1. (10) Show $\text{Var}(Y) = \text{E}(Y^2) - [\text{E}(Y)]^2$ starting from the definition $\text{Var}(Y) = \text{E}(Y - \text{E}(Y))^2$ by expanding and properties of expectation.

2. (30) Let (X, Y) have the joint density $f_{X,Y}(x, y) = (x+y)$ over $R = \{(x, y) : 0 \leq x \leq 1, 0 \leq y \leq 1\}$, the unit square in the plane.

(a) Find $\text{E}(X)$, $\text{Var}(X)$ and $\text{E}(XY)$.

(b) Find $\text{Corr}(X, Y)$. Are X and Y independent?

(c) Find $\text{Cov}(X, X+Y)$.

3. (20) The TSA library in R contains the data set co2, which lists monthly carbon dioxide (CO2) levels in northern Canada from 1/1994 to 12/2004. To load the data in R, that you need to first type

```
install.packages(”TSA”)
library(TSA)
data(co2)
```

Then the data will be under the object named ”co2 ”.

(a) Construct a time series plot of the data. Print the plot and describe all systematic patterns you see in the plot.

(b) Apply a moving average filter of span 12 to the data. You may use the filter function in R and the R code examples from Chapter 1 to implement this. Plot the original data and overlay (superimpose) the moving average, and provide this plot. Discuss whether the moving average filter captures the overall trend in the time series.

4. (40) Suppose $\{e_t\}$ is a normal white noise process with mean 0 and variance σ^2. Let $\{Y_t\}$ be a process defined as (Y_t is a moving average of white noise process):

$$Y_t = \frac{1}{3}(e_t + e_{t-1} + e_{t-2}).$$

(a) Find the mean and variance function of $\{Y_t\}$.

(b) Find the autocovariance function and autocorrelation function of $\{Y_t\}$.

(c) Is the time series $\{Y_t\}$ stationary? Explain your answer.

(d) Simulate and plot the process in R. Provide your R code and print out the plot.