Stat 579 Homework Set # 9

Refer to the calf infection study discussed in lecture. Let n_3 denote the number of calves with primary, secondary, and tertiary infections, n_2 the number with only primary and secondary infections, n_1 the number with only a primary infection, and n_0 the number that were never infected. Let θ be the probability of a primary infection. Consider a model where the rate of infection is constant. That is,

$$P(\text{ tertiary} \mid \text{secondary, primary}) = P(\text{ secondary} \mid \text{primary}) = P(\text{ primary}).$$

1. Derive the maximum likelihood estimator of θ under the constant infection rate model.

2. We observe data $n_0 = 63$, $n_1 = 63$, $n_2 = 25$, $n_3 = 5$. Compute the mle $\widehat{\theta}$ and the estimated probabilities $\pi_o\left(\widehat{\theta}\right)$.

3.
(a) Compute the statistic G^2 and the p-value for testing the constant infection rate model.
(b) Provide an interpretation, stated in the context of the problem.

4.
(a) Compute $\widehat{\pi}$, the estimated probability vector under the full model.
(b) Compute the estimated conditional probabilities of further infection under the full model.
(c) Provide an interpretation, stated in the context of the problem.