Quantification of Beam and Core in Solar Wind Using Higher Order Moments.

Ramiz A Qudsi[1†], Bennett A. Maruca[1], Michael L. Stevens[2], Benjamin L. Alterman[3], Justin C. Kasper[3]

[1]Department of Physics and Astronomy, University of Delaware — [2]Harvard-Smithsonian Center for Astrophysics, Cambridge, MA — †ahmadr@udel.edu
[3]Climate and Space Sciences and Engineering, University of Michigan, Ann Arbor, MI

Abstract

Low collisionality and complex micro-physics processes in solar wind give rise to velocity distribution function (VDF) with multiple population for each species, most often a slow, dense and cold component called core and a faster, hotter and less dense component, beam.

Strength of two population contains important information about the solar wind and the relative abundance of two is conventionally measured using beam fraction, which is the ratio of beam number density to that of total number density.

However, calculation of beam fraction faces mainly two big challenges:

1. No clear distinction between core and beam.

2. Beam isn't always hotter and faster.

Since there is no clear distinction between the two populations, beam fraction often has high level of uncertainty, even though the total number density is very well know and has very small relative uncertainty.

We thus calculated two higher order moments of VDF (Skewness and Kurtosis) and explored the feasibility of using them to get better insight in the micro-physics of the solar wind.

