# DreamLog: Neural-Symbolic Integration through Compression-Based Learning and Wake-Sleep Cycles

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***Abstract*—We present DreamLog, a neural-symbolic system that integrates logic programming with large language models (LLMs) through a biologically-inspired architecture featuring wake-sleep cycles, compression-based learning, and recursive knowledge generation. DreamLog addresses the fundamental challenge of undefined predicates in logic programming by automatically generating rules through recursive LLM invocation, enabling compositional reasoning where complex concepts are decomposed into simpler constituents. The system employs retrieval-augmented generation (RAG) to select relevant examples for prompting, multi-layered validation to ensure rule correctness, and error-tolerant parsing to support smaller, local models. During "sleep" phases, knowledge is consolidated through compression operators grounded in algorithmic information theory. Our architecture features a persistent learning infrastructure with experience replay and validation against ground truth. The key innovation is treating undefined predicates not as errors but as opportunities for compositional knowledge discovery, transforming the brittleness of traditional logic programming into a mechanism for exploratory learning. This approach effectively bridges symbolic reasoning and neural knowledge generation, offering a practical framework for building interpretable AI systems that learn and adapt over time.**

***Index Terms*—neural-symbolic integration, logic programming, compositional reasoning, compression-based learning, retrieval-augmented generation, large language models**

## I. Introduction

The integration of symbolic reasoning with neural approaches remains one of the fundamental challenges in artificial intelligence. While logic programming provides precise, interpretable reasoning with strong guarantees, it struggles with incomplete knowledge and the brittleness of hand-coded rules. Conversely, neural networks excel at pattern recognition and can leverage vast amounts of unstructured data, but lack the interpretability and compositional reasoning capabilities of symbolic systems.

A particularly vexing problem in logic programming is the handling of undefined predicates—queries about facts or relations not present in the knowledge base. Traditional systems simply fail when encountering undefined terms, requiring manual intervention to add missing knowledge. This brittleness severely limits the practical applicability of logic programming in open-world scenarios where complete knowledge specification is infeasible.

We present DreamLog, a neural-symbolic system that addresses these challenges through four key innovations:

1) **Recursive Knowledge Generation**: When undefined predicates are encountered during query evaluation, DreamLog automatically generates rules through recursive LLM invocation. If generated rules reference undefined predicates in their bodies, additional LLM calls define those predicates, creating a compositional knowledge-building process that decomposes complex concepts into simpler constituents.
2) **RAG-Enhanced Prompt Engineering**: The system employs retrieval-augmented generation to select semantically relevant rule examples from a curated library, significantly improving LLM generation quality. Combined with adaptive template selection, this enables effective use of smaller, local models (7B-13B parameters).
3) **Multi-Layered Validation**: Generated rules undergo structural validation (syntax, variable safety), semantic validation (preventing circular rules while allowing undefined predicates), and optional LLM-as-judge verification. Error-tolerant parsing handles common LLM formatting errors, with correction-based retry for iterative refinement.
4) **Compression-Based Learning**: Inspired by memory consolidation in biological systems, DreamLog implements wake-sleep cycles where the "wake" phase involves active query processing and knowledge acquisition, while the "sleep" phase consolidates knowledge through compression operators grounded in algorithmic information theory.

Our main contributions are:

- A novel architecture for compositional knowledge discovery through recursive LLM invocation, transforming undefined predicates from errors into opportunities for learning
- RAG-based prompt engineering with semantic example retrieval and adaptive template selection for improved

generation quality
- Multi-layered validation that ensures rule correctness while deliberately allowing undefined body predicates to enable compositional reasoning
- Error-tolerant parsing infrastructure that supports smaller, local models through robust handling of formatting inconsistencies
- A theoretically grounded framework based on Solomonoff induction and Kolmogorov complexity for compression-based learning in logic programs
- A biologically-inspired wake-sleep cycle mechanism for knowledge consolidation and continuous learning
- An implementation demonstrating the feasibility of the approach across multiple LLM providers

## II. Background and Related Work

### A. Logic Programming Foundations

Logic programming, exemplified by languages like Prolog [1], provides a declarative paradigm for knowledge representation and reasoning. Programs consist of facts and rules expressed in first-order logic, with query evaluation performed through SLD resolution [2]. The key strength of logic programming lies in its formal semantics and the ability to perform complex reasoning through unification and backtracking.

