Stat 581, Homework Set #7 Solutions

(1.) treatment = operator (a=2), block = fuse (b=10), response = measured running time (in secs.)

(a) $t_0 = -16.295$, $p = .000$

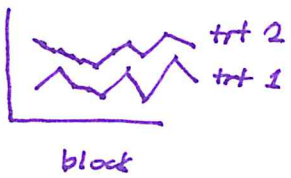
The experiment finds that operator has an effect on the measured running time of a fuse.

(b) $\bar{D} = -0.211$, $S_D = 0.04$, CI for $\mu_D = [-0.24, -0.18]$

(c) $F_0 = 265.53$. Note that $t_0^2 = F_0$

A paired difference analysis is equivalent to a block design analysis, when $a = 2$.

(d) see interaction plot.



(2.) treatment = machine tip (a=4), block = metal specimen (b=4), response = measured hardness (in C-scale units)

(a) model: $Y_{ij} = \mu + \tau_i + \beta_j + \varepsilon_{ij}$ $\begin{cases} i = 1, \ldots, a \\ j = 1, \ldots, b \end{cases}$

where

$$\beta_j \sim N(0, \sigma_\beta^2), \quad \varepsilon_{ij} \sim N(0, \sigma^2)$$

(b) $$MS_{tr} = \frac{b \sum_1^a (\bar{y}_{i\cdot} - \bar{y}_{\cdot\cdot})^2}{a-1}, \quad MS_{BL} = \frac{a \sum_1^b (\bar{y}_{\cdot j} - \bar{y}_{\cdot\cdot})^2}{b-1}$$

$$MS_E = \frac{\sum_1^a \sum_1^b (y_{ij} - \bar{y}_{i\cdot} - \bar{y}_{\cdot j} + \bar{y}_{\cdot\cdot})^2}{(a-1)(b-1)}$$

$$E(MS_{Tr}) = \frac{b\sum_{i=1}^{a}\tau_i^2}{a-1} + \sigma^2, \quad E(MS_{Bl}) = a\sigma_\beta^2 + \sigma^2$$

$$E(MS_E) = \sigma^2$$

(c) We are testing whether or not an observed effect is generalizable to a larger population. How the effect depends on the experimental units determines how well the effect can be generalized.

(d) $F_0 = 14.439$, $p = .001$

The experiment finds that machine tip has an effect on the measured hardness.

(e) see output for pairwise comparison p-values

see output for box plots

grouping information: (smallest → largest)

tip	3	1	2	4
	a	ab	b	c

σ_β^2 is between specimen variance

σ^2 is repeat measurement variance

(f) $\hat{\sigma}_\beta^2 = 0.0665$, $\hat{\sigma}^2 = 0.0089$

A block design is better than a completely randomized design when between block variance is large relative to within block variance.

(g) $\hat{\mu} = 49.625$, $\hat{\tau}_1 = -0.050$, $\hat{\tau}_2 = -0.025$, $\hat{\tau}_3 = -0.175$, $\hat{\tau}_4 = 0.250$

(i) Because specimens have no identifiable features, we model block level effect through a probability distribution.

(ii) Since levels for a block are not identifiable, we can only estimate the main effect (average effect / aggregate effect) of the treatment variable.

