Stat 581, Problem Set #8 Solutions

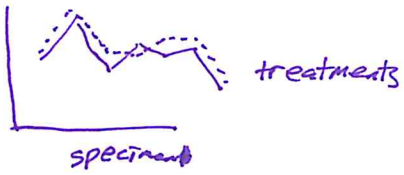
(1.) treatment = device ($a=2$), block = specimen ($b=10$), response = hardness measurement ($n=3$)

(a) $$Y_{ijk} = \mu + \tau_i + \beta_j + (\tau\beta)_{ij} + \varepsilon_{ijk} \begin{cases} i = 1,\dots,a \\ j = 1,\dots,b \\ k = 1,\dots,n \end{cases}$$

Fixed effect parameters: $\tau_1,\dots,\tau_a$ $\left(\sum_1^a \tau_i = 0\right)$

random effect parameters: σ^2_β, $\sigma^2_{\tau\beta}$, σ^2

(b) see interaction plot



(c) $F_0 = 0.3121$, $p = .5789$

The experimenter finds that device does not have an effect on hardness measurement.

(d) $\hat{\mu} = 49.95$, $\hat{\tau}_1 = 0.083$, $\hat{\tau}_2 = -0.083$

(e) $$F_A = \frac{MS_A}{MS_{AB}}$$

(f) $$F_0 = \frac{b\sum_{i=1}^{a}(\bar{y}_{i\cdot\cdot} - \bar{y}_{\cdot\cdot\cdot})^2/(a-1)}{\sum_{i=1}^{a}\sum_{j=1}^{b}(\bar{y}_{ij\cdot} - \bar{y}_{i\cdot\cdot} - \bar{y}_{\cdot j\cdot} + \bar{y}_{\cdot\cdot\cdot})^2/(a-1)(b-1)}$$

(g) F_A [redacted] $= 0.69$ (see R code)

(h) Repeat measurements are summarized by a sample mean. The test statistic for a block design on the sample means leads to the interaction mean squares as the error term.

(2.) treatment = operator ($a=3$), block = machine ($b=4$), response = strength of fiber ($n=2$)

(a) $E(MS_A) = \sigma^2 + n\sigma^2_{\tau\beta} + \dfrac{bn\sum_1^a \tau_i^2}{a-1}$

$E(MS_B) = \sigma^2 + n\sigma^2_{\tau\beta} + an\sigma^2_\beta$

$E(MS_{AB}) = \sigma^2 + n\sigma^2_{\tau\beta}$, $E(MS_E) = \sigma^2$

(b) $\hat{\sigma}^2_\beta = \dfrac{MS_B - MS_{AB}}{an} = -0.5486$

$\hat{\sigma}^2_{\tau\beta} = \dfrac{MS_{AB} - MS_E}{n} = 1.8264$, $\hat{\sigma}^2 = MS_E = 3.79$

(c) Under the null hypothesis ($H_0: \tau_1 = \dots = \tau_a = 0$), $E(MS_A) = \sigma^2 + n\sigma^2_{\tau\beta}$. The appropriate scaling for the test statistic requires a denominator with the same expected value.

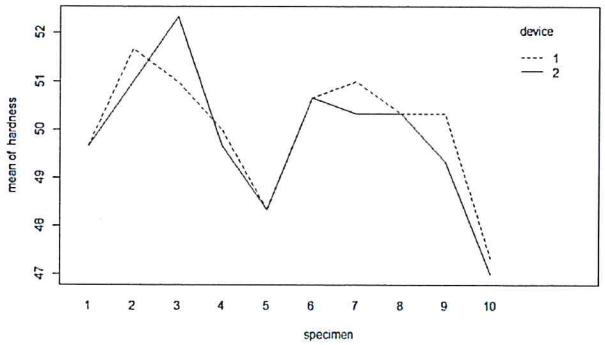
(d) $F_A = 10.77$, $P = .010$

The experiment finds that operator does have an effect on the fiber strength.

