of T'
 - u : vertex of $L(T')$ with depth h' (bottom-most)
 - w : vertex on $L(T'')$ with depth $= h'+1$
- Case 3: $\text{height}(T') > \text{height}(T'')$: similar to case2



Implementing Postorder Traversal in Linear Time

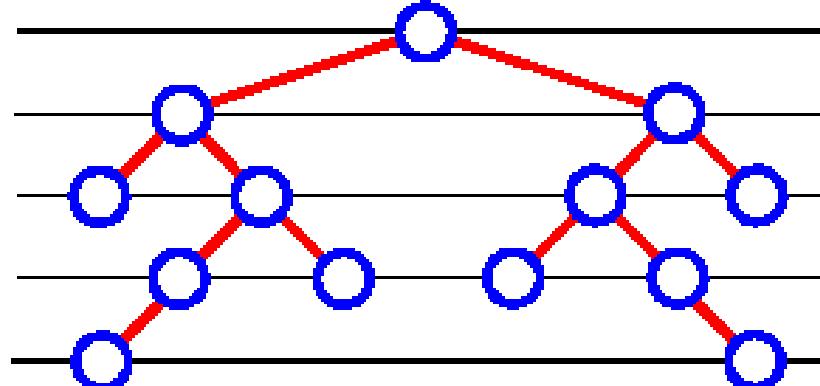
- It is necessary to travel down the contours of two subtrees T' and T'' only as far as the height of the subtree of *lesser height*.
- the time spent processing vertex v is proportional to the minimum heights of T' and T'' .
- The sum over all vertices v of the minimum height of the subtrees of v is no more than the number of vertices of the tree.



[Theorem]

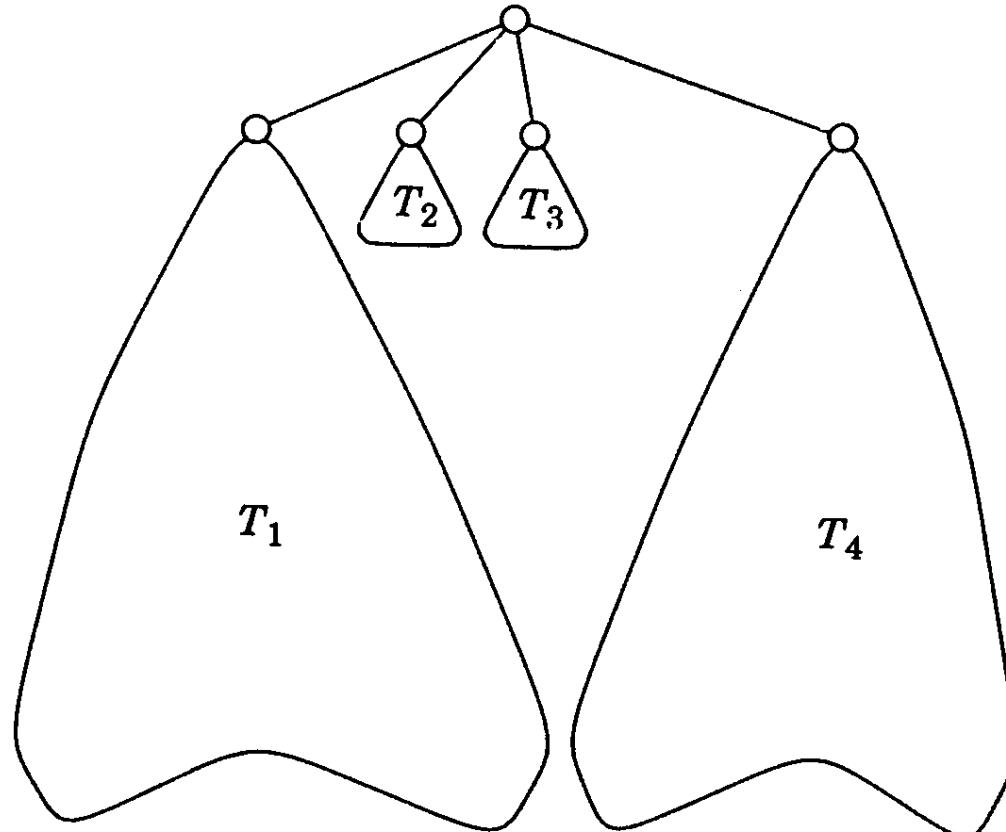
Tidier Drawing Algorithm constructs a drawing of a binary tree T in linear time such that the drawing is

- layered, planar, straight-line and strictly downward
- $O(n^2)$ area
- two vertices are at horizontal & vertical distance at least 1
- parent vertex is centered with respect to its children
- isomorphic subtrees have congruent drawing up to a translation
- axially isomorphic subtrees have congruent drawings, up to a translation & a reflection in y-axis



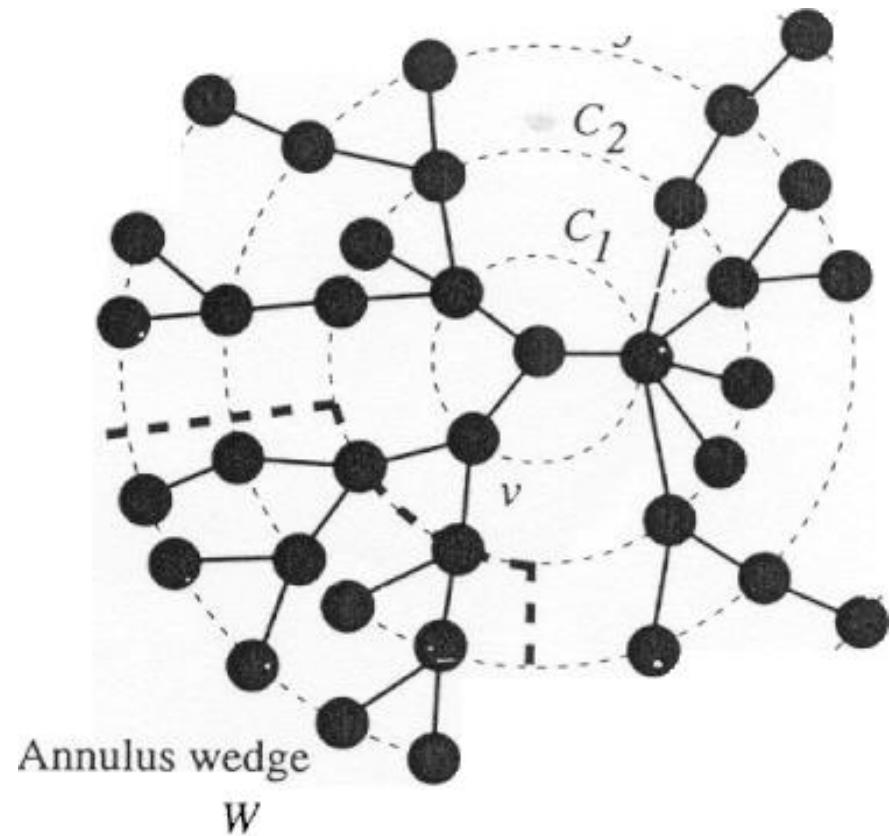
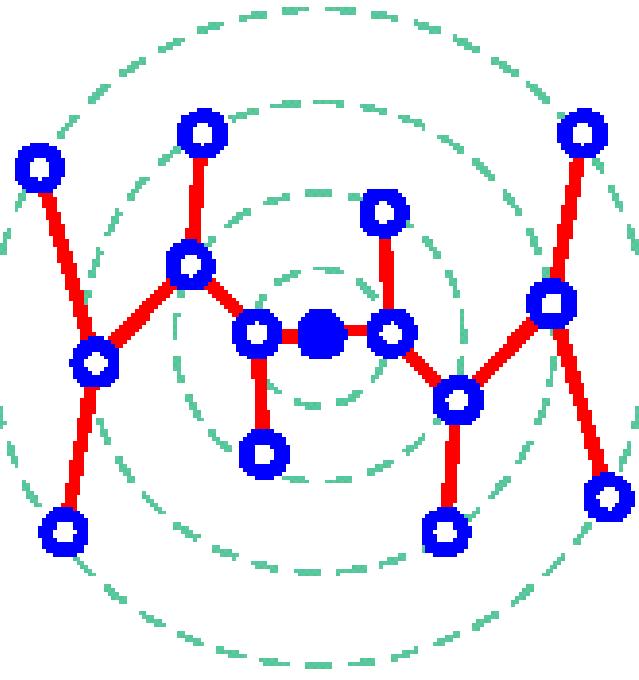
Generalization to Rooted Trees

- root is placed at the average x-coordinates of its children
- small imbalance problem: T₂ and T₃ much closer to T₁ than T₄.
- Modify conquer step or postprocessing
 - [Walker 90]: layered drawing for trees



