1. (12pt) Suppose that simple exponential smoothing is being used to forecast the process $y_t = \mu + e_t$, where where $\{e_t\}$ are white noise with mean 0 and variance $\sigma^2$. At the start of period $t^*$, the mean of the process experiences a transient; that is, it shifts to a new level $\mu + \delta$, but reverts to its original level $\mu$ at the start of the next period $t^* + 1$. The mean remains at this level for subsequent time periods.

(a) Find the expected value of the simple exponential smoother

$$\tilde{y}_T = (1 - \theta) \sum_{t=0}^{\infty} \theta^t y_{T-t}.$$

(b) For $\theta = 0.5$, Determine the number of periods that it will take following the impulse for the expected value of $\tilde{y}_T$ to return to within $0.1\delta$ of the original level $\mu$.

2.(10pt) Let $\{Y_t\}$ be an AR(1) process with $|\phi| < 1$. That is $Y_t = \phi Y_{t-1} + e_t$, where $\{e_t\}$ are white noise with mean 0 and variance $\sigma^2$. Also note $e_t$'s are independent of $Y_{t-1}, Y_{t-2}$,...

(a) Find the autocorrelation function for $W_t = Y_t - Y_{t-1}$ in terms of $\phi$ and $\sigma^2$.

(b) Calculate $\mathrm{Var}\,(W_t)$, where $W_t = Y_t - Y_{t-1}$.

3. (12pt) Suppose $Y_t = X_t + e_t$, where $\{e_t\}$ are normal white noise with mean 0 and variance $\sigma_e^2$ . The $\{X_t\}$ process is a stationary AR(1) defined by $X_t = \phi X_{t-1} + Z_t$ , where $\{Z_t\}$ is a zero mean normal white noise process with variance $\sigma_Z^2$. As usual, in the AR(1) process, assume that $Z_t$ is independent of $X_{t-1}$, $X_{t-2}$, ... Assume additionally that $E(e_t Z_s) = 0$ for all $t$ and $s$.

(a) Show that $\{Y_t\}$ is stationary and find its auto-covariance function, $\gamma_k$.

(b) Show that the process $\{U_t\}$, where $U_t = Y_t - \phi Y_{t-1} = (1 - \phi B)Y_t$, has nonzero correlation only at lag 1 (excluding lag 0, of course!).

4. (16pt) Suppose that $\{e_t\}$ is a zero mean white noise process with variance $\sigma^2$. Consider:
(i) $y_t = 0.80y_{t-1} - 0.15y_{t-2} + e_t - 0.30e_{t-1}$
(ii) $y_t = y_{t-1} - 0.50y_{t-2} + e_t - 1.2e_{t-1}$.

(a) Identify each model as an ARMA(p, q) process; that is, specify p, and q. (watch out for parameter redundancy).

(b) Determine whether each model is stationary and/or invertible.