(e) Taking repeat measurements at each randomly selected level may serve to increase the measurement accuracy, but does not increase the pertinent sample size.

```
> library("readxl")
> setwd("C:/Users/aneath/iCloudDrive/Lexar/stat581 fall2021")
>
> hw8.data = read_excel("handout8data.xlsx")
> str(hw8.data)
Classes ‘tbl_df’, ‘tbl’ and 'data.frame':    120 obs. of  18 variables:
 $ operator   : num  1 1 2 2 3 3 1 1 2 2 ...
 $ part       : num  1 1 1 1 1 1 2 2 2 2 ...
 $ measurement: num  21 20 20 20 19 21 24 23 24 24 ...
 $ o          : num  1 1 1 1 1 1 1 1 1 1 ...
 $ p          : num  1 2 3 4 5 6 7 8 9 10 ...
 $ means      : num  20.5 23.5 20.5 27 18.5 22 21.5 18 23.5 24 ...
 $ device     : num  1 1 1 2 2 2 1 1 1 2 ...
 $ specimen   : num  1 1 1 1 1 1 2 2 2 2 ...
 $ hardness   : num  50 49 50 50 48 51 52 52 51 51 ...
 $ d          : num  1 1 1 1 1 1 1 1 1 1 ...
 $ s          : num  1 2 3 4 5 6 7 8 9 10 ...
 $ h          : num  49.7 51.7 51 50 48.3 ...
 $ temperature: num  800 800 800 825 825 825 850 850 850 800 ...
 $ position   : num  1 1 1 1 1 1 1 1 1 2 ...
 $ density    : num  570 565 583 1063 1080 ...
 $ op         : num  1 1 2 2 3 3 1 1 2 2 ...
 $ mach       : num  1 1 1 1 1 1 2 2 2 2 ...
 $ strength   : num  109 110 110 112 116 114 110 115 110 111 ...
>
> library("lme4")
> library("lmerTest")

>
> device = as.factor(na.omit(hw8.data$device))
> specimen = as.factor(na.omit(hw8.data$specimen))
> hardness = na.omit(hw8.data$hardness)
>
> interaction.plot(specimen,device,hardness)
>
> contrasts(device)=contr.sum
>
> mixed.mod = lmer(hardness ~ device + (1|specimen) + (1|device:specimen))
boundary (singular) fit: see ?isSingular
>
> anova(mixed.mod)
Type III Analysis of Variance Table with Satterthwaite's method
       Sum Sq Mean Sq NumDF DenDF F value Pr(>F)
device 0.41667 0.41667     1    49  0.3121 0.5789

> summary(mixed.mod)

Random effects:
 Groups          Name        Variance Std.Dev.
 device:specimen (Intercept) 0.000    0.000
 specimen        (Intercept) 1.611    1.269
 Residual                    1.335    1.155
Number of obs: 60, groups:  device:specimen, 20; specimen, 10

Fixed effects:
            Estimate Std. Error       df t value Pr(>|t|)
(Intercept) 49.95000    0.42821  9.00000 116.648 1.27e-15 ***
device1      0.08333    0.14917 49.00000   0.559    0.579
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1


>
```

```
> mixed.test = function(A,B,y)
+ {
+   av=anova(lm(y~A*B))
+   F.a = av$`Mean Sq`[1]/av$`Mean Sq`[3]
+   p.value = pf(F.a,df1=av$Df[1],df2=av$Df[3],lower.tail = FALSE)
+   table1 = matrix(c(av$`Sum Sq`[1],av$`Sum Sq`[2],av$`Sum Sq`[3],av$`Sum Sq
`[4],
+                     av$Df[1],av$Df[2],av$Df[3],av$Df[4],
+                     av$`Mean Sq`[1],av$`Mean Sq`[2],av$`Mean Sq`[3],av$`Mea
n Sq`[4]),nrow = 4)
+   dimnames(table1) = list(c("Fixed Effect A","Random Effect B","Interaction
AB","Error"),
+                           c("SS","df","MS"))
+   print(table1)
+
+   table2 = matrix(c(F.a,p.value),nrow = 1)
+   dimnames(table2) = list(c(""),c("F-test for fixed effect","p-value"))
+   print(table2)
+
+   a=nlevels(A)
+   b=nlevels(B)
+   n=length(y) / a / b
+
+   var.hat = av$`Mean Sq`[4]
+   var.interaction.hat = (av$`Mean Sq`[3]-av$`Mean Sq`[4])/n
+   var.block = (av$`Mean Sq`[2]-av$`Mean Sq`[3])/n/a
+
+   table3 = matrix(c(var.hat,var.interaction.hat,var.block),nrow=1)
+   dimnames(table3) = list(c(""),c("error.var","interaction.var","block.var"
))
+   print(table3)
+ }
>
> mixed.test(device,specimen,hardness)
                          SS df         MS
Fixed Effect A    0.4166667  1  0.4166667
Random Effect B 99.0166667  9 11.0018519
Interaction AB   5.4166667  9  0.6018519
Error           60.0000000 40  1.5000000
 F-test for fixed effect   p-value
               0.6923077 0.4269057
 error.var interaction.var block.var
       1.5      -0.2993827  1.733333
>
```



```
> d = as.factor(na.omit(hw8.data$d))
> s = as.factor(na.omit(hw8.data$s))
> h = na.omit(hw8.data$h)
>
> rcbd.mod = lmer(h ~ d + (1|s))
> anova(rcbd.mod)
Type III Analysis of Variance Table with Satterthwaite's method
   Sum Sq Mean Sq NumDF DenDF F value Pr(>F)
d 0.13945 0.13945     1     9  0.6954 0.4259
>
>
>
> operator = as.factor(na.omit(hw8.data$op))
> machine = as.factor(na.omit(hw8.data$mach))
> strength = na.omit(hw8.data$strength)
>
> mixed.test(operator,machine,strength)
                        SS df        MS
Fixed Effect A  160.33333  2 80.166667
Random Effect B  12.45833  3  4.152778
Interaction AB   44.66667  6  7.444444
Error            45.50000 12  3.791667
 F-test for fixed effect    p-value
                10.76866 0.01034401
 error.var interaction.var  block.var
  3.791667        1.826389 -0.5486111
>
> interaction.plot(machine,operator,strength)
```