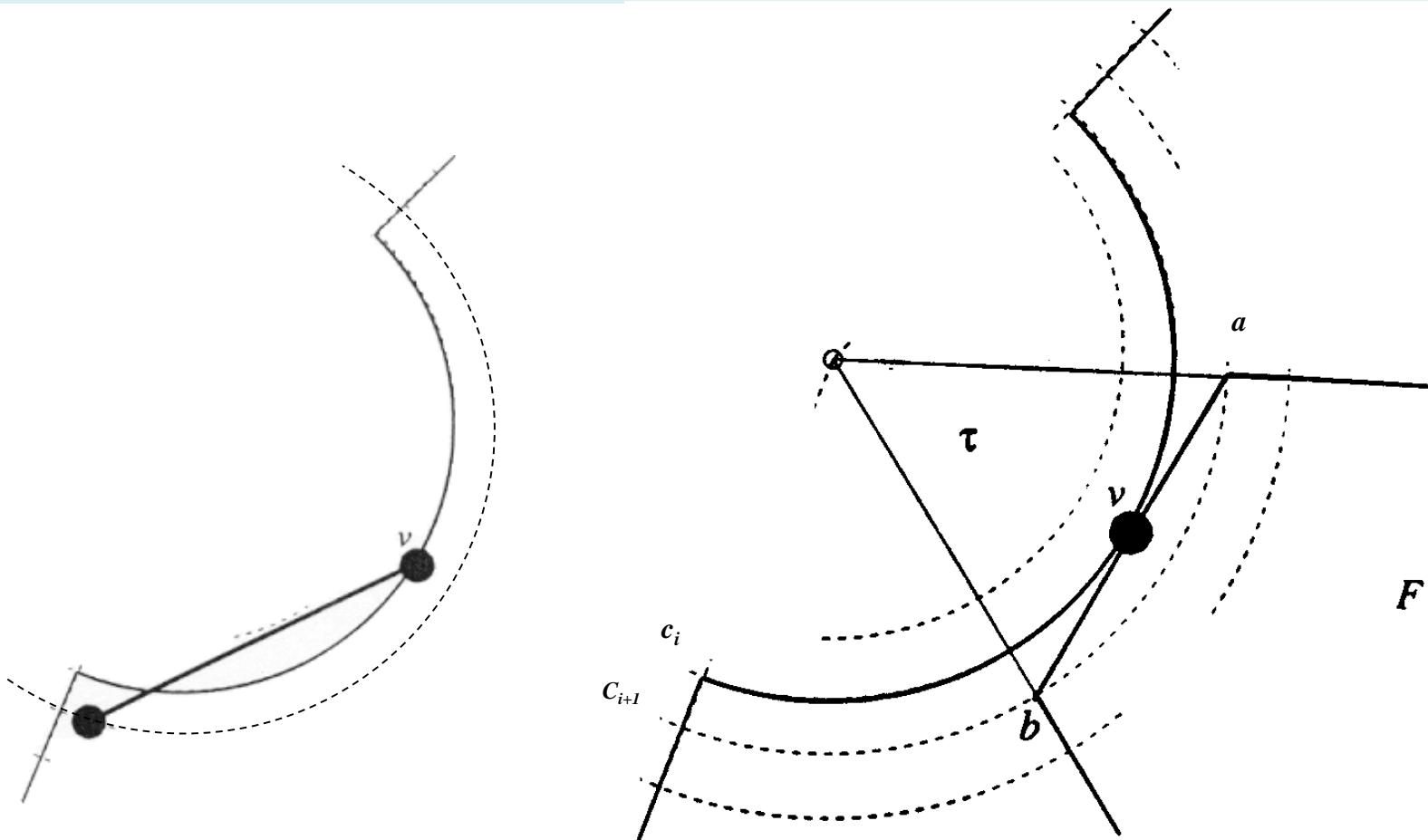
3. Radial Drawing

- Layers are represented as concentric circles.
- Draw each subtree in annulus wedge W.
- Angle of wedge: proportional to # of leaves of each subtree



Planar Drawing

- to guarantee planarity, define convex subset F of the wedge.
- Draw a subtree inside F

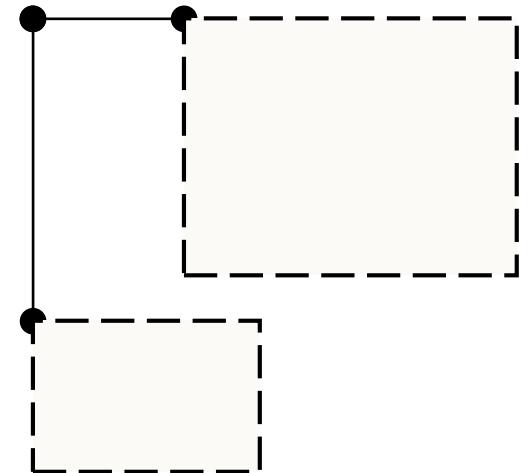
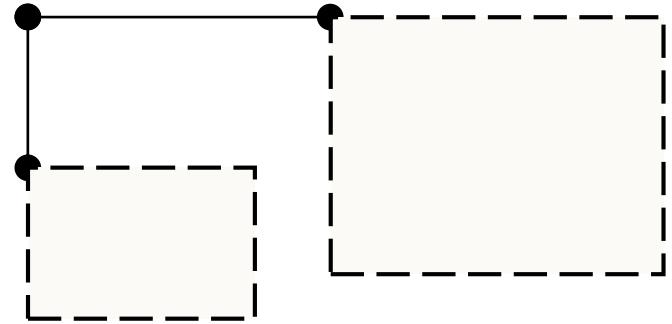
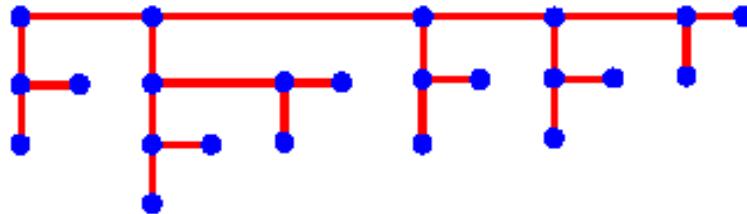


Radial Drawing

- Running time: linear
- used for free trees
 - select the center as a root
- can be used to display symmetry
 - Symmetry detection: [Manning, Atallah 88]
- Variations: [Eades 92], [Bernard 81], [Esposito 88]
 - choice of root
 - radii of the circles
 - how to determine the size of the wedge

4. hv-Drawing

- hv-drawing of a binary tree T :
straight-line grid drawing such that for each u ,
a child of u is either
 - horizontally aligned to the right of u , or
 - vertically aligned below u
 - bounding rectangles of the subtrees of u
do not intersect
- planar, straight-line, orthogonal, and
downward



Divide & Conquer Algorithm

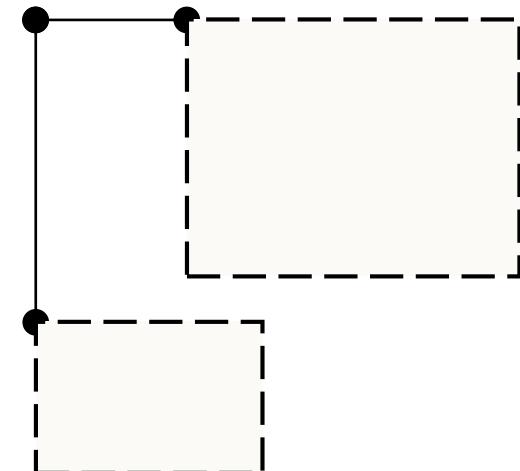
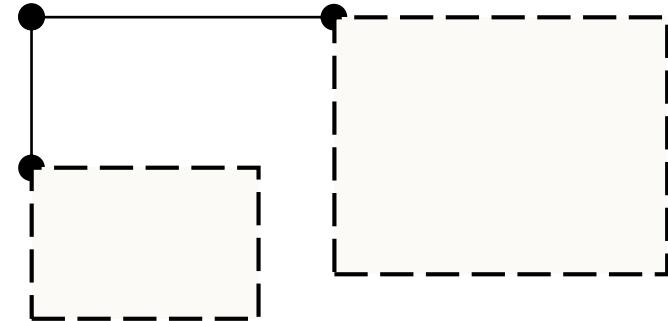
- **Divide:**

recursively construct hv-drawings for the left & right subtrees

- **Conquer:**

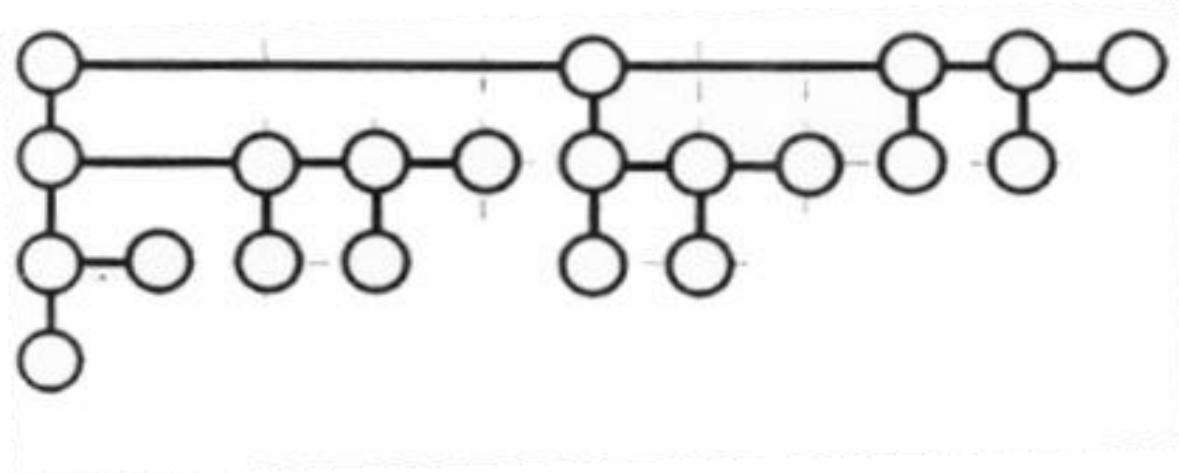
perform either horizontal combination or a vertical combination

- The height & width are each at most $n-1$



Algorithm Right-Heavy-HV-Tree-Draw

1. Recursively construct drawing of the left & right subtrees.
2. Using only **horizontal** combination, place the subtree with the **largest** number of vertices to the **right** of the other one.



Algorithm Right-Heavy-HV-Tree-Draw

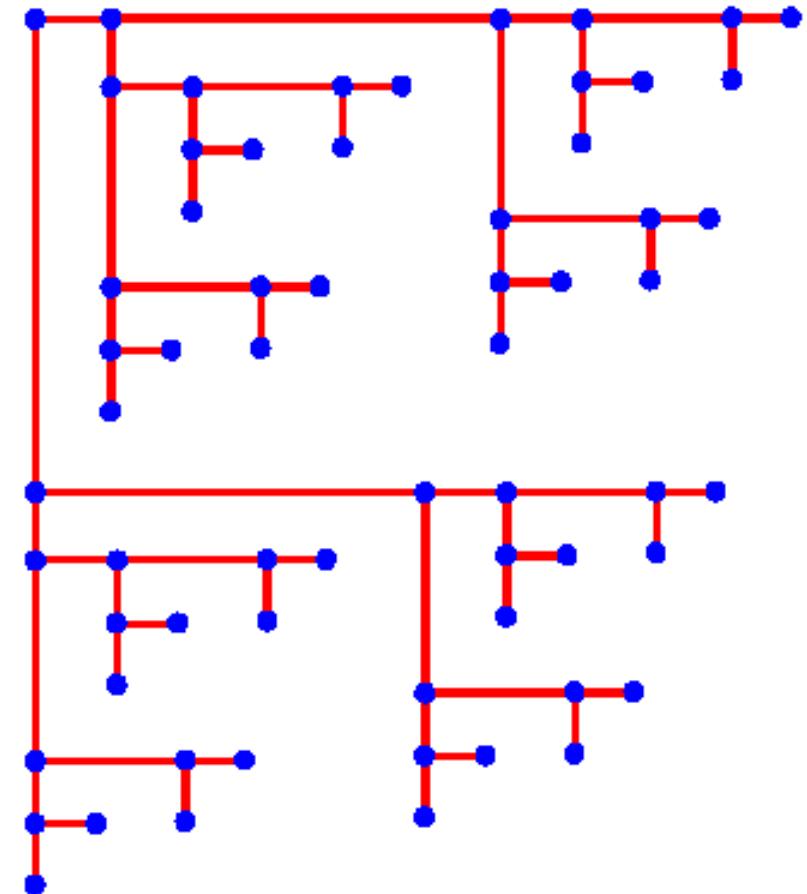
[Theorem]

Algorithm **Right-Heavy-HV-Tree-Draw** construct a drawing of binary tree T with n vertices such that the drawing is

- hv-drawing (downward, planar, grid, straight-line and orthogonal)
- area $\mathcal{O}(n \log n)$
- width is at most $n-1$
- height is at most $\log n$
- simply and axially isomorphic subtrees have congruent drawings, up to a translation

Algorithm Right-Heavy-HV-Tree-Draw

- Good area bound, but bad aspect ratio
- **Better aspect ratio**: use both horizontal/vertical combinations for odd/even depth.
- **Complete binary tree**: $O(n)$ area and constant aspect ratio.



