1. (20) Suppose that simple exponential smoothing is being used to forecast a process, $y_t = \mu + \epsilon_t$. However, at the start of period t^*, the mean of the process shifts to a new mean level $\mu + \delta$. The mean remains at this new level for subsequent time periods. Calculate the expected value of the simple exponential moving average. (Note: you need to discuss cases for $T > t^*$ and $T < t^*$.)

2. (30) Consider the time series data in the following table:

Time	y_T	MA	EMA1	EMA2	$\hat{y}_T = 2\tilde{y}_T^{(1)} - \tilde{y}_T^{(2)}$	$y_T - \hat{y}_T$
1	14	X				
2	19	X				
3	18					
4	22					
5	17					
6	28					
7	43					
8	45					
9	62					
10	60					

(a) Calculate the simple moving average with span=3. Enter this into the MA column of the table.

(b) Calculate the simple (1st order) exponential moving average, $\tilde{y}_T^{(1)}$, of the data using $\lambda = 0.2$ and initial value $= y_1$. Enter this into the EMA1 column of the table.

(c) Calculate the 2nd order exponential moving average, $\tilde{y}_T^{(2)}$, using $\lambda = 0.2$ and initial values $= y_1$. Enter this into the EMA2 column of the table.

(d) Calculate the linear trend estimator $\hat{y}_T = 2\tilde{y}_T^{(1)} - \tilde{y}_T^{(2)}$. Enter this into the table.

(e) Finish the table by calculate the errors. What is the SSE?

(g) Make a one-step-ahead forecast for Time=11 based on the linear trend process. If the true observation is 72, what's your prediction error? Also provide the prediction interval.

3. (50) Consider the Dow Jones Index data on Blackboard. The dataset contains yearly Dow Jones closing index from year 1981 to 2016.

(a) Read the data into R. Then construct a time plot (Dj index v.s. time).

(b) Calculate the simple exponential moving average of the data using $\lambda = 0.1$, and initial value equals the 1st observation. Impose the 1st order EMA on the time plot. Comment on what you observe.

(c) Calculate the sum of squared error (SSE) of your 1st order EMA.

(d) Make a one-step-ahead forecast for year 2017 based on 1st order EMA. Also provide the prediction interval.

(e) Fit a linear regression $y_t = \beta_0 + \beta_1 t + \epsilon_t$ with the data and report the OLS estimators $\hat{\beta}_0$ and $\hat{\beta}_1$.

(f) Set the initial values for 2nd order smoothing based on part (e)

(g) Calculate the 2nd order exponential moving average of the data using $\lambda = 0.1$, and use initial values in part (f). Calculate the unbiased estimates, by $\hat{y}_T = 2\tilde{y}_T^{(1)} - \tilde{y}_T^{(2)}$. Plot the estimates on the original time plot. Compare with the 1st order EMA approach. Comment on what you observe.

(h) Calculate the sum of squared error (SSE) of your estimators in part g.

(i) Make a one-step-ahead forecast for year 2017 based on 2nd order EMA. Also provide the prediction interval.

(j) The true index for 2017 is 24719.22. What is your forecast errors in part (d) and (i). Is the 2nd order approach an apparent improvement over the use of simple exponential smoothing?