1. $Var(Y) = E(Y - E(Y))^2$

$= E(Y^2 - 2E(Y)Y + (E(Y))^2)$

$= E(Y^2) - 2E(Y)E(Y) + [E(Y)]^2$

$= E(Y^2) - E(Y)^2$

Note $E(Y)$ is a constant, so $E(E(Y)) = E(Y)$.

2. $E(X) = \int_0^1 \int_0^1 x(x+y)\,dx\,dy$

$= \int_0^1 x^3/3 + x^2y/2 \Big]_0^1 dy$

$= \int_0^1 \frac{1}{3} + \frac{y}{2}\,dy$

$= \frac{1}{3}y + \frac{1}{4}y^2 \Big]_0^1$

$= \frac{1}{3} + \frac{1}{4} = \frac{7}{12}$

$Var(X) = EX^2 - (EX)^2$

$= \int_0^1 \int_0^1 x^2(x+y)\,dx\,dy - \left(\frac{7}{12}\right)^2$

$= \int_0^1 \frac{x^4}{4} + \frac{x^3y}{3} \Big]_0^1 dy - \frac{49}{144}$

$= \int_0^1 \frac{1}{4} + \frac{1}{3}y\,dy - \frac{49}{144}$

$= \frac{1}{4}y + \frac{1}{6}y^2 \Big]_0^1 - \frac{49}{144}$

$= \frac{1}{4} + \frac{1}{6} - \frac{49}{144} = \frac{36}{144} + \frac{24}{144} - \frac{49}{144} = \frac{11}{144}$

$E(XY) = \int_0^1 \int_0^1 xy(x+y)\,dx\,dy$

$= \int_0^1 \frac{x^3}{3}y + \frac{x^2}{2}y^2 \Big]_0^1 dy$

$= \int_0^1 \frac{1}{3}y + \frac{1}{2}y^2\,dy$

$= \frac{1}{6}y^2 + \frac{1}{6}y^3 \Big]_0^1 = \frac{1}{3}$

b). $\text{corr}(X,Y) = \dfrac{\text{cov}(X,Y)}{\sqrt{Var(X)Var(Y)}}$

$= \dfrac{E(XY) - EXEY}{\sqrt{Var(X)Var(Y)}}$

$= \dfrac{1/3 - \frac{7}{12}\cdot\frac{7}{12}}{\sqrt{\frac{11}{144} \cdot \frac{11}{144}}}$

$= \dfrac{\frac{1}{3} - \frac{49}{144}}{\frac{11}{144}} = \dfrac{48-49}{11} = -\dfrac{1}{11}$

Not indep.

c) $Cov(X, X+Y)$

$= E(X(X+Y)) - E(X)E(X+Y)$

$= E(X^2+XY) - E(X)(E(X)+E(Y))$

$= E(X^2) + E(XY) - (E(X))^2 - E(X)E(Y)$

$= E(X^2) - (E(X))^2 + E(XY) - E(X)E(Y)$

$= Var(X) + cov(X,Y)$

$= \dfrac{11}{144} + \left(\dfrac{1}{3} - \dfrac{49}{144}\right) = \dfrac{11+48-49}{144} = \dfrac{10}{144} = \dfrac{5}{72}$

See R code for #3.

4.(a) $E(Y_t) = \frac{1}{3}\left(E(e_t) + E(e_{t-1}) + E(e_{t-2})\right)$

$= 0$

$Var(Y_t) = \left(\frac{1}{3}\right)^2\left[Var(e_t) + Var(e_{t-1}) + Var(e_{t-2})\right] = \frac{1}{9}(3\sigma^2) = \frac{\sigma^2}{3}$

(b) $cov(Y_t, Y_{t+k}) = \begin{cases} \sigma^2/3 & k=0 \\ \frac{2}{9}\sigma^2 & k=1 \\ \frac{1}{9}\sigma^2 & k=2 \\ 0 & k>2 \end{cases}$ $\Rightarrow$ free of t

stationary.

$ACF = \begin{cases} 1 & k=0 \\ \frac{2}{3} & k=1 \\ \frac{1}{3} & k=2 \\ 0 & k>2 \end{cases}$

See R code for (d)