General Binary Tree

[Eades, Lin, Lin 92] [Eades, Lin, Lin 93]

It is possible to construct an **hv-drawing** of a general binary tree that is optimal with respect to “area” or “perimeter” in $O(n^2)$ time.

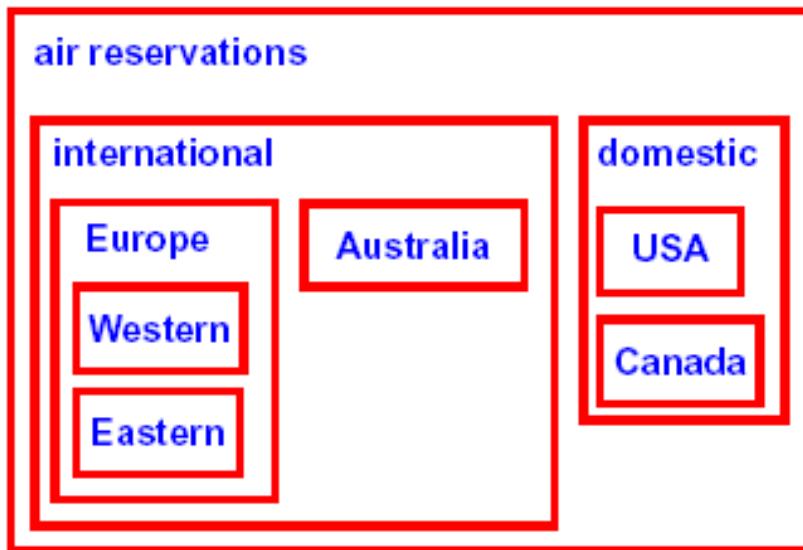
- use dynamic programming algorithm.

5. Inclusion Tree Drawing

[Eades, Lin, Lin 93] Inclusion Drawing of Rooted Trees

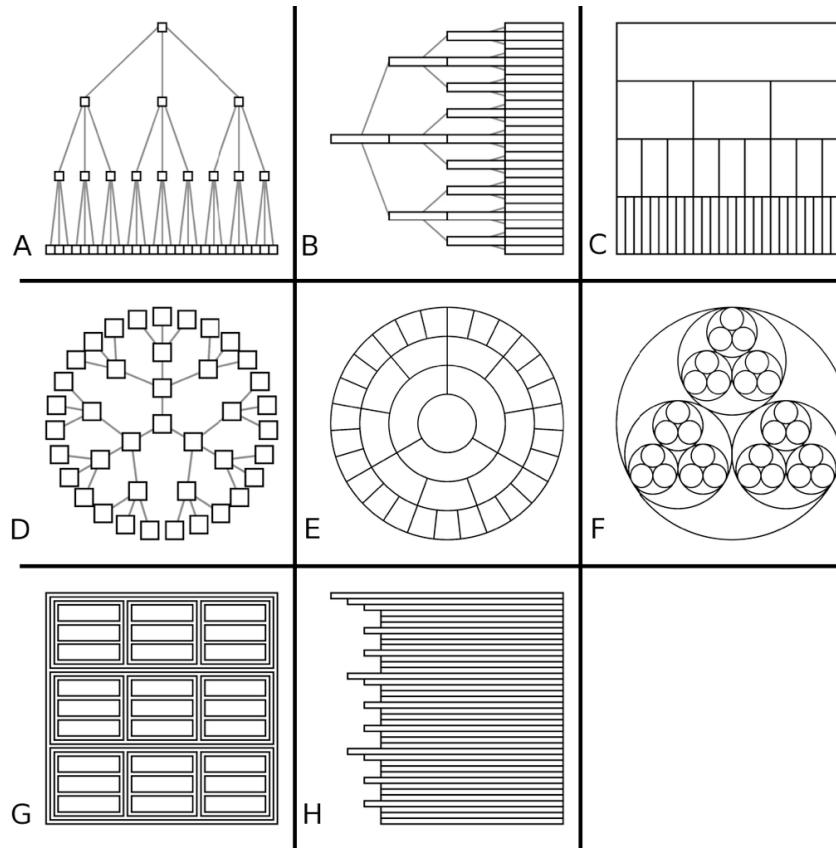
Display the parent-child relationship by the inclusion between isothetic rectangles.

- Minimization of area (perimeter, width, height)
 - NP-hard for general trees
 - Polynomial time algorithm for balanced trees



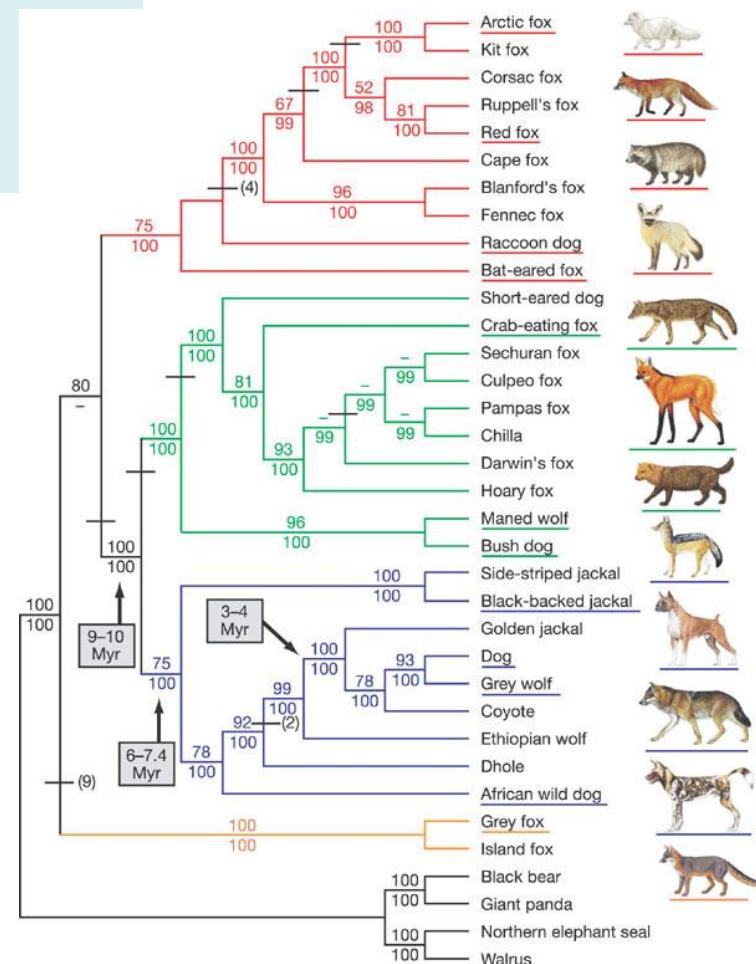
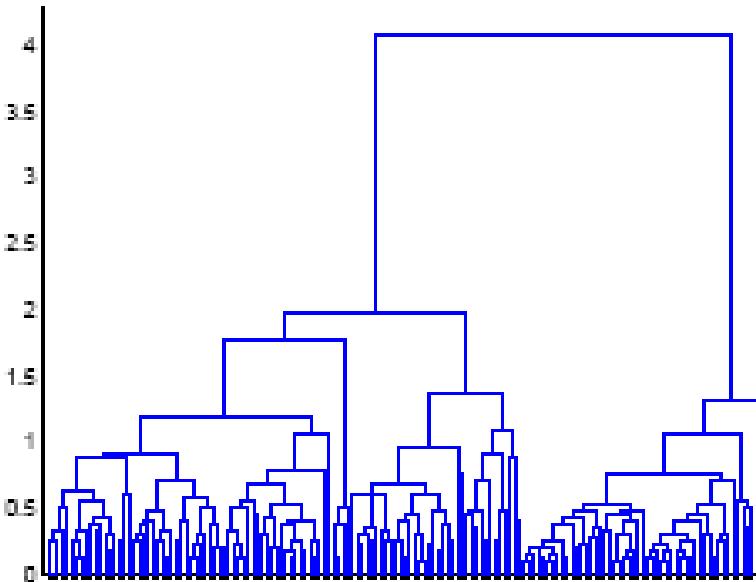
- used for compounds graphs (union of a graph and a tree)
- allow better fit the drawing in a prescribed region

