Time Series Analysis - STAT 478 - Final Exam - Part 1 Q4

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Part 1: Problem 4

Consider the following models where $\{e_t\}$ is a 0 mean white noise process with variance σ^2.

i $Y_t = 1.9Y_{t-1} - 0.9Y_{t-2} + e_t - 0.5e_{t-1}$

ii $Y_t = 0.5Y_{t-1} + e_t - 0.2e_{t-1} - 0.15e_{t-2}$

Part (a)

Write each model above using backshift notation.

(i) $(1 - 1.9\,\mathrm{B} + 0.9\,\mathrm{B}^2)Y_t = (1 - 0.5\,\mathrm{B})e_t$.

Working on both sides of the equation simultaneously, we use the following sequence of transformations to arrive at a canonical form.

$$
\begin{aligned}
(1 - 1.9\,\mathrm{B} + 0.9\,\mathrm{B}^2)Y_t &= (1 - 1/2\,\mathrm{B})e_t \\
1/10(10 - 19\,\mathrm{B} + 9\,\mathrm{B}^2)Y_t &= (1 - 1/2\,\mathrm{B})e_t \\
1/10(1 - \mathrm{B})(10 - 9\,\mathrm{B})Y_t &= (1 - 1/2\,\mathrm{B})e_t \\
1/100(1 - \mathrm{B})(1 - 9/10\,\mathrm{B})Y_t &= (1 - 1/2\,\mathrm{B})e_t \\
(1 - \mathrm{B})(1 - 9/10\,\mathrm{B})Y_t &= (1 - 1/2\,\mathrm{B})100e_t
\end{aligned}
$$

We let $\eta_t = 10^2 e_t$, which is white noise with a mean 0 and variance $10^4\sigma^2$, thus

$$(1 - \mathrm{B})(1 - 9/10\,\mathrm{B})Y_t = (1 - 1/2\,\mathrm{B})\eta_t.$$

(ii) $(1 - 0.5\,\mathrm{B})Y_t = (1 - 0.2\,\mathrm{B} - 0.15\,\mathrm{B}^2)e_t$. Working on both sides of the equation simultaneously, we use the following sequence of transformations to arrive at a canonical form.

$$
\begin{aligned}
(1 - 0.5\,\mathrm{B})Y_t &= (1 - 0.2\,\mathrm{B} - 0.15\,\mathrm{B}^2)e_t \\
(1 - 1/2\,\mathrm{B})Y_t &= (1 - 1/5\,\mathrm{B} - 3/20\,\mathrm{B}^2)e_t \\
(1 - 1/2\,\mathrm{B})Y_t &= (1 - 1/2\,\mathrm{B})(1 + 3/10)e_t \\
Y_t &= (1 + 3/10)e_t
\end{aligned}
$$

Part (b)

Characterize these models as models in the ARIMA(p, d, q) family, that is, identify p, d and q.

(i) By the form, $\{Y_t\}$ is an ARIMA$(1, 1, 1)$ process, or equivalently, $\{\nabla Y_t\}$ is an ARMA$(1, 1)$ process.

(ii) By the form, $\{Y_t\}$ is an ARIMA$(0, 0, 1)$ process, or equivalently, MA(1) process.

Part (c)

$\boxed{\text{Determine if each model corresponds to a stationary process or not.}}$

(i) $\{Y_t\}$ is an ARIMA$(1, 1, 1)$ process, which means that its characteristic function has a unit root. Recall that ARIMA$(1, 1, 1)$ denotes a $\{Y_t\}$ is therefore a non-stationary process.[1]

(ii) $\{Y_t\}$ is an MA(1) process, which are necessarily stationary.

[1] $\nabla\{Y_t\}$ is a stationary ARMA$(1, 1)$ process since it does not have a unit root.