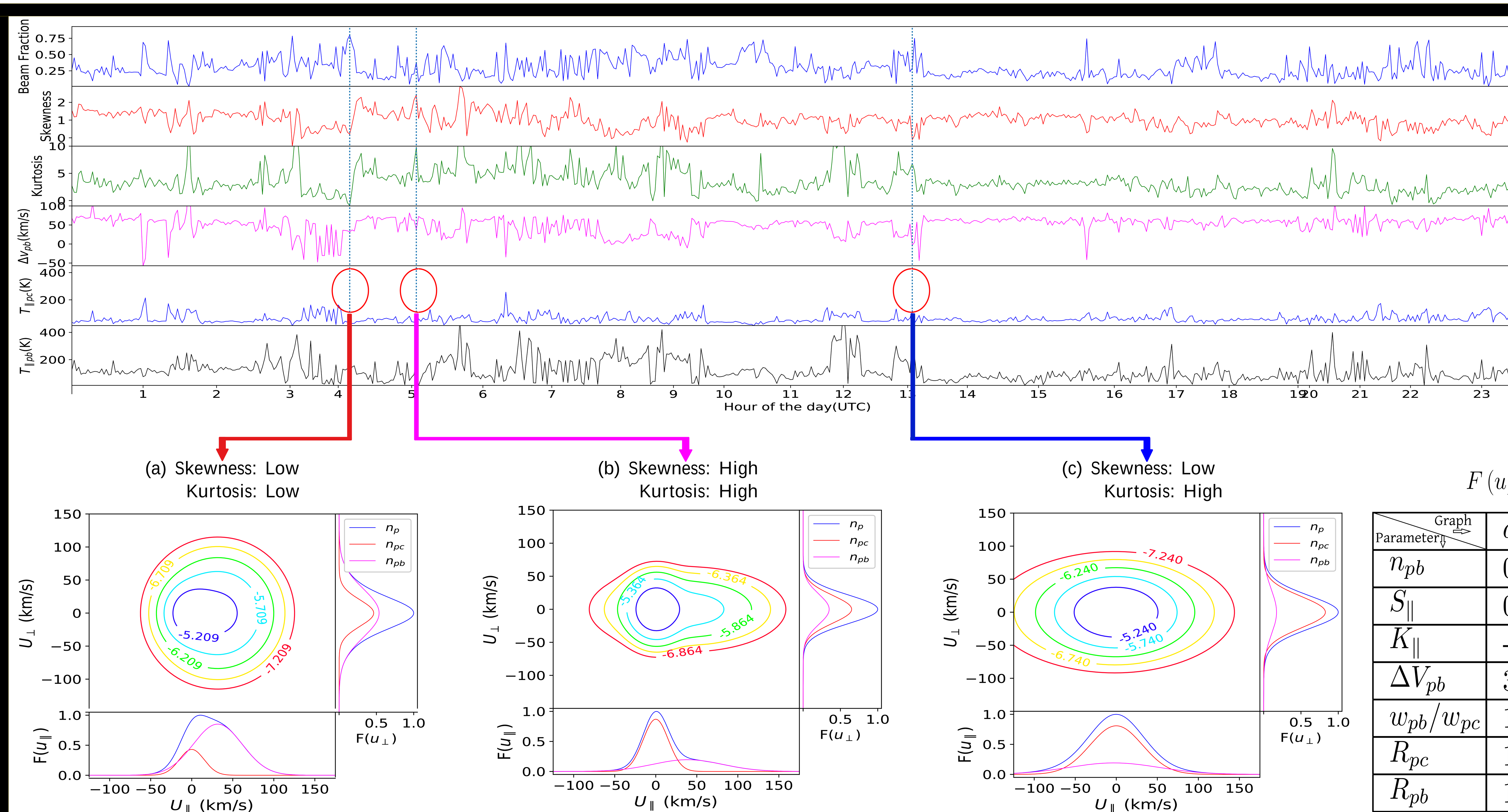


Figure shows the stack plot of six important parameters for 17th of January, 2006 (24 hr, data set 2).

Low (high) value of both skewness and kurtosis were defined as the value which is less than (more than) one standard deviation from the mean.

The line plots are the directional integrated VDF given by:

$$F(u_\perp) = \sum_j \int_{\forall u_\parallel} du_\parallel \, f_j^{(b)}(\vec{u})$$

Parameter \ Graph	a	b	c
n_{pb}	0.80	0.38	0.29
$S_\parallel$	0.20	2.42	-0.11
$K_\parallel$	-0.95	9.82	6.80
ΔV_{pb}	31.65	38.77	-2.94
w_{pb}/w_{pc}	1.98	2.76	1.74
R_{pc}	1.75	1.9	0.57
R_{pb}	1.51	0.29	0.37

Theoretical Background

To model the space plasma, we often use a drifting bi-Maxwellian VDF which has the following form:

$$f_j^{(b)}(\vec{u}) = \frac{n_j}{(2\pi)^{3/2} w_{j\parallel} w_{j\perp}^2} \exp\left(-\frac{(u_\parallel - v_{j\parallel})^2}{2w_{j\parallel}^2} - \frac{(u_{\perp 1} - v_{j\perp 1})^2}{2w_{j\perp}^2} - \frac{(u_{\perp 2} - v_{j\perp 2})^2}{2w_{j\perp}^2}\right)$$

Skewness is defined as the 3rd order moment of VDF:

$$S = \frac{1}{n}\left\langle \frac{|\vec{u}-\vec{v}|^3}{w^3} \right\rangle = \frac{1}{n}\int_{\forall \vec{u}} d^3\vec{u} \left(\frac{|\vec{u}-\vec{v}|}{w}\right)^3 \sum_j f_j^{(b)}(\vec{u})$$

Skewness tells us about the asymmetry of the VDF and after a fashion, is also a measure of heat flux through the system.

And the Kurtosis is defined as :

$$K = \frac{1}{n}\left\langle \frac{|\vec{u}-\vec{v}|^4}{w^4} \right\rangle = \frac{1}{n}\int_{\forall \vec{u}} d^3\vec{u} \left(\frac{|\vec{u}-\vec{v}|}{w}\right)^4 \sum_j f_j^{(b)}(\vec{u})$$

