Stat 581 Problem Set # 8

1. An experiment is designed to test for systematic differences in the hardness measurements provided by two devices (fixed effect, factor A). Ten specimens are randomly selected (random effect, factor B). Each device is used to make $n = 3$ hardness measurements on each specimen. The data is available on Blackboard as an Excel File.

(a) Write the model for this mixed effects design, defining the fixed effect parameters, and the random effect parameters.

(b) Create an interaction plot to display the device effect on hardness measurement.

(c) Use a mixed model likelihood approach to test for a systematic difference in the measurements of the two devices. Compute the F_o statistic, and the p-value. Provide an interpretation, stated in the context of the problem.

(d) Compute estimates of the fixed effect parameters.

(e) Now write an F_A statistic as a ratio of mean squares.

(f) Write the algebraic formula for the F_o statistic from a block design on the sample means.

(g) Show computationally that (e) and (f) lead to the same test statistic.

(h) Use the result from (g) to argue why interaction mean squares is the appropriate error term.

2. A mixed effects design is used to investigate the effects of operator (fixed effect, factor A) and machine (random effect, factor B) on the breaking strength of a synthetic fiber. There are $a = 3$ operators under investigation. A random sample of $b = 4$ machines is selected, and each operator produces $n = 2$ samples on each of the selected machines. The data is available on Blackboard as an Excel File.

(a) State the expected value for each of the mean squares.

(b) Compute unbiased estimates for the random effect parameters.

(c) Use the result from (a) to argue why interaction mean squares is the appropriate error term.

(d) Perform a test for operator effects. Compute the F_A statistic, and the p-value. Provide an interpretation, stated in the context of the problem.

(e) Now, use the idea of random factors as an experimental unit to explain why interaction mean squares is the appropriate error term. In particular, comment on how taking repeat measurements on a selected level of a random factor does not increase the pertinent sample size.