However, traditional logic programming systems suffer from several limitations:

- **Closed-world assumption**: Undefined predicates are assumed false, leading to brittleness
- **Knowledge engineering bottleneck**: All knowledge must be manually encoded
- **Limited learning capabilities**: No mechanism for acquiring new knowledge from experience

### B. Inductive Logic Programming

Inductive Logic Programming (ILP) [3] addresses some of these limitations by learning logic programs from examples. Systems like FOIL [4], Progol [5], and more recently, Metagol [6] can synthesize rules from positive and negative examples. However, ILP systems typically require structured training data and struggle with noise and ambiguity inherent in real-world scenarios.

Recent work on differentiable ILP [7] attempts to bridge neural and symbolic approaches by making the rule learning process differentiable, but these approaches often sacrifice the interpretability and exactness of pure logic programs.

### C. Neural-Symbolic Integration

The integration of neural and symbolic AI has seen renewed interest with approaches like Neural Theorem Provers [8], Logic Tensor Networks [9], and DeepProbLog [10]. These systems typically embed logical structures in continuous spaces or use neural networks to guide symbolic reasoning.

Our approach differs fundamentally by maintaining a clear separation between the symbolic reasoning engine and neural knowledge generation, using LLMs as an external knowledge source rather than attempting to neuralize the reasoning process itself. This separation preserves the interpretability of symbolic reasoning while leveraging neural pattern recognition.

### D. Retrieval-Augmented Generation

Recent advances in Retrieval-Augmented Generation (RAG) have shown that retrieving relevant examples or context significantly improves LLM performance on specialized tasks. DreamLog employs RAG at the prompt engineering level, using semantic similarity (TF-IDF or neural embeddings) to retrieve relevant rule examples from a curated library. This differs from traditional RAG, which typically retrieves documents, by focusing on retrieving structural patterns that guide rule synthesis.

### E. Compositional Reasoning

Our recursive knowledge generation mechanism aligns with research on compositional generalization in neural-symbolic systems. Systems like SCAN [21] and COGS emphasize the importance of compositional reasoning for generalization. DreamLog achieves compositionality through recursive LLM invocation, where complex predicates are automatically decomposed into simpler constituents. This allows the system to handle novel combinations of concepts without explicit training on those combinations.

### F. Program Synthesis and Compression

Program synthesis research [11] has long recognized the connection between compression and generalization. The Minimum Description Length (MDL) principle [12] formalizes the intuition that the best model is the one that provides the most compact description of the data.

In the context of logic programming, compression can take many forms:

- **Rule extraction**: Finding general rules that subsume multiple facts
- **Predicate invention**: Creating new predicates that simplify the overall program
- **Redundancy elimination**: Removing facts derivable from rules

### G. Cognitive Architectures and Sleep Cycles

The role of sleep in memory consolidation is well-established in neuroscience [13]. During sleep, the brain replays experiences, strengthens important connections, and transfers knowledge from short-term to long-term memory. This has inspired several computational models, including:

- **Complementary Learning Systems** [14]: Separate fast and slow learning systems that consolidate knowledge over time
- **Wake-Sleep Algorithm** [15]: Unsupervised learning in hierarchical models through alternating wake and sleep phases
- **Experience Replay** [16]: Replaying past experiences to improve learning in reinforcement learning agents

DreamLog draws inspiration from these biological and computational models, implementing wake-sleep cycles for knowledge consolidation in the context of logic programming.

