Homework Set # 1 Solutions

1. (a) $Y_1 \sim BIN(n=50, \pi_1 = \frac{1}{6})$,

probability function: $$f(n_1) = \frac{50!}{n_1!(50-n_1)!}\left(\frac{1}{6}\right)^{n_1}\left(\frac{5}{6}\right)^{50-n_1}$$

(b) $(Y_1, \ldots, Y_6) \sim MULT(n=50, \pi_1 = \frac{1}{6}, \ldots, \pi_6 = \frac{1}{6})$

probability function: $$f(n_1, \ldots, n_6) = \frac{50!}{n_1! \cdots n_6!}\left(\frac{1}{6}\right)^{n_1} \cdots \left(\frac{1}{6}\right)^{n_6}$$

$(n_1 + \cdots + n_6 = 50)$

2. joint probability function

(a) $$f(n_1, n_2, n_3) = e^{-1}\frac{1^{n_1}}{n_1!} \cdot e^{-2}\frac{2^{n_2}}{n_2!} \cdot e^{-3}\frac{3^{n_3}}{n_3!}$$

$$= e^{-6}\frac{1^{n_1}2^{n_2}3^{n_3}}{n_1!\,n_2!\,n_3!}$$

(b) If Y_1, Y_2, Y_3 are indep. $POI(\mu_i)$, then $Y_+ \sim POI(\mu_+)$

probability function: $$f(n) = e^{-6}\frac{6^n}{n!}$$

since $\mu_+ = \mu_1 + \mu_2 + \mu_3$

$= 1 + 2 + 3 = 6$

(c) $(Y_1, Y_2, Y_3) \,\big|\, Y_+ = n \sim MULT(n, \pi_1, \pi_2, \pi_3)$

where $\pi_i = \dfrac{\mu_i}{\mu_+}$

Here, $\pi_1 = \frac{1}{6}$, $\pi_2 = \frac{2}{6}$, $\pi_3 = \frac{3}{6}$.

probability function: $f(n_1, n_2, n_3 \mid n) = \dfrac{n!}{n_1!\, n_2!\, n_3!} \left(\frac{1}{6}\right)^{n_1} \left(\frac{2}{6}\right)^{n_2} \left(\frac{3}{6}\right)^{n_3}$