1. Consider the following integration
$$\int_{-\infty}^{\infty} e^{-x^2}dx.$$

(a) Evaluate the integral in closed form using π
(b) Estimate the above integral using Riemman's Rule. Give a estimate of π. Does it at least provide a couple digits worth of accuracy?
(c) Redo part (b) to estimate π using Gauss-Hermite quadrature. You may use the **fastGHQuad()** function in R.

2. Use Monte Carlo simulation to evaluate the confidence (coverage) level of 95% CI for regression slope in the model
$$y_i = 3x_i + \epsilon_i, \epsilon_i \sim N(0, 1)$$
In each Monte Carlo sample, first generate a vector of x (you may pick x from any distribution, say a normal or a uniform). Then generate ϵ from $N(0, 1)$ and then y according to the regression formula. Use lm() to fit the regression model, and confint() to get the 95% confidence interval for the slope parameter. Run the MC iterations for 10000 times and get the proportion of CI that covers the true slope $\beta_1 = 3$. Verify the proportion is close to 0.95.

3. Let $Y \sim Bernoulli(0.7)$ and the conditional distribution of X given Y is $X|Y \sim N(\mu_Y, 1)$, where $\mu_0 = -2$ and $\mu_1 = 2$.

(a) Derive the marginal pdf of X.
(b) Use iterated expectation and variance to find $\mathrm{E}(X)$ and $\mathrm{Var}(X)$ exactly.
(c) Obtain a Monte Carlo sample of size $m = 10000$. Use this sample to compute (i) $\mathrm{E}(X)$, (ii) $\mathrm{Var}(X)$, (iii) 90th percentile of X.