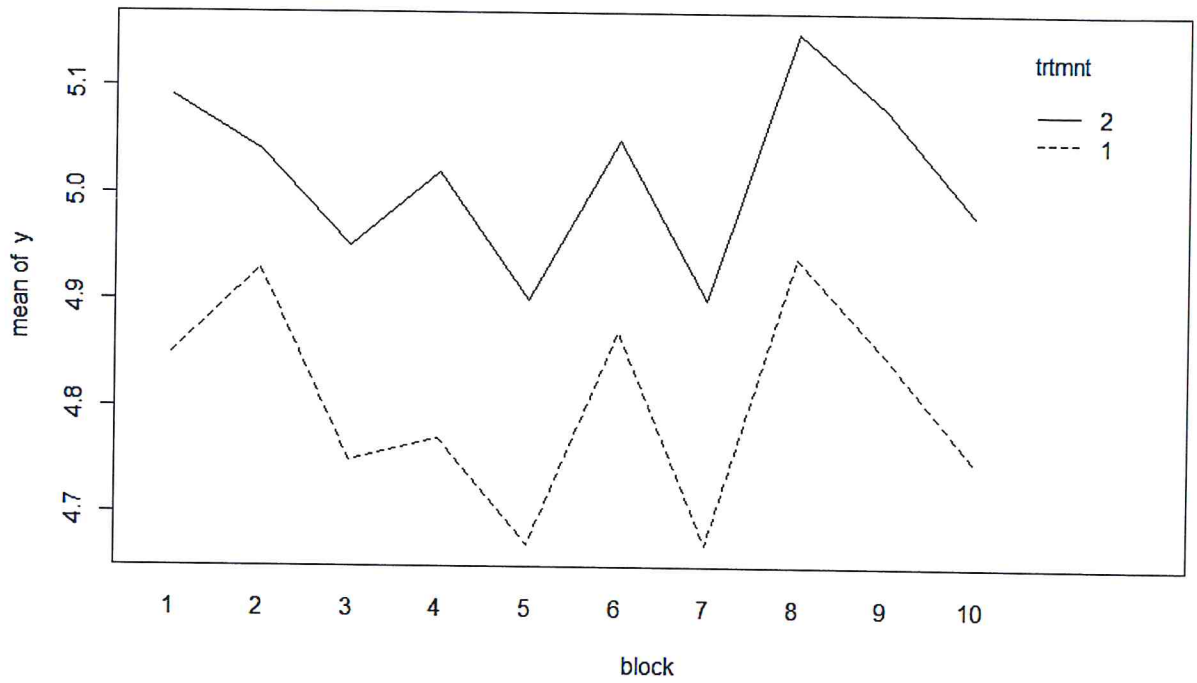
```
> op1 = na.omit(hw7.data$`operator 1`)
> op2 = na.omit(hw7.data$`operator 2`)
>
> t.test(op1,op2,paired = TRUE)

        Paired t-test

data:  op1 and op2
t = -16.295, df = 9, p-value = 5.484e-08
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
 -0.2402918 -0.1817082
sample estimates:
mean of the differences
                 -0.211

>
> paired.test = function(y1,y2,alpha=.05)
+ {
+   d = y1 - y2
+
+   n = length(d)
+   d.bar = mean(d)
+   s.d = sd(d)
+   SE = s.d/sqrt(n)
+
+   t.0 = d.bar / SE
+   p.value = 2*pt(abs(t.0),df=n-1,lower.tail = FALSE)
+
+   t.mult = qt(alpha/2,lower.tail = FALSE, df=n-1)
+   lower.est = d.bar - t.mult*SE
+   upper.est = d.bar + t.mult*SE
+
+   table1 = matrix(c(n,d.bar,s.d),nrow = 1)
+   dimnames(table1) = list(c(""),c("sample.size","mean.diff","sd.diff"))
+   print(table1)
+
+   table2 = matrix(c(t.0,p.value),nrow = 1)
+   dimnames(table2) = list(c(""),c("test statistic","p-value"))
+   print(table2)
+
+   table3 = matrix(c(d.bar,lower.est,upper.est),nrow = 1)
+   dimnames(table3) = list(c(""),c("estimated difference","lower limit","upp
er limit"))
+   print(table3,digits = 3)
+
+ }
>
> paired.test(op1,op2)
 sample.size mean.diff    sd.diff
          10    -0.211 0.04094712
 test statistic      p-value
      -16.29518 5.484094e-08
 estimated difference lower limit upper limit
               -0.211       -0.24      -0.182
>
```

```
> trtmnt = as.factor(na.omit(hw7.data$o))
> block = as.factor(na.omit(hw7.data$fuse))
> y = na.omit(hw7.data$time)
>
> rcbd.mod = aov(y ~ block + trtmnt)
> summary(rcbd.mod)
            Df  Sum Sq Mean Sq F value   Pr(>F)
block        9 0.14014 0.01557   18.57 8.78e-05 ***
trtmnt       1 0.22261 0.22261  265.53 5.48e-08 ***
Residuals    9 0.00755 0.00084
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
>
> interaction.plot(block,trtmnt,y)
```



```
> specimen = as.factor(na.omit(hw7.data$specimen))
> hardness = na.omit(hw7.data$hardness)
>
> contrasts(tip)=contr.sum
>
> random.mod = lmer(hardness ~ (1|specimen) + tip)
> anova(random.mod)
Type III Analysis of Variance Table with Satterthwaite's method
    Sum Sq Mean Sq NumDF  DenDF F value    Pr(>F)
tip  0.385 0.12833     3 9.0004  14.439 0.0008709 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
>
> comps = glht(random.mod,linfct = mcp(tip="Tukey"))
> summary(comps,test=univariate())

         Simultaneous Tests for General Linear Hypotheses

Multiple Comparisons of Means: Tukey Contrasts


Fit: lmer(formula = hardness ~ (1 | specimen) + tip)

Linear Hypotheses:
           Estimate Std. Error z value Pr(>|z|)
2 - 1 == 0  0.02500    0.06666   0.375   0.7076
3 - 1 == 0 -0.12500    0.06666  -1.875   0.0608 .
4 - 1 == 0  0.30000    0.06666   4.500 6.79e-06 ***
3 - 2 == 0 -0.15000    0.06666  -2.250   0.0244 *
4 - 2 == 0  0.27500    0.06666   4.125 3.70e-05 ***
4 - 3 == 0  0.42500    0.06666   6.375 1.83e-10 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
(Univariate p values reported)

> cld(summary(comps,test=univariate()))
   1    2    3    4
"ab"  "b"  "a"  "c"
> plot(cld(summary(comps,test=univariate())))
>
> summary(random.mod)
Linear mixed model fit by REML. t-tests use Satterthwaite's method ['lmerModL
merTest']
Formula: hardness ~ (1 | specimen) + tip

Scaled residuals:
    Min      1Q  Median      3Q     Max
-1.1293 -0.6266 -0.0212  0.3379  1.5225

Random effects:
 Groups   Name        Variance Std.Dev.
 specimen (Intercept) 0.066545 0.25796
 Residual             0.008888 0.09428
Number of obs: 16, groups:  specimen, 4

Fixed effects:
            Estimate Std. Error       df t value Pr(>|t|)
(Intercept) 49.62500    0.13112  2.99890 378.478 4.09e-08 ***
tip1        -0.05000    0.04082  9.00038  -1.225  0.25174
tip2        -0.02500    0.04082  9.00038  -0.612  0.55543
tip3        -0.17500    0.04082  9.00038  -4.287  0.00203 **
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

```
Correlation of Fixed Effects:
     (Intr) tip1   tip2
tip1  0.000
tip2  0.000 -0.333
tip3  0.000 -0.333 -0.333
>
> estimates = summary(random.mod)
> estimates.tip = c(estimates$coefficients[1:4,1],0-sum(estimates$coefficient
s[2:4,1]))
> estimates.tip
(Intercept)        tip1        tip2        tip3
     49.625      -0.050      -0.025      -0.175       0.250
```