Stat 581, Problem Set #11 Solutions

(1.) A = machine, X = thickness, Y = strength

(a) $F_A = \dfrac{R(A)/(a-1)}{SSE_{(A)}/(N-a)} = 4.09$, $p = .044$

(b) The experiment finds that machine has an effect on strength. The covariate thickness plays no role in this analysis.

(c) $F_{A|X} = \dfrac{R(A|X)/(a-1)}{SSE_{(A,X)}/(N-a-1)} = 2.61$, $p = .118$

(d) The experiment finds that machine does not have an effect on strength, after adjusting for the effect of thickness.

(e) machine 1: $\hat{y} = 17.4 + 0.954\,x$

machine 2: $\hat{y} = 18.4 + 0.954\,x$

machine 3: $\hat{y} = 15.8 + 0.954\,x$

(f) see output for scatterplot

(g)

machine	$\bar{x}$	$\bar{y}$		$\bar{y}_{adj}$
m1	25.2	41.4	↓	40.4
m2	26.0	43.2	↓	41.4
m3	21.2	36.0	↑	38.8

$(\bar{x}_{..} = 24.13)$

$\boxed{\bar{Y}_{i,adj} = \bar{Y}_{i.} + \hat{\beta}(\bar{X}_{..} - \bar{X}_{i.})}$

$(\hat{\beta} = 0.954)$

$(\hat{\beta} > 0)$

(h) If $\bar{x}_{i.} < \bar{x}_{..}$, then $\bar{Y}_{i,adj} > \bar{Y}_{i.}$ (adjust upward for less favorable conditions)

If $\bar{x}_{i.} > \bar{x}_{..}$, then $\bar{Y}_{i,adj} < \bar{Y}_{i.}$ (adjust downward for more favorable conditions)

2. A = temperature , B = pressure , y = yield

(a) $$F_A = \frac{R(A)/(a-1)}{SSE_{(A)}/(N-a)} = 7.30, \quad p = .074$$

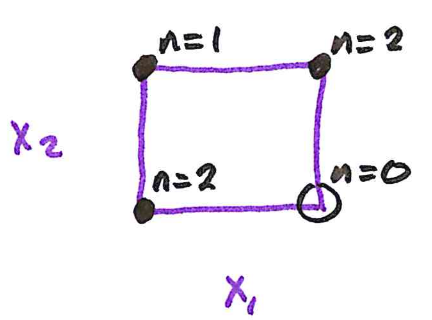
(b) $$F_{B|A} = \frac{R(B|A)/(b-1)}{SSE(A,B)/(N-a-b+1)} = 8.53, \quad p = .10$$

(c) A marginal effect is computed by averaging over the level of the other factor.

A partial effect is computed by holding the level of the other factor fixed.