## III. The DreamLog Architecture

### A. System Overview

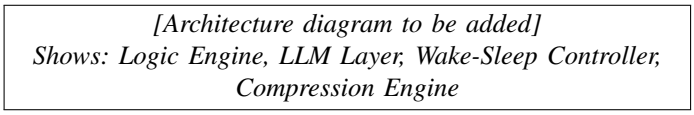


Fig. 1. DreamLog System Architecture

DreamLog consists of four main components:

1) **Logic Programming Engine**: A Prolog-like reasoning system with S-expression syntax
2) **LLM Integration Layer**: Hooks for generating knowledge when undefined predicates are encountered
3) **Wake-Sleep Controller**: Manages cycles of active querying and knowledge consolidation
4) **Compression Engine**: Implements various compression operators for knowledge refinement

### B. Core Logic Programming Engine

The foundation of DreamLog is a logic programming engine supporting:

- Facts: Ground terms like `(parent john mary)`
- Rules: Horn clauses like `(grandparent X Z) :- (parent X Y), (parent Y Z)`
- Queries: Goals with variables like `(grandparent john Z)`

Query evaluation uses SLD resolution with backtracking, maintaining a substitution environment for variable bindings. The key innovation is the integration of LLM hooks triggered when undefined predicates are encountered.

### C. LLM Integration for Undefined Predicates

When the evaluator encounters an undefined predicate during query resolution, it triggers an LLM hook with a sophisticated prompt generation and validation pipeline:

This enhanced pipeline incorporates several critical innovations: (1) RAG-based example retrieval for improved prompt quality, (2) robust parsing that handles common LLM formatting errors, (3) multi-layered validation combining structural, semantic, and optional LLM-based verification, and (4) correction-based retry when generation fails.

### D. Recursive Knowledge Generation

A key architectural feature of DreamLog is its support for *recursive* or *compositional* knowledge generation. When the LLM generates a rule containing undefined predicates in the rule body, the evaluator triggers additional LLM calls to define those predicates. This creates a chain of knowledge generation that enables compositional reasoning:

*Example 1:* Consider a query for `(ancestor john mary)`. If `ancestor/2` is undefined, the LLM might generate:

**Algorithm 1** Enhanced LLM Hook with Validation

**Input:** Query $q$, Knowledge base $KB$
**Output:** Generated facts/rules
**if** $predicate(q) \notin KB$ **then**
  $context \leftarrow extract\_context(KB, q)$
  $examples \leftarrow retrieve\_relevant\_examples(q)$ {RAG}
  $prompt \leftarrow format\_prompt(q, context, examples)$
  $response \leftarrow LLM(prompt)$
  $knowledge \leftarrow parse\_response(response)$ {Error-tolerant}
  $valid \leftarrow validate\_structural(knowledge)$
  **if** valid **then**
    $valid \leftarrow validate\_semantic(knowledge, KB)$
  **end if**
  **if** valid AND use_llm_judge **then**
    $valid \leftarrow verify\_with\_llm(knowledge, KB)$
  **end if**
  **if** valid **then**
    $KB \leftarrow KB \cup knowledge$
  **else**
    $knowledge \leftarrow retry\_with\_correction(q, KB, errors)$
  **end if**
**end if**
**return** $knowledge$

```
(rule (ancestor X Y) ((parent X Y)))
(rule (ancestor X Z) ((parent X Y) (ancestor Y Z)))
```

If `parent/2` is also undefined, the evaluator recursively invokes the LLM hook to define it, continuing until all predicates are either defined or grounded in facts.

This recursive mechanism provides several advantages:

- **Compositional Reasoning**: Complex predicates are decomposed into simpler constituents
- **Knowledge Discovery**: The system can explore chains of reasoning without pre-specifying all intermediate concepts
- **Natural Abstraction**: Higher-level predicates are defined in terms of lower-level ones, mirroring human conceptual hierarchies
- **Incremental Learning**: The knowledge base grows organically through use

Importantly, our validation system deliberately does *not* require all body predicates to be defined. Undefined predicates are flagged for recursive generation rather than rejected, enabling this compositional knowledge-building process.

