1. (8pt) True or False:
(a) You have performed a Ljung-Box test on the residuals to determine if an ARMA(1,1) model is suitable for a dataset. The p-value = 0.329. You should conclude the model is suitable.
(b) You have performed the augmented Dickey-Fuller unit test to determine if a series needs to be differenced or not. The p-value = 0.004. You should conclude the time series needs to be differenced.
(c) The width of $l$-step head prediction intervals from fitting an AR(1) model increases as $l$ increases.
(d) $\{Y_t\}$ follows a non-stationary ARIMA$(p, d, q)$ process. Then $\{\nabla Y_t\}$ must be stationary.

2. (8pt) Consider a linear trend process $Y_t = \beta_0 + \beta_1 t + e_t$, where $\{e_t\}$ is a 0 mean white noise process with variance $\sigma^2$. Let $\tilde{Y}_T$ be the simple exponential smoother, i.e.

$$\tilde{Y}_T = (1-\theta)\sum_{t=0}^{\infty}\theta^t Y_{T-t}.$$

Show that the simple exponential smoother is a biased estimator for the linear trend process by calculating $\mathrm{E}(Y_T) - \mathrm{E}(\tilde{Y}_T)$.

3. (10pt) Consider the model $Y_t = \beta_1 t + X_t$.

(a) $\{X_t\}$ is a zero 0 white noise process with $\mathrm{Var}(X_t) = \sigma^2$. Find the least square estimator of $\beta_1$.
(b) Suppose $\{X_t\}$ is a process of the form $X_t = X_{t-1} + e_t - \theta e_{t-1}$. Derive the ACF for $\nabla Y_t$ and show that $\{\nabla Y_t\}$ is stationary. What is the name of the process identified by $\{\nabla Y_t\}$?

4. (12pt) Consider the following models where $\{e_t\}$ is a 0 mean white noise process with variance $\sigma^2$.
i. $Y_t = 1.9Y_{t-1} - 0.9Y_{t-2} + e_t - 0.5e_{t-1}$
ii. $Y_t = 0.5Y_{t-1} + e_t - 0.2e_{t-1} - 0.15e_{t-2}$

(a) Write each model above using backshift notation.
(b) Characterize these models as models in the ARMA$(p, d, q)$ family, that is, identify $p$, $d$ and $q$.
(c) Determine if each model corresponds to a stationary process or not.

5.(12pt) Suppose that $\{Y_t\}$ is a seasonal ARIMA process in the form

$$Y_t = (1-\theta B)(1-\Theta B^4)e_t$$

where $\{e_t\}$ is a zero mean white noise process with variance $\sigma^2$.

(a) Derive expressions for $\mathrm{E}(Y_t)$ and $\mathrm{Var}(Y_t)$.
(b) Derive the autocovariance function, that is, calculate $\mathrm{Cov}(Y_t, Y_{t-k})$, for $k = 1, 2, \cdots$
(c) Characterize this models as SARIMA$(p, d, q) \times (P, D, Q)_s$, that is, identify $p$, $d$ and $q$, $P$, $D$ and $Q$, and $s$.