(4C) Tree visualization methods

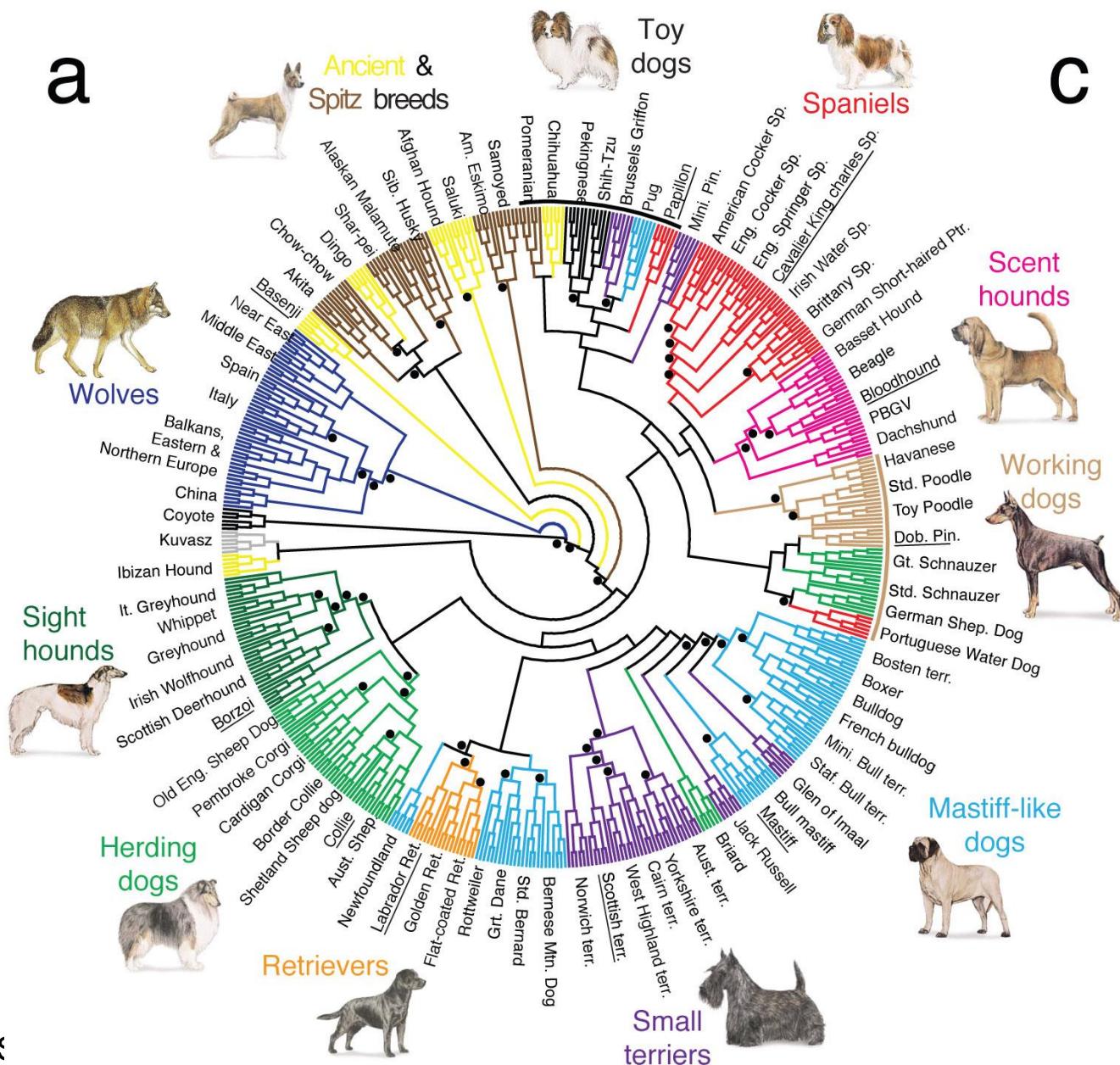


Dendrogram

- layered drawing with bended orthogonal edges.
- all the leaves are on the same layer.
- good for drawing large trees in small area.
- used in bioinformatics to represent
 - hierarchical clustering
 - phylogenetic trees



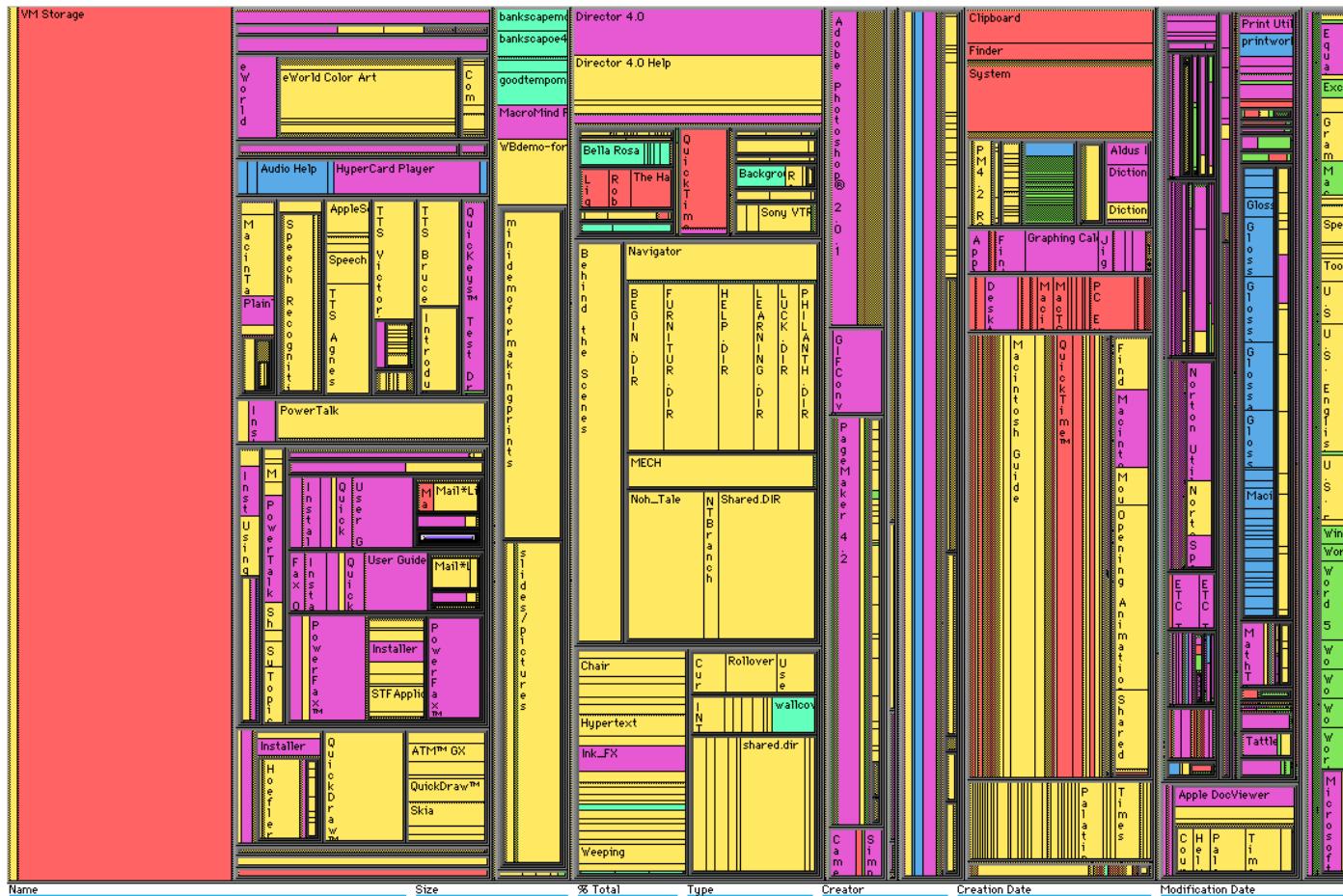
Radial Dendrogram



Treemap

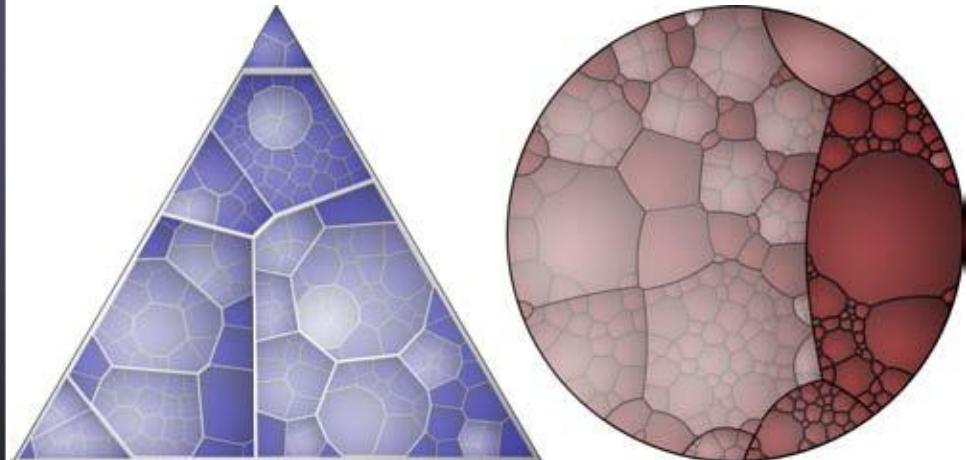
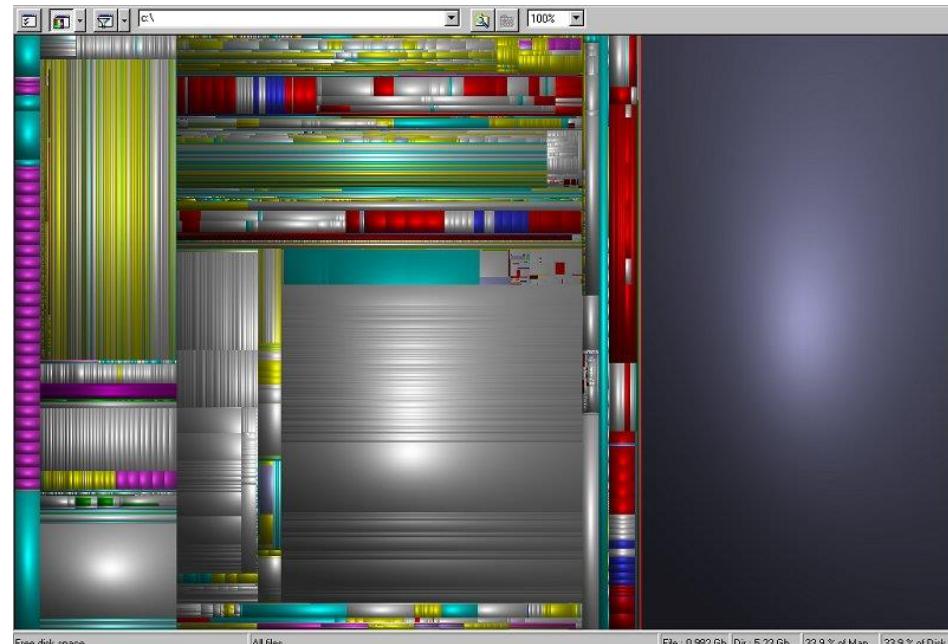
[Shneiderman 92] use containment to show the hierarchy.

- partition the space recursively according to the size of subtrees.
- space-efficient compare to node-link diagram.
- difficult to follow parent-child relationship.