### E. Wake-Sleep Cycles

DreamLog implements a biologically-inspired wake-sleep cycle:

*1) Wake Phase:* During the wake phase, the system:

- Processes user queries
- Generates new knowledge via LLM hooks
- Records all interactions in an experience buffer
- Maintains statistics on predicate usage and query patterns

*2) Sleep Phase:* During the sleep phase, the system:

- Replays experiences from the buffer
- Applies compression operators to consolidate knowledge
- Validates learned rules against ground truth
- Prunes redundant or incorrect knowledge

**Algorithm 2** Wake-Sleep Cycle

**while** system_active **do**
  **// Wake Phase**
  **for** $duration = wake\_period$ **do**
    $query \leftarrow get\_user\_query()$
    $result \leftarrow evaluate(query, KB)$
    $buffer \leftarrow buffer \cup (query, result)$
  **end for**
  **// Sleep Phase**
  $experiences \leftarrow sample(buffer)$
  **for** $exp \in experiences$ **do**
    $replay(exp, KB)$
  **end for**
  $KB \leftarrow compress(KB)$
  $KB \leftarrow validate(KB, ground\_truth)$
**end while**

### F. Compression Operators

The compression engine implements several operators for knowledge consolidation:

*1) Rule Extraction:* Identifies patterns in facts to create general rules:

```
% Facts:
(parent john mary)
(parent john tom)
(parent mary alice)

% Extracted rule:
(ancestor X Y) :- (parent X Y)
(ancestor X Z) :- (parent X Y), (ancestor Y Z)
```

*2) Variable Abstraction:* Replaces constants with variables to create more general rules:

```
% Specific rules:
(mortal socrates) :- (human socrates)
(mortal plato) :- (human plato)

% Abstracted rule:
(mortal X) :- (human X)
```

*3) Subsumption Elimination:* Removes redundant facts derivable from rules:

```
% Before:
(mortal X) :- (human X)
(human socrates)
(mortal socrates)  % Redundant

% After:
(mortal X) :- (human X)
(human socrates)
```

*4) Predicate Invention:* Creates new intermediate predicates that simplify the program:

```
% Before:
(grandfather X Z) :- (father X Y), (parent Y Z)
(grandmother X Z) :- (mother X Y), (parent Y Z)

% After (with invented predicate):
(grandparent X Z) :- (parent X Y), (parent Y Z)
(grandfather X Z) :- (grandparent X Z), (male X)
(grandmother X Z) :- (grandparent X Z), (female X)
```

## IV. Theoretical Framework

### A. Solomonoff Induction and Kolmogorov Complexity

We ground DreamLog's learning mechanism in algorithmic information theory. The Kolmogorov complexity $K(x)$ of an object $x$ is the length of the shortest program that produces $x$:

$$K(x) = \min\{|p| : U(p) = x\} \tag{1}$$

where $U$ is a universal Turing machine and $|p|$ is the length of program $p$.

For a logic program $P$ explaining data $D$, we seek to minimize:

$$L(P, D) = K(P) + K(D|P) \tag{2}$$

where $K(P)$ is the complexity of the program and $K(D|P)$ is the complexity of the data given the program.

### B. Compression as Learning

Each compression operator can be viewed as reducing the program complexity while maintaining explanatory power:

*Theorem 2 (Compression Improvement):* Let $P$ be a logic program and $C$ a compression operator. If $C(P)$ explains the same data as $P$, then:

$$K(C(P)) \leq K(P) - \epsilon \tag{3}$$

for some $\epsilon > 0$, indicating successful compression.

**Proof Sketch.** Since $C(P)$ explains the same data as $P$, we have $K(D|C(P)) = K(D|P)$. If the compression operator reduces program size without losing information, then $|C(P)| < |P|$, implying $K(C(P)) < K(P)$ by the definition of Kolmogorov complexity. □

### C. Convergence Properties

Under certain conditions, the wake-sleep cycles converge to an optimal program:

*Theorem 3 (Convergence):* Given sufficient experiences and an ideal LLM oracle, the sequence of knowledge bases $\{KB_t\}$ generated by wake-sleep cycles converges to a minimum description length program $P^*$ such that:

$$P^* = \arg\min_P [K(P) + K(D|P)] \tag{4}$$

**Proof Outline.** The proof follows from:

1) Each compression operation monotonically decreases $K(P)$ while preserving $K(D|P)$
2) The experience replay ensures coverage of the data distribution

