Stat 579 Final Exam Part 1: A Logistic Regression Estimation Problem

A sample of elderly patients were given a psychiatric examination to determine whether symptoms of senility are present. One explanatory variable is a patient's score on the Wechsler Adult Intelligence Scale (WAIS).

1. Define the logistic regression model, including notation for the input matrix X, the response vector y, the parameter vector β, and the probability vector $\pi(\beta)$.

2.
(a) State the likelihood equation for the MLE $\widehat{\beta}$ as a normal equation.
(b) State the equation for $\widehat{V} = \widehat{Cov}\left(\widehat{\beta}\right)$, the estimated covariance matrix for $\widehat{\beta}$.
(c) Compute $\widehat{\beta}$ and $\widehat{V}$ from the WAIS data.

3.
(a) State the equations for a 95% confidence interval for β_j.
(b) Compute a 95% confidence interval for β_1 from the WAIS data, and provide an interpretation in the context of the problem.

4.
(a) State the equations for a 95% confidence interval for the odds ratio θ.
(b) Compute a 95% confidence interval for θ from the WAIS data.

5.
(a) State the equations for a 95% confidence interval for the logit L_o at input level x_o.
(b) Compute a 95% confidence interval for L_o at input level $x_o = 10$ from the WAIS data.

6.
(a) State the equations for a 95% confidence interval for the probability π_o at input level x_o.
(b) Compute a 95% confidence interval for π_o at input level $x_o = 10$ from the WAIS data, and provide an interpretation in the context of the problem.

wais	**n**	**senile**
4	2	1
5	1	1
6	2	1
7	3	2
8	2	2
9	6	2
10	6	2
11	6	1
12	2	0
13	6	1
14	7	2
15	3	0
16	4	0
17	1	0
18	1	0

Stat 579 Final Exam Part 2: A Logistic Regression Testing Problem

Applicants for graduate school are classified according to department, sex, and admission status. A goal of the study is to determine the role an applicant's sex plays in the determination of admission status.

1. Define a main effects logistic regression model M having two binary input variables. Include notation for the design matrix X, and the parameter vector β. Provide an interpretation for each of the effect parameters in β, stated in the context of the problem.

2. Provide notation for the design matrix X_S and parameter vector β_S for the saturated model M_S. Provide a brief description of an interaction effect.

3. For each of the models M_O, M_1, M_2, provide notation for the design matrix and a brief description of the model effects, stated in the context of the problem.

4. Compute the deviance statistic D, and give degrees of freedom Δdf, for each of the models M_O, M_1, M_2, M, M_S from the grad school data. Provide a general form for the statistic G^2, and the degrees of freedom for the reference chi-square distribution, for testing a reduced model M_R against a full model M_F.

5. Compute the likelihood statistic G^2 for testing reduced model M against full model M_S from the grad school data, and provide an interpretation in the context of the problem.

6. Compute the likelihood statistic G^2 for testing reduced model M_O against full model M_2 from the grad school data, and provide an interpretation in the context of the problem.

7. Compute the likelihood statistic G^2 for testing reduced model M_1 against full model M from the grad school data, and provide an interpretation in the context of the problem. Include an explanation of how this test differs from that of the previous problem.

8. Compute estimates of the response probabilities based on model M_1 from the grad school data, and provide an interpretation in the context of the problem.

department	sex	(x_1, x_2)	**admit yes**	**admit no**
science	**male**	$(0, 0)$	235	35
	female	$(0, 1)$	38	7
nonscience	**male**	$(1, 0)$	122	93
	female	$(1, 1)$	103	69