

1. The *surface finish* of metal parts made on  $a = 4$  machines is under investigation. Each machine can be run by one of  $b = 3$  operators. Because of the location of the machines, operators are specific to a particular machine. Therefore, a nested design with fixed factors is used. Each operator produces  $n = 2$  samples. The data is available on Blackboard as an Excel File.

- (a) Explain the difference between crossed factors and nested factors.
- (b) Write the model for a nested design. Provide algebraic formulas for the estimates  $\hat{\tau}_i$  and  $\hat{\beta}_{j(i)}$ .
- (c) Compute the  $F_A$  statistic for testing factor A effects, and the  $F_{B(A)}$  statistic for testing nested factor B effects. Compute the p-values. Provide an overall interpretation, stated in the context of the problem.
- (d) Compute estimates for each of the effect parameters. Identify which machine performs best, and which operator performs best on each machine. (Higher scores of response are preferred.)
- (e) Explain why it is not possible to directly compare operators across machines in the above design.

2. A nested design is used to study the *burning rate* of propellant from three production processes (fixed effect, factor A). Four batches of propellant are randomly selected from each of the processes (random effect, factor B), and  $n = 3$  determinations of burning rate are made on each batch. The data is available on Blackboard as an Excel File.

- (a) Provide the algebraic formulas for  $MSA$ ,  $MSB(A)$ , and  $MSE$ .
- (b) State the expected value for each of the mean squares.
- (c) Test for differences between the production processes. Write the  $F_A$  statistic as a ratio of mean squares. Compute  $F_A$  and the p-value. Provide an interpretation, stated in the context of the problem.
- (d) Explain why  $MSE$  is the incorrect error term to use when the nested factor is random. In particular, comment on the pertinent sample size.
- (e) Illustrate how evidence in favor of a process effect would be overstated if  $MSE$  is used when computing the test statistic.
- (f) Compute estimates of the batch variance and the measurement variance.