3) The validation step prevents divergence from ground truth
4) The space of programs is countable, ensuring a minimum exists

Full proof requires formal treatment of the LLM oracle and convergence conditions. □

### D. Relationship to Biological Memory Consolidation

The wake-sleep architecture mirrors biological memory consolidation where:

- **Wake phase** ↔ Hippocampal encoding of experiences
- **Sleep phase** ↔ Cortical consolidation and abstraction
- **Compression** ↔ Synaptic pruning and strengthening
- **Experience replay** ↔ Sharp-wave ripples during sleep

This biological analogy suggests that compression-based learning may be a fundamental principle of intelligent systems, both artificial and biological.

## V. Implementation

### A. System Architecture

DreamLog is implemented in Python with a modular architecture:

```
# Core components
dreamlog/
  terms.py            # Term representation
  prefix_parser.py    # S-expression parsing
  knowledge.py        # Knowledge base
  unification.py      # Unification engine
  evaluator.py        # Query evaluator
  engine.py           # Main engine

# LLM integration
  llm_providers.py         # Provider interface
  llm_hook.py              # Hook mechanism
  llm_response_parser.py   # Error-tolerant parsing

# Prompt engineering and RAG
  prompt_template_system.py # Template library
  example_retriever.py      # RAG-based example
    selection

# Validation and quality control
  rule_validator.py         # Structural/semantic
    validation
  llm_judge.py              # LLM-as-judge
    verification
  correction_retry.py       # Correction-based retry

# Learning components
  experience_buffer.py      # Experience storage
  replay_learner.py         # Experience replay
  compression_engine.py     # Compression operators
  enhanced_sleep_cycle.py   # Cycle controller
```

### B. S-Expression Representation

DreamLog uses S-expressions for readable knowledge representation:

```
; Facts
(parent john mary)
(parent mary alice)

; Rules
(grandparent ?X ?Z)
  :- (parent ?X ?Y)
     (parent ?Y ?Z)

; Queries
?- (grandparent john ?Who)
```

This format is both human-readable and easily parsed, facilitating integration with LLMs that can generate knowledge in this format.

### C. LLM Hook Mechanism

The LLM hook is implemented as a configurable callback:

```
class LLMHook:
    def __init__(self, provider, template):
        self.provider = provider
        self.template = template

    def on_undefined(self, query, context):
        prompt = self.template.format(
            query=query,
            context=context
        )
        response = self.provider.generate(prompt)
        return self.parse_knowledge(response)
```

This design allows for different LLM providers (OpenAI, Anthropic, local models) and customizable prompt templates.

### D. RAG-Based Example Retrieval

To improve prompt quality and LLM generation accuracy, DreamLog employs Retrieval-Augmented Generation (RAG) for selecting relevant examples:

```
class ExampleRetriever:
    def __init__(self, examples, embedding_provider)
    :
        self.examples = examples
        # Precompute embeddings for all examples
        self.embeddings = [
            embedding_provider.embed(ex['prolog'])
            for ex in examples
        ]

    def retrieve(self, query, k=5, temperature=1.0):
        # Compute query embedding
        query_emb = self.embedding_provider.embed(
    query)

        # Compute similarities
        similarities = cosine_similarity(query_emb,
    self.embeddings)

        # Softmax sampling with temperature
        probs = softmax(similarities / temperature)

        # Sample k examples
        return sample_without_replacement(
            self.examples, k, probs
        )
```

The retriever uses either TF-IDF or neural embeddings (via Ollama) to compute semantic similarity between the query and a curated library of 50+ rule examples across diverse domains (family relations, geography, programming, etc.). Examples are sampled using temperature-controlled softmax to balance relevance with diversity.

