1. (30) Consider the N-span moving average applied to data that is uncorrelated with mean μ and variance σ^2.
a. Show that the variance of the moving average is $\operatorname{Var}(M_t) = \sigma^2/N$.

b. Show that $\operatorname{Cov}(M_t, M_{t+k}) = \sigma^2 \sum_{j=1}^{N-k} (1/N)^2$.

c. Show the ACF is

$$\rho_k = 1 - \frac{|k|}{N}, \text{ for } k < N.$$

2. (40) Suppose that Z_1 and Z_2 are uncorrelated random variables with $\operatorname{E}(Z_1) = \operatorname{E}(Z_2) = 0$ and $\operatorname{Var}(Z_1) = \operatorname{Var}(Z_2) = 1$. Consider the process defined by

$$Y_t = Z_1 \cos(\omega t) + Z_2 \sin(\omega t) + e_t,$$

where e_t's are iid and independent of both Z_1 and Z_2. $e_t \sim N(0, \sigma^2)$.

a. Prove that $\{Y_t\}$ is stationary. (Hint: you may need to use the identity $\cos(\alpha - \beta) = \cos\alpha \cos\beta + \sin\alpha \sin\beta$.)

b. Let Z_1 and Z_2 be independent $N(0, 1)$ random variables, and set $\sigma^2 = 1$ and $\omega = 0.5$. Use R to simulate n = 250 observations from the $\{Y_t\}$ process. Plot $\{Y_t\}$ and describe the appearance of your time series.

c. Now consider the process

$$X_t = \beta_0 + \beta_1 t + Z_1 \cos(\omega t) + Z_2 \sin(\omega t) + e_t,$$

Show the time series $\{X_t\}$ is not stationary. Then Use R to simulate a realization of $\{X_t\}$. Plot $\{X_t\}$ and describe the appearance of your time series. Does your $\{X_t\}$ process appear to be stationary?

d. Consider the differenced time series $\{\nabla X_t\}$, where $\nabla X_t = X_t - X_{t-1}$. Show that the first difference $\{\nabla X_t\}$ is actually stationary. Plot the first differences ∇X_t, you may use diff() in R to get the difference. Describe the appearance of this difference process $\{\nabla X_t\}$. Does it appear to be stationary?

3. (30) The monthly values of the average hourly wages for U.S. apparel and textile workers for July 1981 to June 1987 are in the wages object in the TSA package. Type library(TSA); data(wages); print(wages) in R to see the data set.

a. Plot the time series. What basic pattern do you see from the plot?

b. Fit a linear time trend model using least squares. Give the plot of the linear trend overlain on the data, and give the estimated regression equation.

c. Plot the standardized residuals from the linear regression over time. Comment on any notable pattern.

d. Fit a quadratic time trend model using least squares. Give the plot of the quadratic trend overlain on the data, and give the estimated regression equation.

e. Plot the standardized residuals from the quadratic regression over time. Comment on any notable pattern.