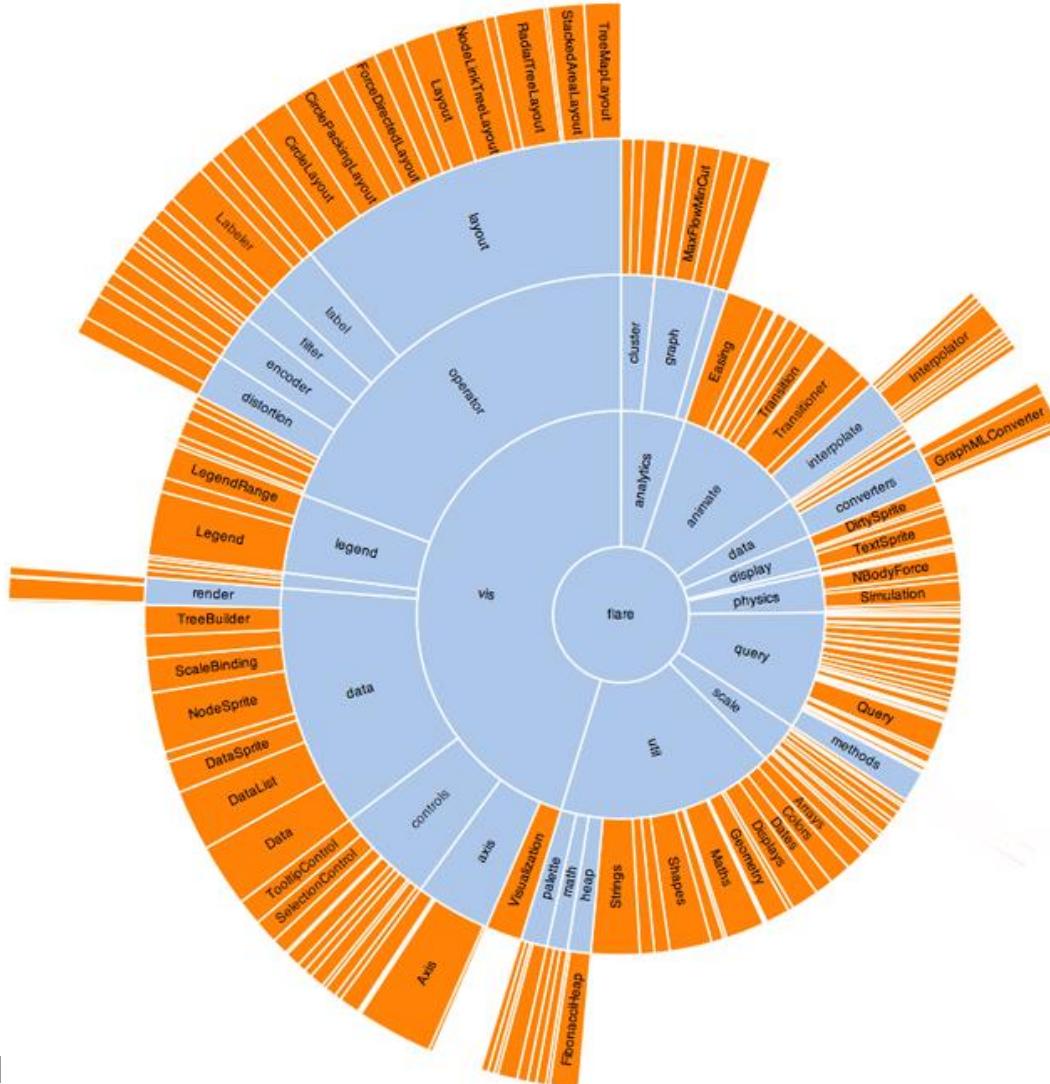
Variations of Treemap

- Cushion treemap [Wijk 00]
 - uses shading to help identify the levels in a treemap.
- Voronoi treemap
 - uses voronoi diagram as partition.

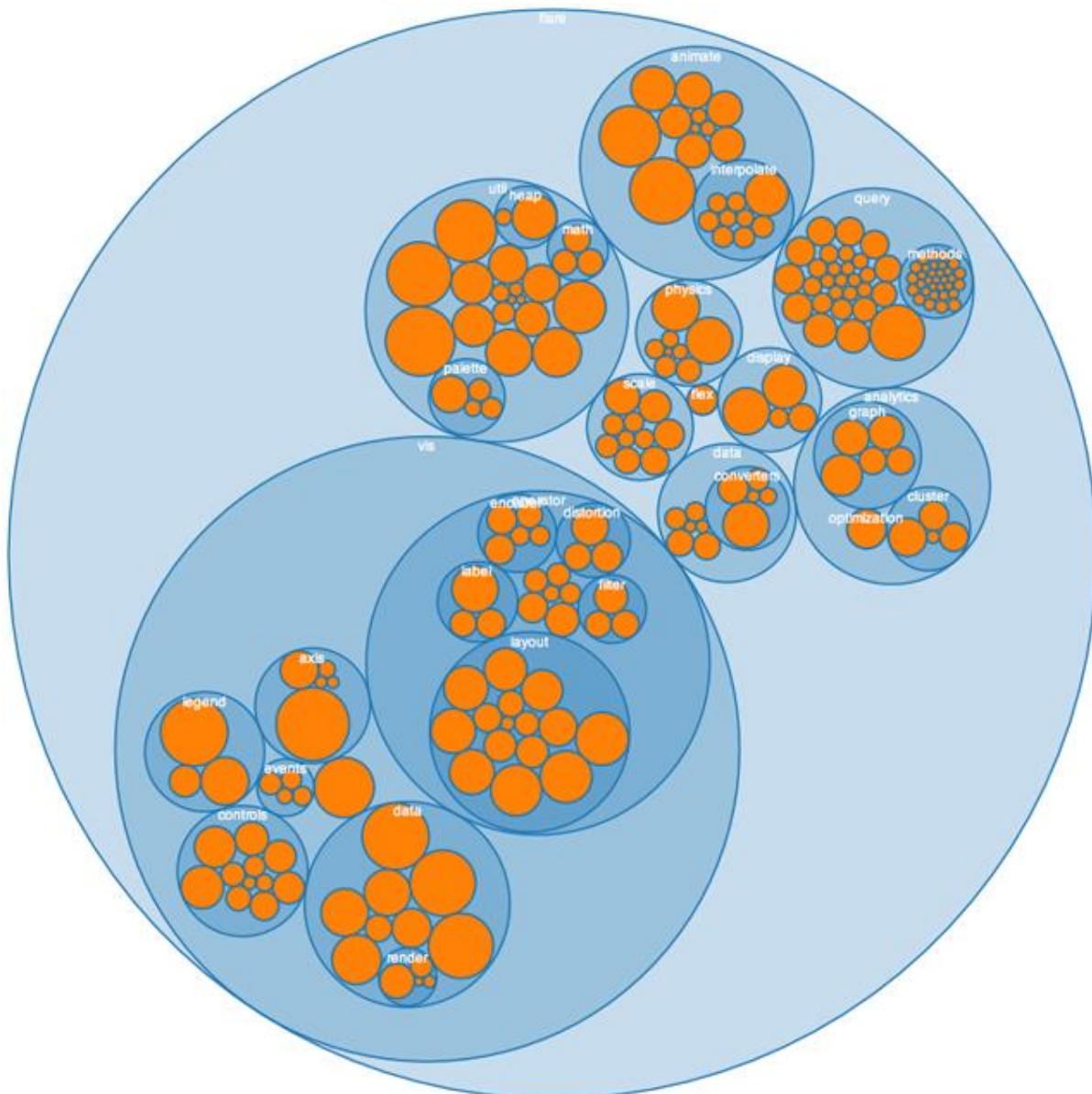


Sunburst Diagrams

[Stasko 00] space-filling visualization method
– radial version of tree map



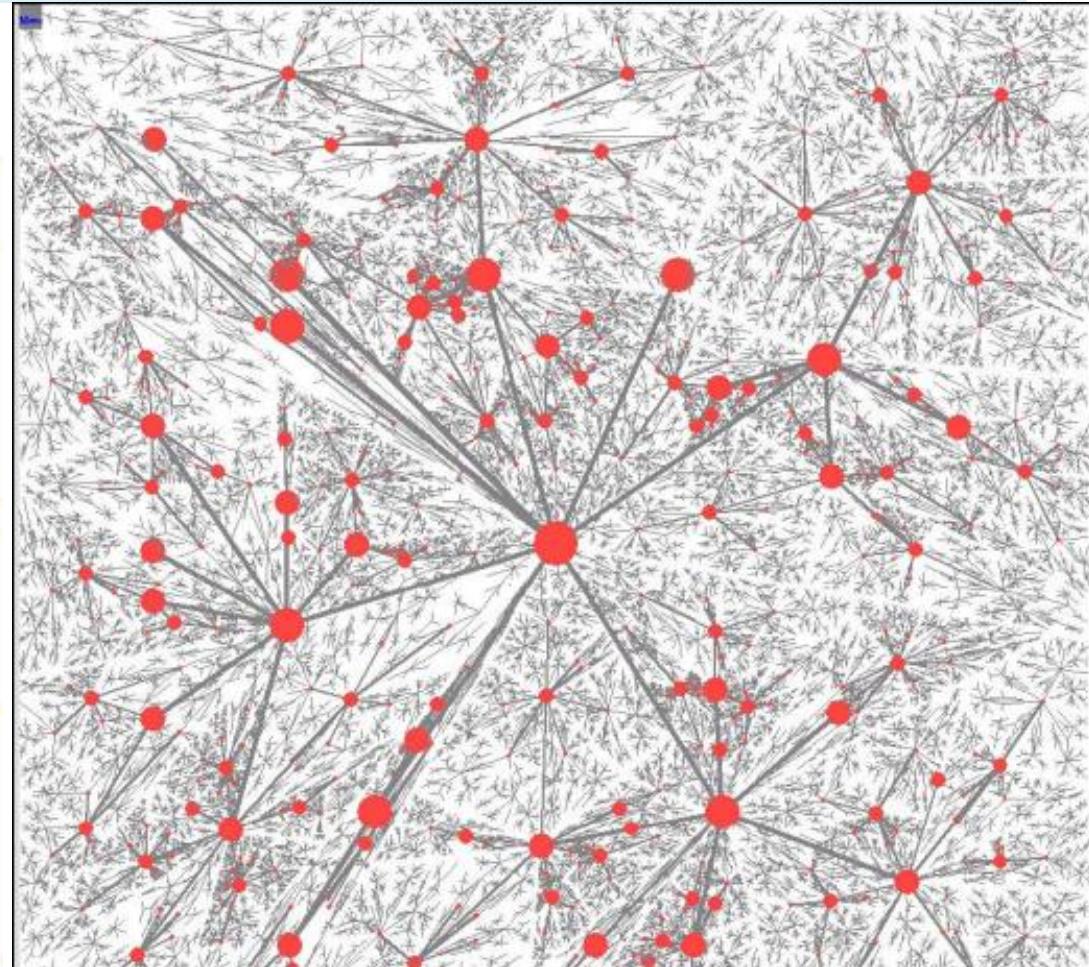
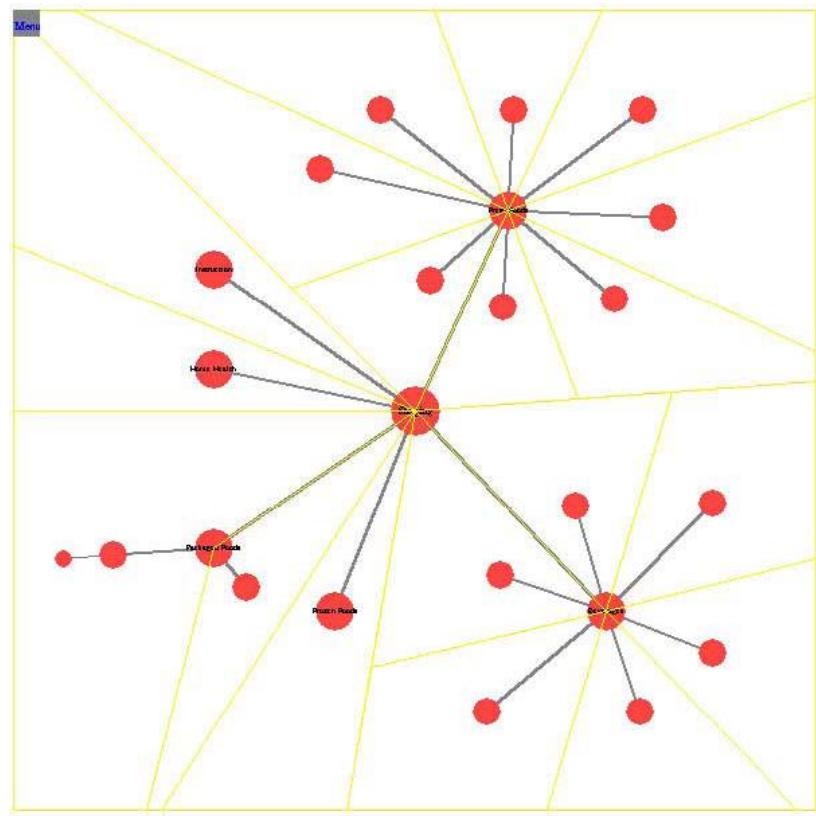
Space-filling by Circle Packing