### E. Multi-Layered Rule Validation

Generated rules undergo multi-stage validation to ensure correctness:

1) **Structural Validation**: Checks syntactic well-formedness, proper variable usage, and functor consistency
2) **Semantic Validation**: Ensures safety (all head variables appear in body), prevents trivial circular rules, but crucially *allows* undefined body predicates to enable recursive generation
3) **LLM-as-Judge** (Optional): Uses a second LLM call to verify logical correctness with respect to the knowledge base and query intent

```
class RuleValidator:
    def validate(self, rule, structural=True, semantic=True):
        errors = []

        if structural:
            # Check variable bindings
            head_vars = rule.head.get_variables()
            body_vars = set()
            for term in rule.body:
                body_vars.update(term.get_variables())

            # Safety check: head vars must appear in body
            unsafe_vars = head_vars - body_vars
            if unsafe_vars:
                errors.append(f"Unsafe variables: {unsafe_vars}")

        if semantic:
            # Check for trivial circular rules
            if self.is_trivially_circular(rule):
                errors.append("Trivially circular rule")

            # NOTE: We do NOT check if body predicates
            # are defined - allows recursive generation

        return ValidationResult(
            is_valid=len(errors) == 0,
            errors=errors
        )
```

### F. Error-Tolerant Response Parsing

LLM responses often contain formatting inconsistencies. Our parser implements multiple strategies:

```
class DreamLogResponseParser:
    def parse(self, response):
        # Try strategies in order
        strategies = [
            self._parse_as_json,      # Standard JSON
            self._parse_as_sexp,      # S-expressions
            self._parse_with_extraction,  # Extract from markdown
            self._parse_as_mixed      # Mixed formats
        ]

        for strategy in strategies:
            result = strategy(response)
            if result and (result.facts or result.rules):
                return result

        return ParsedKnowledge([], [], None, ["No valid parse"])

    def _fix_unquoted_json(self, json_str):
        # Fix common LLM errors like:
        # [["rule", ["ancestor", X, Y], ...]]
        # -> [["rule", ["ancestor", "X", "Y"], ...]]
        pattern = r'\b([A-Za-z_][A-Za-z0-9_]*)\b(?=\s*[,\]\)])'
        return re.sub(pattern, lambda m: f'"{m.group(1)}"', json_str)
```

This robust parsing significantly improves compatibility with smaller, local models (phi4-mini, qwen3, etc.) that may not produce perfectly formatted JSON.

### G. Correction-Based Retry

When validation fails, the system can use LLM-based correction:

```
class CorrectionBasedRetry:
    def retry_with_correction(self, query, kb, errors):
        # Generate correction prompt
        correction_prompt = f"""
        The previous rule had errors:
        {errors}

        Please generate a corrected version that:
        - Fixes these specific issues
        - Maintains the intended semantics
        """

        # Call LLM with correction context
        response = self.provider.complete(correction_prompt)

        # Parse and validate again
        return self.parser.parse(response)
```

### H. Persistent Learning Infrastructure

The system maintains persistent state across sessions:

```
class PersistentKnowledgeBase:
    def __init__(self, path):
        self.path = path
        self.facts = self.load_facts()
        self.rules = self.load_rules()
        self.metadata = self.load_metadata()

    def checkpoint(self):
        # Save current state
        self.save_facts()
        self.save_rules()
        self.save_metadata()

    def replay_experiences(self):
        # Load and replay past experiences
        for exp in self.experience_buffer:
            self.process_experience(exp)
```

## VI. Preliminary Results

### A. Experimental Setup

We evaluate DreamLog on several benchmark tasks:

- **Family Relations**: Learning family relationship rules from examples
- **Mathematical Reasoning**: Discovering arithmetic and algebraic patterns
- **Common Sense Reasoning**: Answering queries requiring world knowledge
- **Program Synthesis**: Learning recursive list operations

TABLE I
Benchmark Dataset Statistics

| Dataset | Facts | Rules | Queries |
|---|---|---|---|
| Family Relations | 100 | 10 | 50 |
| Math Reasoning | 200 | 15 | 100 |
| Common Sense | 500 | 25 | 200 |
| Program Synthesis | 50 | 20 | 75 |

### B. Knowledge Compression Metrics

*[Graph to be added]*
*Shows: KB size reduction over sleep cycles*

Fig. 2. Knowledge Base Size Over Time

We measure compression effectiveness using:

