1. $\tilde{y}_T = (1-\theta)\sum_{t=0}^{\infty}\theta^t y_{T-t}$

for $T < t^*$

$$E(\tilde{y}_T) = (1-\theta)\sum_{t=0}^{\infty}\theta^t E(y_{T-t})$$
$$= (1-\theta)\sum_{t=0}^{\infty}\theta^t \mu$$
$$= (1-\theta)\cdot\left(\frac{1}{1-\theta}\right)\mu$$
$$= \mu$$

for $T > t^*$

$$E(\tilde{y}_T) = (1-\theta)\sum_{t=0}^{\infty}\theta^t E(y_{T-t})$$
$$= (1-\theta)\left[\sum_{t=0}^{T-t^*+1}\theta^t E(y_{T-t}) + \sum_{t=T-t^*}^{\infty}\theta^t E(y_{T-t})\right]$$
$$= (1-\theta)\left(\sum_{t=0}^{T-t^*+1}\theta^t(\delta+\mu) + \sum_{t=T-t^*}^{\infty}\theta^t\mu\right)$$
$$= (1-\theta)\left(\delta\sum_{t=0}^{T-t^*+1}\theta^t + \mu\sum_{t=0}^{\infty}\theta^t\right)$$
$$= (1-\theta)\left(\delta\cdot\frac{1-\theta^{T-t^*+1}}{1-\theta} + \mu\cdot\frac{1}{1-\theta}\right)$$
$$= \delta\left(1-\theta^{T-t^*+1}\right) + \mu$$

or $= \delta - \delta(1-\lambda)^{T-t^*+1} + \mu$ Since $\theta = 1-\lambda$.

2.

Time	y_T	MA	EMA1	EMA2	$\hat{y}_T$	$y_T - \hat{y}_T$
1	14	×	14	14	14	0
2	19	×	15	14.2	15.8	3.2
3	18	17	15.6	14.48	16.72	1.28
4	22	19.67	16.88	14.96	18.80	3.2
5	17	19	16.904	15.349	18.459	−1.459
6	28	22.33	19.123	16.104	22.143	5.857
7	43	29.33	23.899	17.662	30.134	12.866
8	45	38.67	28.119	19.754	36.484	8.516
9	62	50	34.895	22.782	47.008	14.992
10	60	55.67	39.916	26.209	53.623	6.377

$$SSE = \sum (y_T - \hat{y}_T)^2 = 562.0258$$

$$\hat{y}_{11}(10) = \left(2 + \frac{\lambda}{1-\lambda}\right)\tilde{y}_{10}^{(1)} - \left(1 + \frac{\lambda}{1-\lambda}\right)\tilde{y}_{10}^{(2)}$$

$$= \left(2 + \frac{0.2}{0.8}\right) 39.916 - \left(1 + \frac{0.2}{0.8}\right) 26.209$$

$$= 57.050$$

PI: $57.050 \pm Z_{\alpha/2}\, \hat{\sigma}_e$

$$= 57.050 \pm 1.96 \sqrt{562.0258/10}$$

$$= (42.356,\ 71.744)$$