Stat 482, Homework #10 Solutions

Y is sales, X is pre-sales, categorical input is promotion type ("1", "2", "3")

(1.) $I_1 = \begin{cases} 1, & \text{if type} = \text{"1"} \\ 0, & \text{otherwise} \end{cases}$, $I_2 = \begin{cases} 1, & \text{if type} = \text{"2"} \\ 0, & \text{otherwise} \end{cases}$

(2.) model: $E(Y) = \beta_0 + \beta_1 X + \beta_2 I_1 + \beta_3 I_2$

(3.) $E(Y) = \begin{cases} (\beta_0+\beta_2) + \beta_1 X, & \text{if type} = \text{"1"} \\ (\beta_0+\beta_3) + \beta_1 X, & \text{if type} = \text{"2"} \\ \beta_0 + \beta_1 X, & \text{if type} = \text{"3"} \end{cases}$

(4.) $\beta_1 = \frac{\partial E(Y)}{\partial X_1}$, $\beta_2 = E(Y|1,x) - E(Y|3,x)$

$\beta_3 = E(Y|2,x) - E(Y|3,x)$, $\beta_2 - \beta_3 = E(Y|1,x) - E(Y|2,x)$

(5.) β_1 is the difference in mean sales from a 1 unit increase in pre-sales, with promotion type held constant.

β_2 is the difference in mean sales between promotion types "1" and "3", with pre-sales held constant.

β_3 and $\beta_2 - \beta_3$ interpretations are analogous to β_2.

(6.) CI for $\beta_1 = [0.673, 1.124]$

CI for $\beta_2 = [10.32, 15.63]$, CI for $\beta_3 = [5.285, 10.518]$

CI for $\beta_2 - \beta_3 = [2.37, 7.78]$

(7.) see attached for scatterplot

HW 10 Computing

Data from Table 22.1

A company wishes to study the effects of three different types of promotions on sales of its crackers. For each store in the sample, the sales for the promotion period (y) and the sales for the preceeding period (x) are observed.

:

```
hw10.data = read.table(
'http://users.stat.ufl.edu/~rrandles/sta4210/Rclassnotes/data/textdatasets/KutnerData/Chapter%2022%20Data%20Sets/CH22TA01.txt'
  )
colnames(hw10.data)=c("sales","pre.sales","type","obs")
hw10.data$type = as.factor(hw10.data$type)
str(hw10.data)
```

```
## 'data.frame':    15 obs. of  4 variables:
##  $ sales    : int  38 39 36 45 33 43 38 38 27 34 ...
##  $ pre.sales: int  21 26 22 28 19 34 26 29 18 25 ...
##  $ type     : Factor w/ 3 levels "1","2","3": 1 1 1 1 1 2 2 2 2 2 ...
##  $ obs      : int  1 2 3 4 5 1 2 3 4 5 ...
```

```
contrasts(hw10.data$type) = contr.treatment(3,base = 3)
contrasts(hw10.data$type)
```

```
##   1 2
## 1 1 0
## 2 0 1
## 3 0 0
```

```
additive.mod = lm(sales ~ pre.sales + type, data = hw10.data)
```

```
confint(additive.mod)
```

```
##                  2.5 %    97.5 %
## (Intercept) -1.6473329 10.400514
## pre.sales    0.6727716  1.124347
## type1       10.3232717 15.630390
## type2        5.2850286 10.517853
```

```r
summary(additive.mod)
```

```
##
## Call:
## lm(formula = sales ~ pre.sales + type, data = hw10.data)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -2.4348 -1.2739 -0.3362  1.6710  2.4869
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)   4.3766     2.7369   1.599    0.138
## pre.sales     0.8986     0.1026   8.759 2.73e-06 ***
## type1        12.9768     1.2056  10.764 3.53e-07 ***
## type2         7.9014     1.1887   6.647 3.63e-05 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.873 on 11 degrees of freedom
## Multiple R-squared:  0.9403, Adjusted R-squared:  0.9241
## F-statistic: 57.78 on 3 and 11 DF,  p-value: 5.082e-07
```

```r
b.hat = additive.mod$coefficients
intercept.3 = b.hat[1]
intercept.1 = b.hat[1]+b.hat[3]
intercept.2 = b.hat[1]+b.hat[4]
slope = b.hat[2]

dfe = nrow(model.matrix(additive.mod)) - ncol(model.matrix(additive.mod))
V = vcov(additive.mod)
a = c(0,0,1,-1)
b.hat.12 = a %*% b.hat
se.12 = sqrt(a %*% V %*% a)

t.stat.12 = b.hat.12 / se.12
p.value.12 = 2*(1-pt(abs(t.stat.12),dfe))
print(c(t.stat.12,p.value.12))
```

```
## [1] 4.129808111 0.001672662
```

```r
b.hat.12.lower = b.hat.12 - qt(.975,dfe) * se.12
b.hat.12.upper = b.hat.12 + qt(.975,dfe) * se.12
print(c(b.hat.12.lower,b.hat.12.upper))
```

```
## [1] 2.370456 7.780324
```

```
attach(hw10.data)
plot(pre.sales[type == '1'], sales[type == '1'], xlab='pre-sales',
ylab='sales', pch=1, col='blue',xlim = c(min(pre.sales),max(pre.sales)),ylim
= c(min(sales),max(sales)))
points(pre.sales[type == '2'], sales[type == '2'], pch=2, col='red')
points(pre.sales[type == '3'], sales[type == '3'], pch=15, col='green')
abline(intercept.1,slope,col='blue')
abline(intercept.2,slope,col='red')
abline(intercept.3,slope,col='green')
```