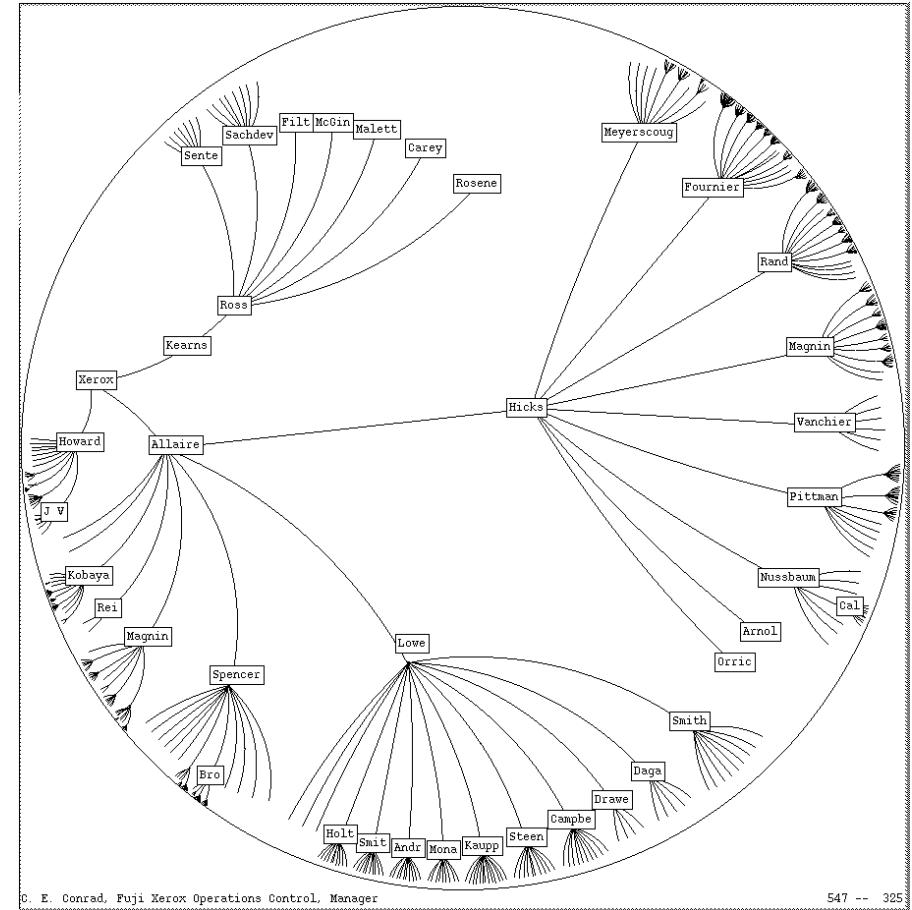
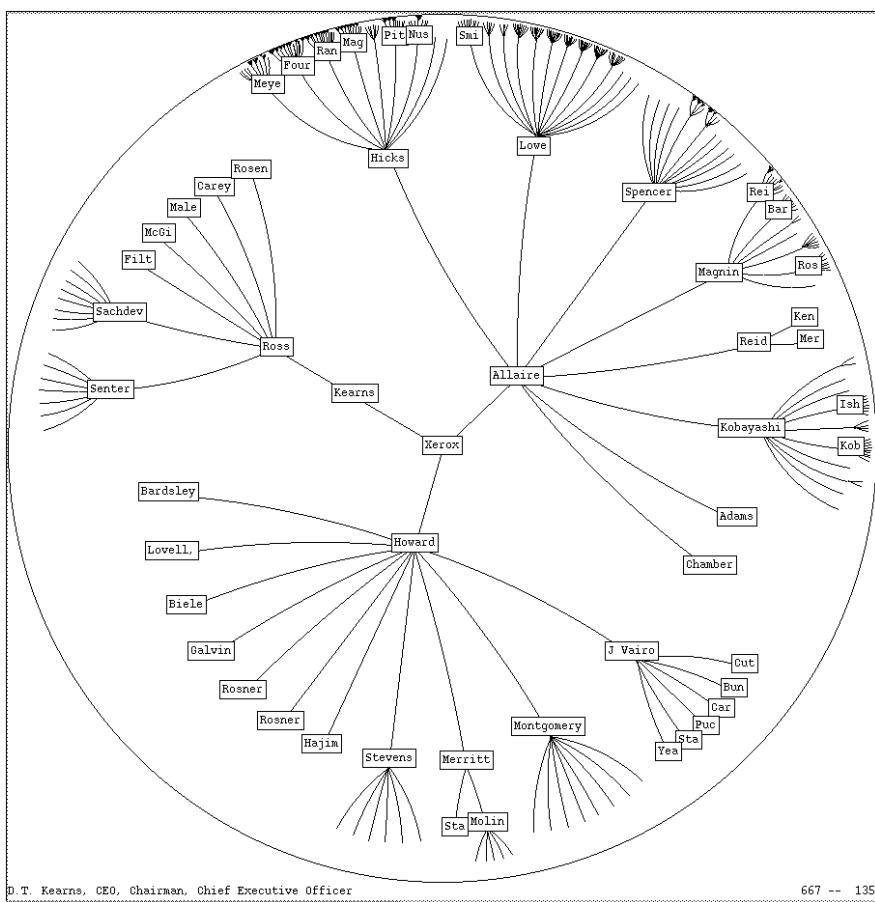
Space-filling Tree Layout

[Huang Nguyen 00]

- Layout a tree according to the recursive partition of the screen space
 - area allocated to a subtree is proportional to its size
- example: 55000 nodes (use all the screen space)



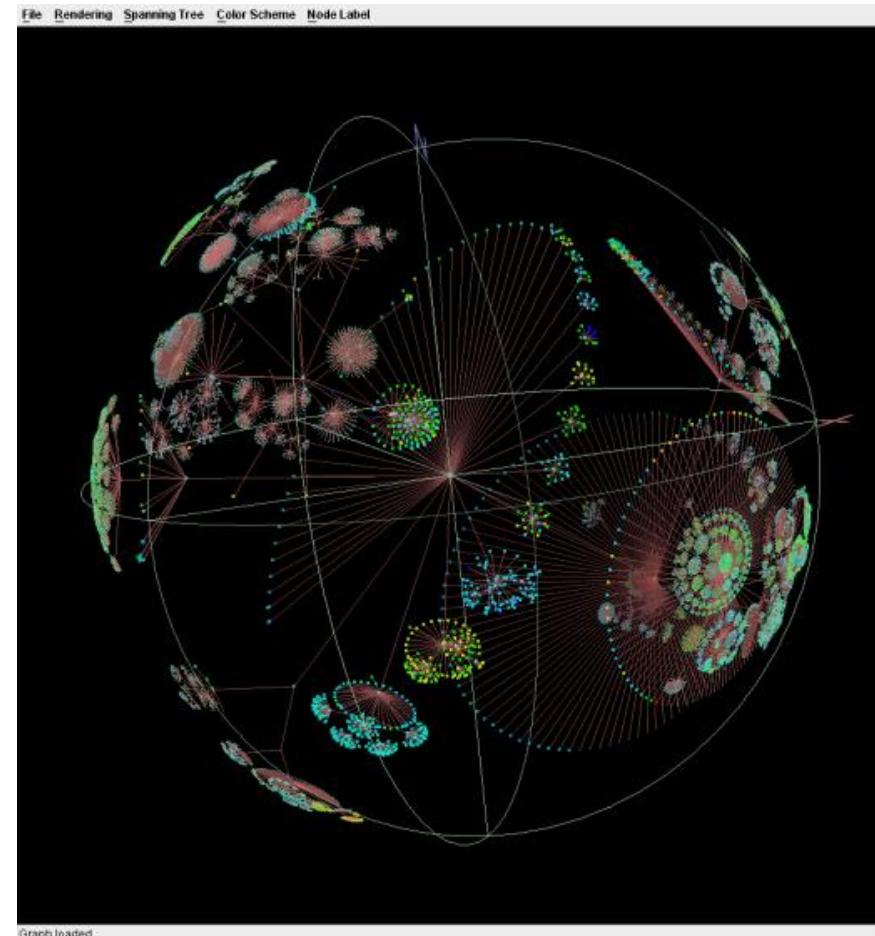
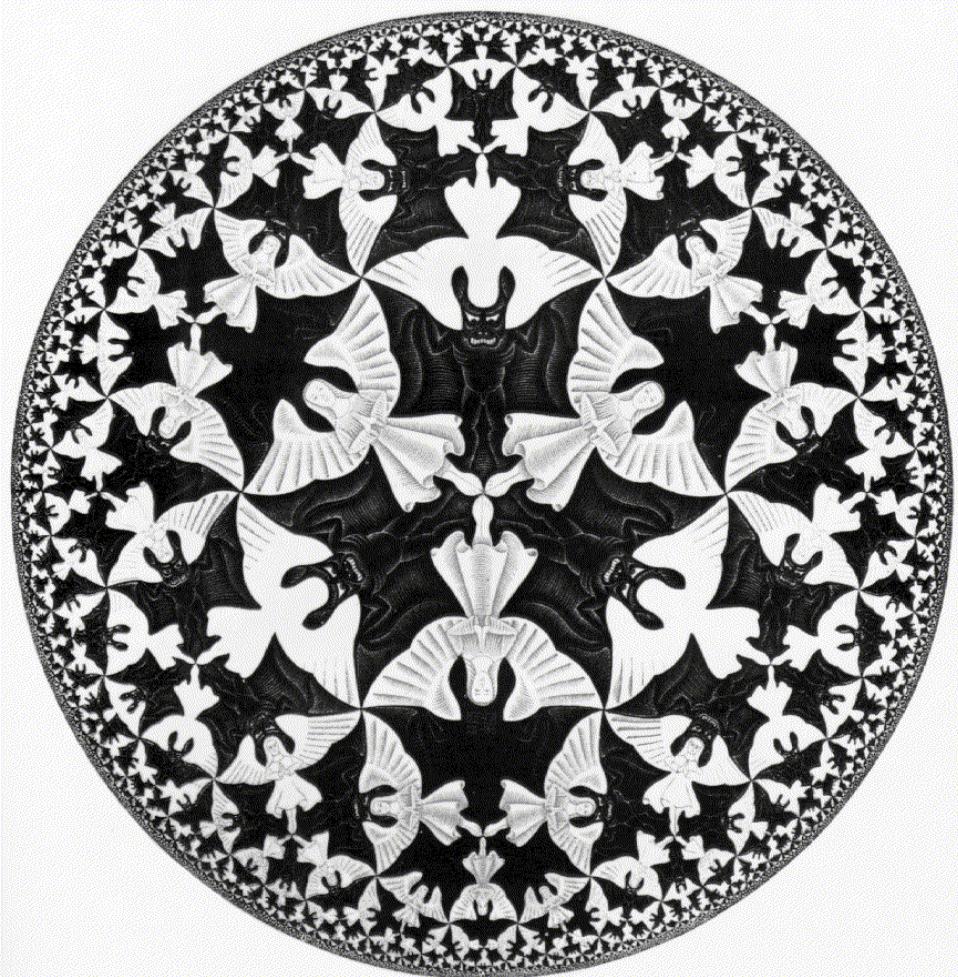
Hyperbolic Tree Browser