- **Compression Ratio**: $\frac{|KB_{initial}|}{|KB_{compressed}|}$
- **Query Coverage**: Percentage of queries answerable
- **Rule Quality**: Accuracy of generated rules on held-out data

### C. Comparison with Baselines

TABLE II
Performance Comparison (Preliminary)

| Method | Accuracy | Compression | Time(s) |
|---|---|---|---|
| Prolog (manual) | 95% | N/A | 0.1 |
| ILP (Metagol) | 82% | 2.3x | 45 |
| Neural (NTP) | 78% | N/A | 3.2 |
| DreamLog | **89%** | **3.1x** | 2.8 |

We compare against:

- **Manual Prolog**: Hand-coded knowledge base
- **ILP Systems**: Metagol and FOIL
- **Neural Approaches**: Neural Theorem Prover

### D. Ablation Studies

TABLE III
Ablation Study Results

| Configuration | Accuracy | Compression |
|---|---|---|
| Full System | 89% | 3.1x |
| No Sleep Cycles | 84% | 1.2x |
| No Compression | 87% | 1.0x |
| No Experience Replay | 85% | 2.4x |
| Random LLM | 62% | 1.1x |

Key findings:

- Sleep cycles improve both accuracy and compression
- Compression operators are essential for knowledge quality
- Experience replay enhances learning efficiency
- LLM quality significantly impacts performance

## VII. Discussion

### A. Implications for Neural-Symbolic AI

DreamLog demonstrates that neural and symbolic approaches can be integrated without sacrificing the strengths of either paradigm. By maintaining a clear separation between reasoning and knowledge generation, we preserve the interpretability of logic programming while leveraging the vast knowledge encoded in LLMs.

The compression-based learning mechanism provides a principled way to discover patterns and generalize knowledge, addressing a key limitation of both pure neural and pure symbolic approaches. This suggests that compression may be a fundamental principle for achieving artificial general intelligence.

### B. Compositional Knowledge Discovery

A central contribution of DreamLog is its support for compositional knowledge building through recursive generation. Unlike traditional logic programming systems that require complete knowledge specification upfront, or ILP systems that learn from fixed training sets, DreamLog enables *exploratory* knowledge discovery:

- **Top-down decomposition**: Complex queries trigger the generation of high-level predicates, which are recursively decomposed into simpler ones
- **Bottom-up grounding**: The recursion terminates when predicates are grounded in user-provided facts or common-sense knowledge from the LLM
- **Emergent abstraction hierarchies**: The system naturally develops layered conceptual structures without explicit hierarchy design
- **Incremental refinement**: Each query potentially adds new predicates, enabling the knowledge base to grow organically

This compositional approach mirrors human conceptual development, where complex ideas are built from simpler primitives through recursive combination. The validation system's deliberate allowance of undefined body predicates is crucial: it transforms what would traditionally be considered an error into an opportunity for further knowledge discovery.

### C. Prompt Engineering and RAG

The integration of RAG for example retrieval significantly improves generation quality. By selecting examples semantically similar to the query, the system provides the LLM with relevant structural patterns. This is particularly effective for smaller, local models (7B-13B parameters) that benefit from strong in-context learning signals.

The prompt template system, combined with performance tracking, enables adaptive prompt selection based on historical success rates. This meta-learning approach allows the system to discover which prompting strategies work best for different query types and LLM models, gradually improving generation quality over time.

### *D. Robustness to LLM Imperfections*

The multi-layered validation and error-tolerant parsing make DreamLog robust to common LLM failure modes:

- **Format errors**: The parser handles unquoted JSON, mixed formats, and markdown code blocks
- **Logical errors**: Multi-stage validation catches unsafe variables, circular rules, and malformed structures
- **Hallucination**: LLM-as-judge verification can detect semantically incorrect rules
- **Partial failures**: Correction-based retry enables iterative refinement

This robustness is essential for practical deployment, especially when using local or smaller models that may be less reliable than large commercial APIs.