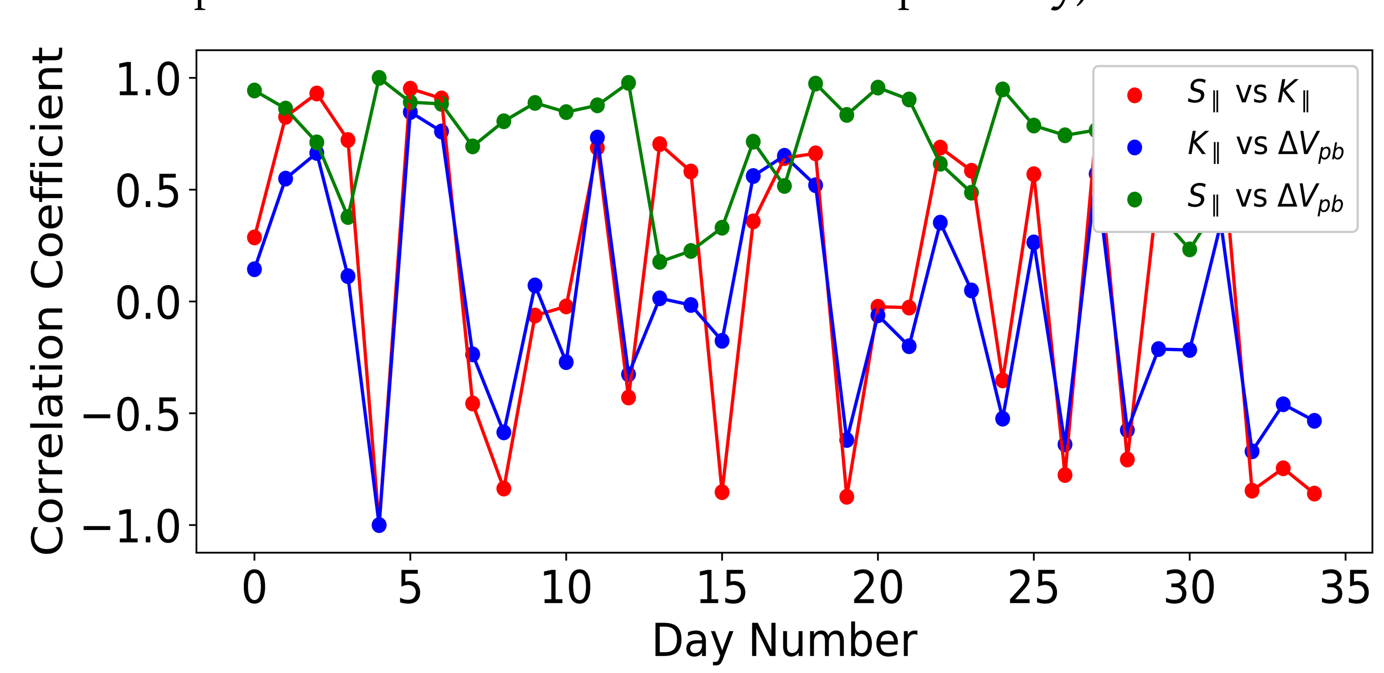
It is a measure of tailiness, or the prominence of tail, of a VDF, and is increasingly negative as the distribution becomes flatter.

In literature, owing to the fact that cumulants are extensive quantity, we often use excess kurtosis, instead of kurtosis, which is kurtosis minus three.

Statistical Analysis, Observations and Conclusion

We carried out similar analysis on two different data sets by fitting a drifting bi-Maxwellian VDF and calculating various parameters. Data set 1 was hourly data from 40 random days in a year, and data set 2 was for every spectra from one of those days for 24 hour period.

All the analysis, including downloading and curve fitting, were done using the software package Janus, which is a publicly available software package for processing in-situ measurement of solar wind ions.



Based on our analysis we found that the beam satisfied all conditions (hotter, faster and less dense) only 75% of times for data set 1 and 63% of times for data set 2. However, for both the data sets, it was almost always the faster component.

We observed that he value of skewness (S) and kurtosis (K) depends not only on beam fraction, but also on the drift velocity of the beam and the core beam temperature ratio.

Drift velocity appears to influence the skewness most and has very high correlation value (90% and 44% for data set 1 and 2 respectively). Both of which are considerably higher than correlation value among various other parameters.

The figure shown on the left side displays the variation in correlation value between proton beam drift velocity, skewness and kurtosis.

We observed that the value of skewness helps to understand the asymmetry and gives a measure of heatflux through the system.

Kurtosis on the other hand tells us about the tailiness of the distribution and is an indication of amount of free energy of the system.

Future Work

Calculation of skewness and kurtosis will help further in:

1. Analysis of turbulence.

2. Understanding wave particle interaction in the solar wind.

3. How the collisionality of the system evolve as a function of space and time.

It is our future plan to study those processes based on our understanding of these two parameters of proton VDF.

References and Acknowledgements

Marsch et al. (1982), Journal of Geophysical Research, 87, 5030

Balakrishnan et al (2012), Journal of Multivariate Analysis, 104, 73

Azzalini et al. (1999), Journal of the Royal Statistical Society, 61, 579

RAQ, BAM, and MLS are supported by NASA Awards NNX17AC72G and NNX17AH88G.