[Lamping, Rao, Pirolli 95]

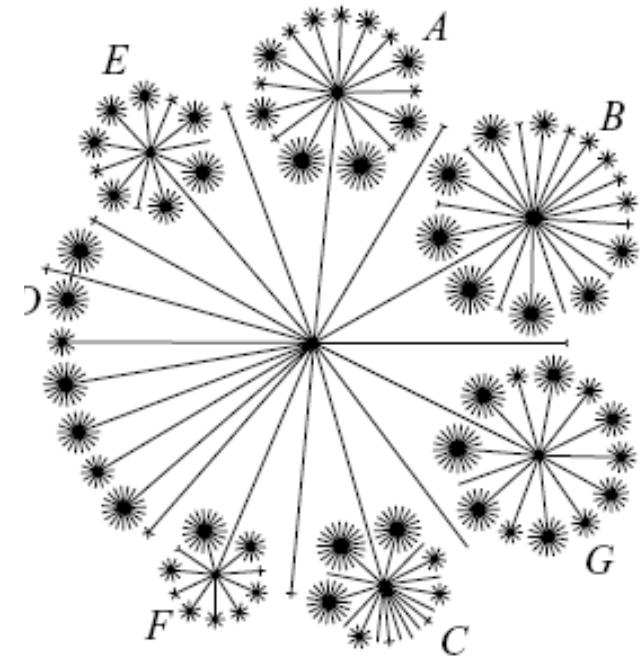
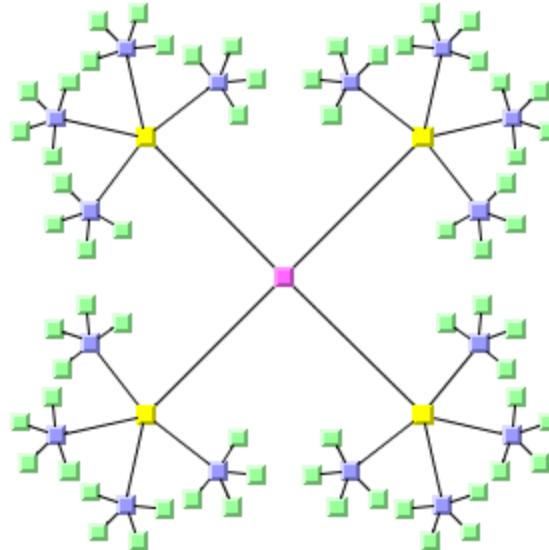
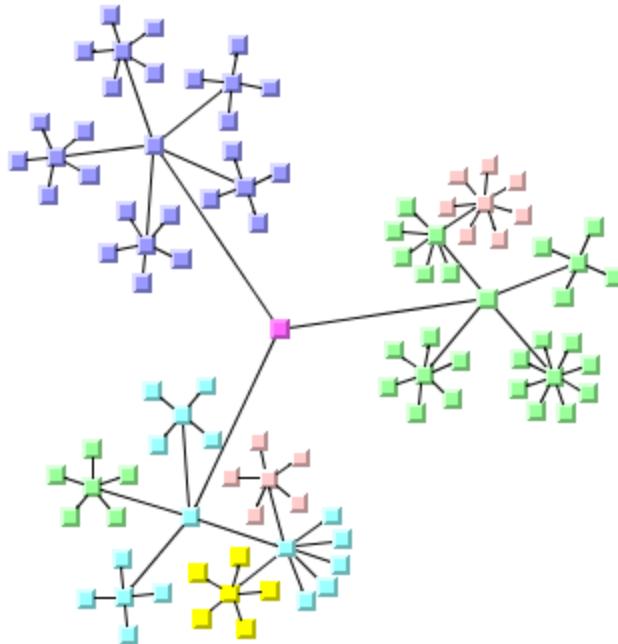
- A Focus+Context Technique Based on Hyperbolic Geometry
- Distortion effect of fisheye lens
- Interaction method

- M.C. Escher, Circle Limit IV (Heaven and Hell)
- 3D hyperbolic tree [Munzner]
 - projecting a graph on hyperbolic sphere
 - produces a distortion effect



Balloon Tree Drawing

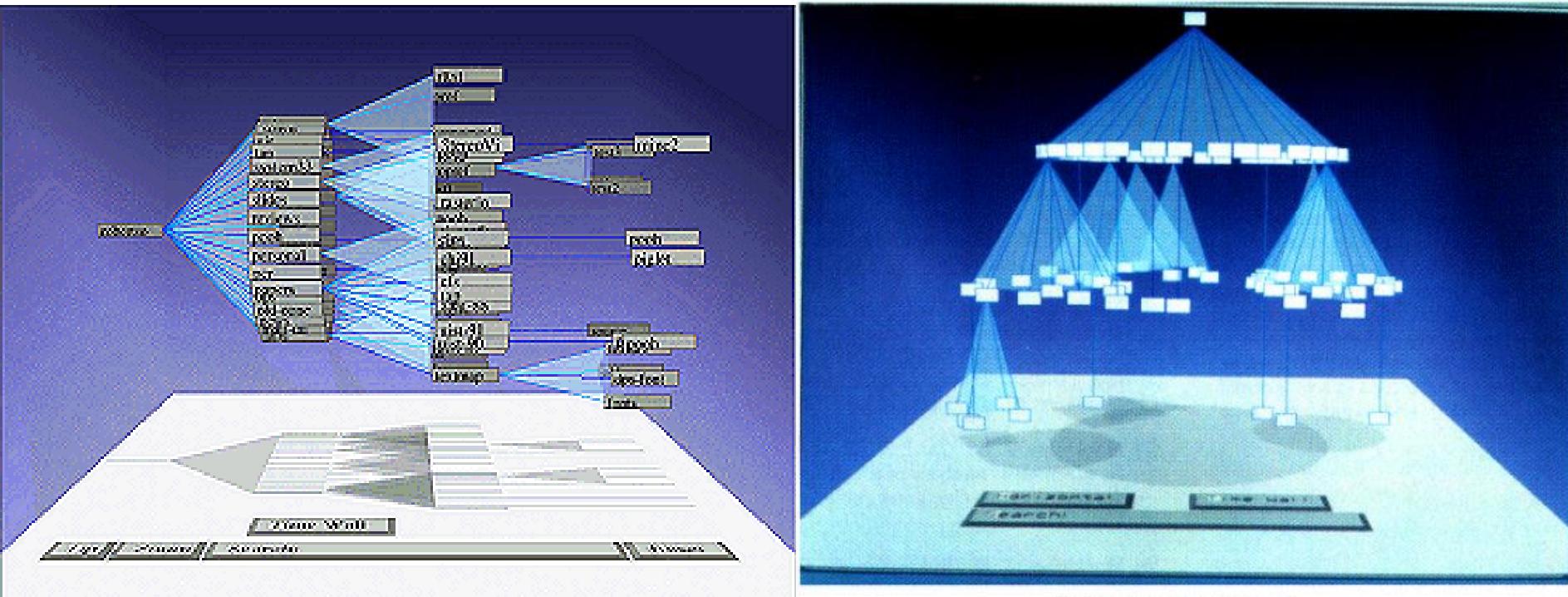
- A variation of radial layout.
- Children are drawn in a circle centered at their parents.
- [Yen 05] Deciding a good ordering: NP-hard problem



Cone Tree

[Roberston et al. 91]