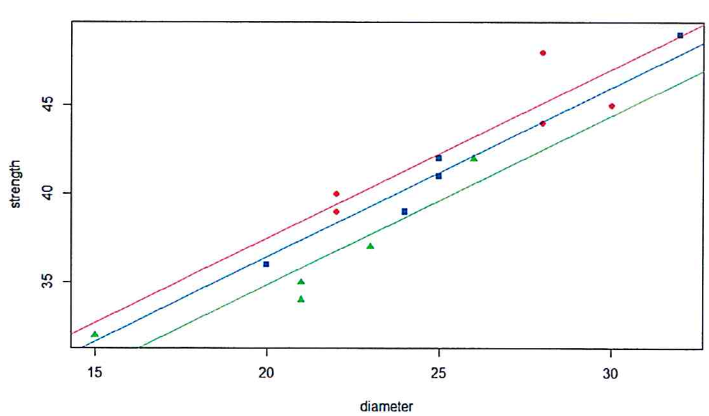
(d), (e), (f) table of fitted values

A	B	Y	$\hat{Y}_A$	$\hat{Y}_B$	$\hat{Y}_{A,B}$
1	1	90.1	90.33	90.2	90.2
1	1	90.3	90.33	90.2	90.2
1	2	90.6	90.33	90.77	90.6
2	2	90.8	90.85	90.77	90.85
2	2	90.9	90.85	90.77	90.85



```
>
> machine = as.factor(na.omit(hw11.data$machine))
> diameter = na.omit(hw11.data$diameter)
> strength = na.omit(hw11.data$strength)
>
> a.mod = lm(strength ~ machine)
> anova(a.mod)
Analysis of Variance Table

Response: strength
          Df Sum Sq Mean Sq F value  Pr(>F)
machine    2  140.4  70.200  4.0893 0.04423 *
Residuals 12  206.0  17.167
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
>
> ancova.mod = lm(strength ~ diameter + machine)
> anova(ancova.mod)
Analysis of Variance Table

Response: strength
          Df  Sum Sq Mean Sq  F value   Pr(>F)
diameter   1 305.130 305.130 119.9330 2.96e-07 ***
machine    2  13.284   6.642   2.6106   0.1181
Residuals 11  27.986   2.544
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
>
> summary(ancova.mod)

Call:
lm(formula = strength ~ diameter + machine)

Residuals:
    Min      1Q  Median      3Q     Max
-2.0160 -0.9586 -0.3841  0.9518  2.8920

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)   17.360      2.961   5.862 0.000109 ***
diameter       0.954      0.114   8.365 4.26e-06 ***
machine2       1.037      1.013   1.024 0.328012
machine3      -1.584      1.107  -1.431 0.180292
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 1.595 on 11 degrees of freedom
Multiple R-squared:  0.9192,	Adjusted R-squared:  0.8972
F-statistic: 41.72 on 3 and 11 DF,  p-value: 2.665e-06

> intercept.1 = coef(ancova.mod)[1]
> intercept.2 = coef(ancova.mod)[1]+coef(ancova.mod)[3]
> intercept.3 = coef(ancova.mod)[1]+coef(ancova.mod)[4]
> slope = coef(ancova.mod)[2]
>
> intercepts = c(intercept.1,intercept.2,intercept.3)
> slopes = c(slope,slope,slope)
> reg.functions = matrix(c(intercepts,slopes),nrow = 3)
> dimnames(reg.functions)=list(c("machine 1","machine 2","machine 3"),c("intercept","sl
```

```
> print(reg.functions,digits = 3)
           intercept slope
machine 1       17.4 0.954
machine 2       18.4 0.954
machine 3       15.8 0.954
>
> plot(diameter[machine == '1'], strength[machine == '1'], xlab='diameter',
+      ylab='strength', pch=15, col='blue',xlim = c(min(diameter),max(diameter)),ylim =
> points(diameter[machine == '2'], strength[machine == '2'], pch=16, col='red')
> points(diameter[machine == '3'], strength[machine == '3'], pch=17, col='green')
>
> abline(intercept.1,slope,col='blue')
> abline(intercept.2,slope,col='red')
> abline(intercept.3,slope,col='green')
>
> xbar.origin = aggregate(diameter, by=list(machine), FUN=mean)
> ybar.origin = aggregate(strength, by=list(machine), FUN=mean)[2]
> means.table = cbind(xbar.origin,ybar.origin)
> colnames(means.table) = c("origin","diameter.mean","strength.mean")
> means.table
  origin diameter.mean strength.mean
1      1          25.2          41.4
2      2          26.0          43.2
3      3          21.2          36.0
>
> mean(diameter)
[1] 24.13333
>
>
> lsmeans(ancova.mod,pairwise ~ machine,adjust="none")
$lsmeans
 machine lsmean    SE df lower.CL upper.CL
 1         40.4 0.724 11     38.8     42.0
 2         41.4 0.744 11     39.8     43.1
 3         38.8 0.788 11     37.1     40.5

Confidence level used: 0.95

$contrasts
 contrast estimate   SE df t.ratio p.value
 1 - 2       -1.04 1.01 11  -1.024  0.3280
 1 - 3        1.58 1.11 11   1.431  0.1803
 2 - 3        2.62 1.15 11   2.283  0.0433
```



```
> A = as.factor(na.omit(hw11.data$temperature))
> B = as.factor(na.omit(hw11.data$pressure))
> y = na.omit(hw11.data$yield)
>
> unbalanced.mod = lm(y ~ A+B)
> a.mod = lm(y~A)
> b.mod = lm(y~B)
>
> anova(a.mod)
Analysis of Variance Table

Response: y
          Df  Sum Sq Mean Sq F value  Pr(>F)
A          1 0.32033 0.32033  7.2987 0.07369 .
Residuals  3 0.13167 0.04389
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
>
> anova(unbalanced.mod)
Analysis of Variance Table

Response: y
          Df  Sum Sq Mean Sq F value  Pr(>F)
A          1 0.32033 0.32033 25.6267 0.03688 *
B          1 0.10667 0.10667  8.5333 0.09993 .
Residuals  2 0.02500 0.01250
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
>
> a.fits = predict(a.mod)
> b.fits = predict(b.mod)
> ab.fits = predict(unbalanced.mod)
>
> fits.table = cbind(A,B,y,a.fits,b.fits,ab.fits)
> colnames(fits.table) = c("A","B","y","a.fits","b.fits","ab.fits")
> fits.table
  A B    y   a.fits   b.fits ab.fits
1 1 1 90.1 90.33333 90.20000   90.20
2 1 1 90.3 90.33333 90.20000   90.20
3 1 2 90.6 90.33333 90.76667   90.60
4 2 2 90.8 90.85000 90.76667   90.85
5 2 2 90.9 90.85000 90.76667   90.85


>
```