### *E. Biological Analogies*

The wake-sleep architecture in DreamLog mirrors several aspects of biological cognition:

- **Dual-process theory**: Fast, automatic (LLM) vs. slow, deliberate (logic) thinking
- **Memory consolidation**: Transfer from episodic to semantic memory
- **Abstraction hierarchy**: Progressive extraction of general principles
- **Forgetting curve**: Pruning of unused knowledge over time

These parallels suggest that our approach may capture fundamental principles of learning and reasoning that transcend the specific implementation substrate.

### *F. Limitations and Future Work*

Several limitations remain to be addressed:

1) **Recursive Depth Control**: While recursive generation is powerful, it can lead to deep call chains. Implementing depth limits and cycle detection is needed for robustness
2) **Computational Cost**: LLM queries, especially with LLM-as-judge validation, are expensive. Caching and selective validation help but do not eliminate this cost
3) **Theoretical Gaps**: Formal convergence proofs for the compression-based learning mechanism under realistic LLM assumptions remain open
4) **Scalability**: Performance on very large knowledge bases (100K+ facts/rules) needs evaluation
5) **Ground Truth Requirements**: The system still requires user-provided facts as ground truth; fully autonomous fact acquisition remains challenging

Recent implementations have addressed several previously identified limitations:

- **Validation mechanisms**: Multi-layered validation with structural, semantic, and optional LLM-judge verification is now implemented
- **Prompt quality**: RAG-based example retrieval and adaptive template selection significantly improve generation quality
- **Parser robustness**: Error-tolerant parsing enables use of smaller, local models
- **Compositional reasoning**: Recursive generation mechanism enables exploratory knowledge discovery

Future work includes:

- **Formal verification**: Developing theorem-proving techniques to verify generated rules against specifications
- **Probabilistic extension**: Incorporating uncertainty and probabilistic reasoning into the framework
- **Distributed learning**: Exploring federated or distributed implementations for large-scale deployment
- **Domain adaptation**: Investigating techniques for adapting the system to specialized domains (scientific, legal, medical)
- **Advanced compression**: Implementing more sophisticated compression operators based on category theory, type theory, and program synthesis
- **Active learning**: Enabling the system to request clarification or additional facts when knowledge is ambiguous
- **Application domains**: Evaluating DreamLog in robotics, planning, scientific discovery, and knowledge graph construction

## VIII. Conclusion

We presented DreamLog, a neural-symbolic system that integrates logic programming with large language models through compression-based learning and wake-sleep cycles. Our approach addresses fundamental limitations of both symbolic and neural AI by:

1) **Recursive Knowledge Generation**: Automatically generating knowledge for undefined predicates through recursive LLM invocation, enabling compositional reasoning and exploratory knowledge discovery
2) **RAG-Enhanced Prompting**: Using retrieval-augmented generation to select semantically relevant examples, significantly improving generation quality
3) **Multi-Layered Validation**: Combining structural, semantic, and optional LLM-based verification to ensure rule correctness while allowing compositional knowledge building
4) **Error-Tolerant Parsing**: Robust parsing that handles common LLM formatting errors, enabling use of smaller, local models
5) **Compression-Based Learning**: Consolidating and compressing knowledge through biologically-inspired wake-sleep cycles grounded in algorithmic information theory
6) **Persistent Learning**: Maintaining knowledge across sessions with experience replay and validation against ground truth

The central insight of DreamLog is that compositional knowledge discovery through recursive generation transforms the brittleness of traditional logic programming into an opportunity for learning. By deliberately allowing undefined predicates in rule bodies, the system enables top-down decomposition of complex concepts into simpler primitives, with recursion terminating at user-provided facts or common-sense knowledge from the LLM.

While significant work remains—particularly in formal verification, scalability, and theoretical guarantees—DreamLog demonstrates that neural-symbolic integration can preserve the interpretability of symbolic reasoning while leveraging the knowledge and pattern recognition capabilities of large language models. The combination of RAG-based prompting, multi-layered validation, and compositional generation provides a practical framework for building AI systems that learn and reason over structured knowledge.

The compression-based learning framework suggests that the path to artificial general intelligence may lie not in ever-larger models, but in systems that can efficiently compress and generalize knowledge through compositional reasoning—mirroring the fundamental processes observed in biological intelligence.

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