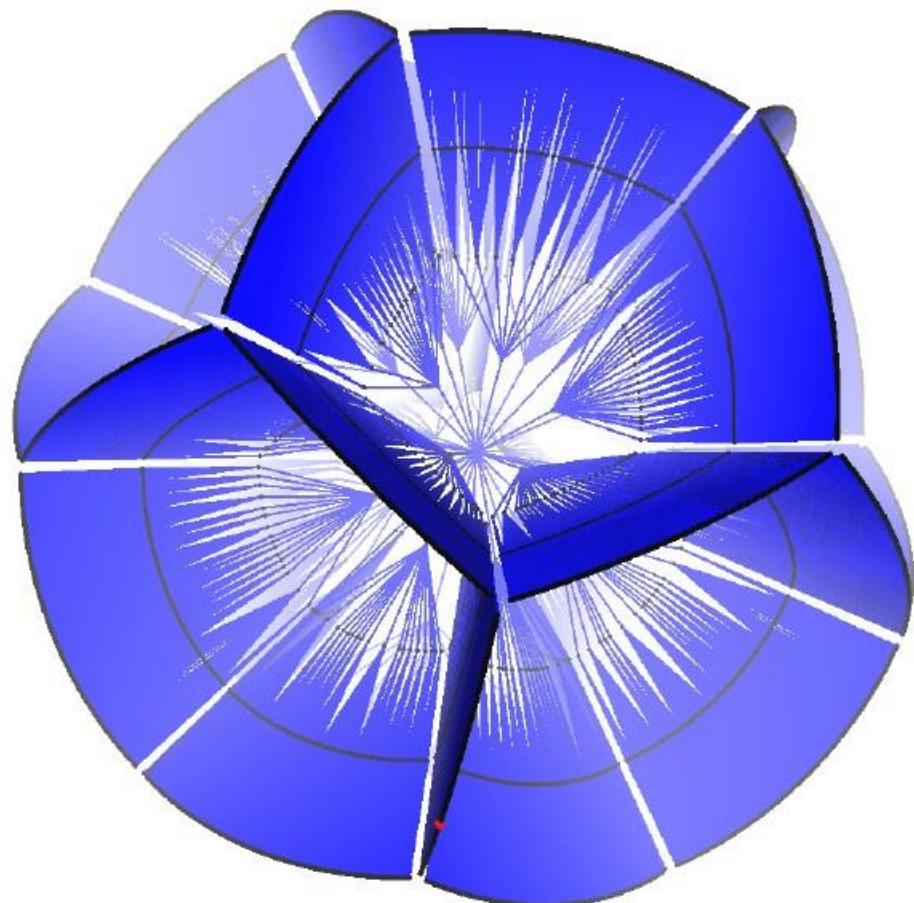
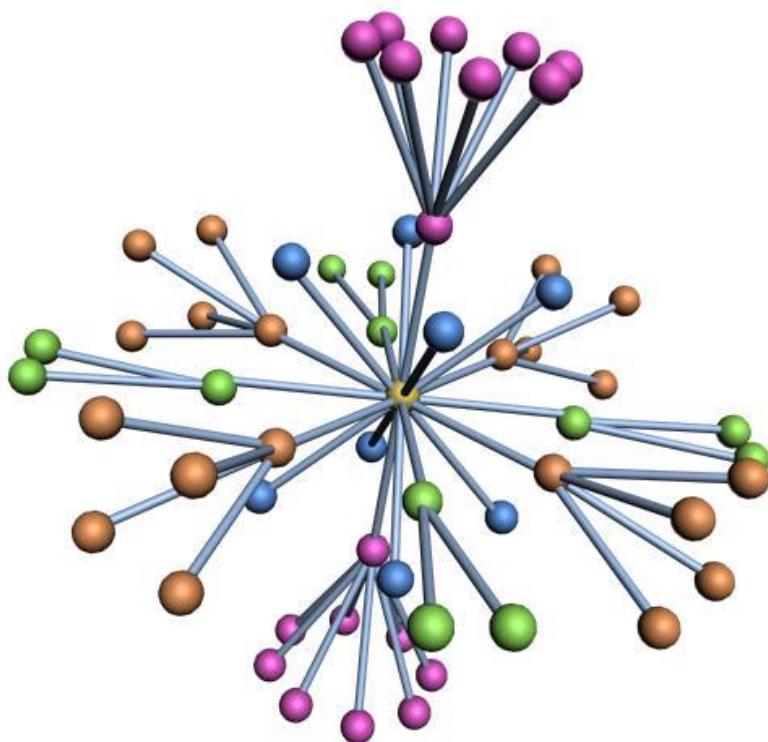
- 3D extension of the 2D layered tree drawing method.
- The extension to 3D does not necessarily means more information can be displayed
 - occlusion problem
 - interaction is essential



Robertson Plate 1

Polyplane

- [Hong Murtagh 04] 2.5D tree visualisation
 - Place subtrees on 2D planes
 - Divide & Conquer algorithm
 - Arrange these planes in 3D to reduce occlusion using Platonic solids



Phyllo Tree

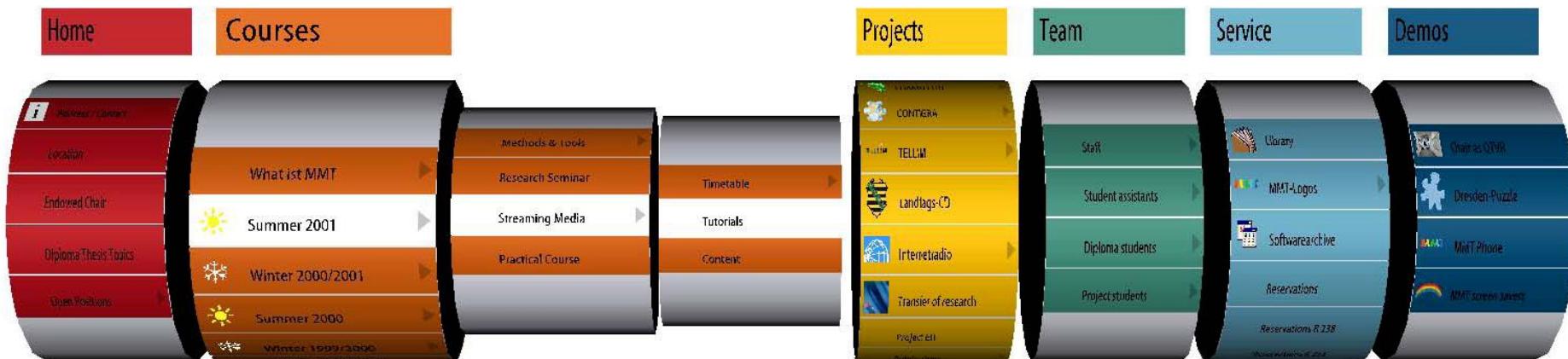
[Carpendale 06]

- use of nature's phyllotactic patterns
- optimal packing effect



Collapsible Cylindrical Tree

- Telescope metaphor: a set of nested cylinders
 - A cylinder is constructed for the children of a node with a smaller radius.
 - It can be pulled out to the right of the parent cylinder or collapsed.
- Only one path of the hierarchy is visible at once



Botanical Tree

- Resembles botanical trees [van Wijk]
 - The root is the tree stem.
 - Non-leave nodes are branches .
 - Leave nodes are “bulbs” at the end of branches.

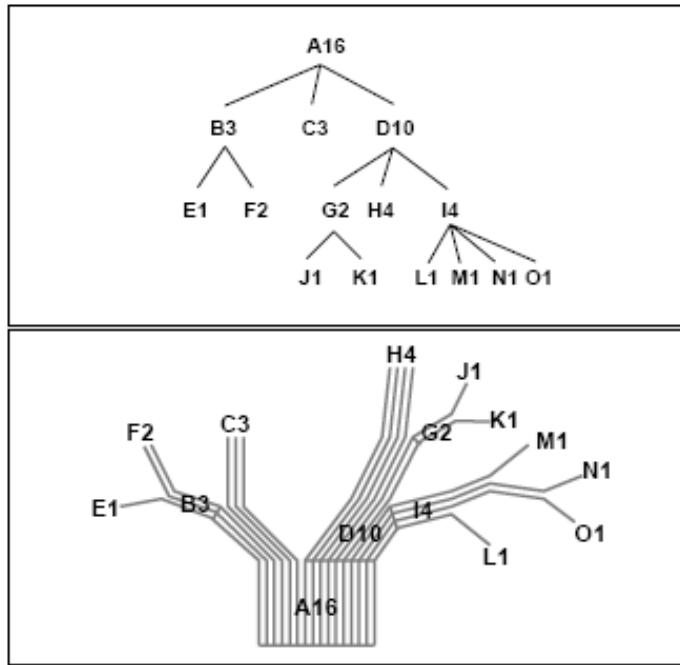
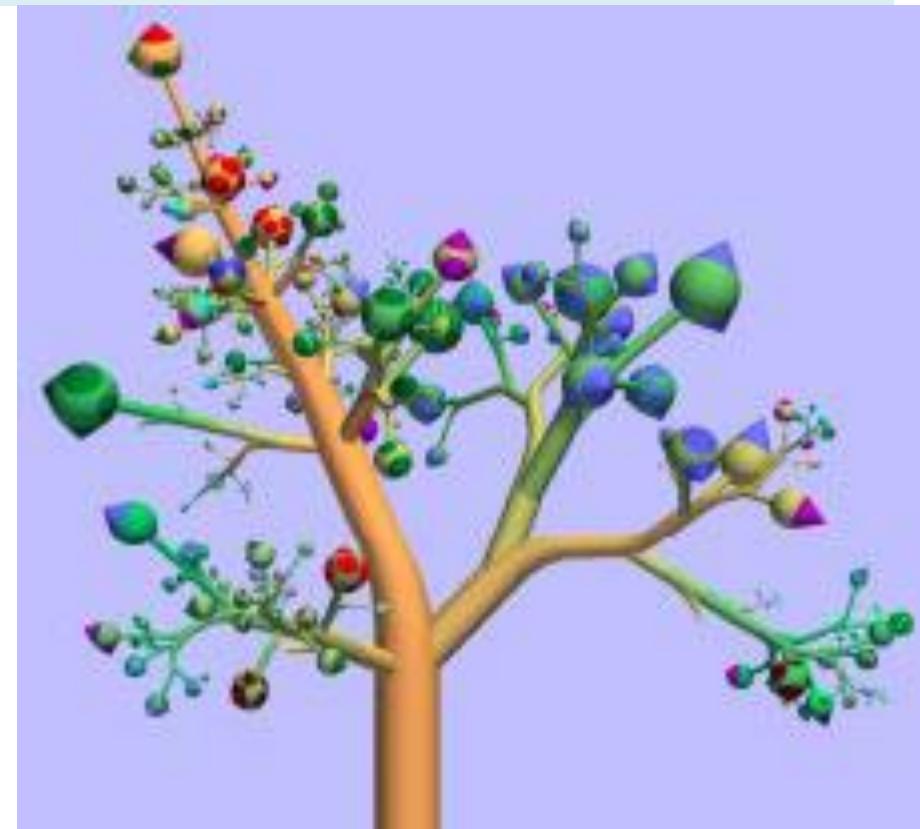


Figure 2. Node and link diagram (t) and corresponding strands model (d).



Summary: Tree Drawing/Visualization

- There are many variations based on
 - edge representation
 - criteria
- Divide and Conquer algorithm
- mostly run in linear time ($O(n)$ time)
- most popular methods
 - tidier tree drawing
 - treemap
 - dendrogram



A Visual Survey of Tree Visualization

<http://vcg.informatik.uni-rostock.de/~hs162/treeposter/poster.html>

Homework

- Tutorial/Homework
- Assumed Knowledge
basic terminology on graphs
 - degree
 - path
 - cycle
 - shortest path
- Reference:
“Graph Drawing: Algorithms for the Visualization of Graphs”
by Di Battista, Eades, Tamassia, Tollis
 - Week 2: chapter 3
 - Week 3: chapter 10
 - Week 4: chapter 9