STAT 581 - HW #2

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A product developer is investigating the tensile strength of a new synthetic fiber that will be used to make cloth for men's shirts. Strength may be affected by the percentage of cotton used in the blend of material for the fiber. A completely randomized experiment with five levels of cotton content is performed.

Part 1

State the statistical hypothesis of interest.

The hypothesis of interest is whether the percentage of cotton at five different levels effects the tensile strength of a new synthetic fiber.

We may formulate this as a hypothesis test of the form

$$
\begin{aligned}
H_0 &: \mu_1 = \cdots = \mu_5 \\
H_A &: \mu_i \neq \mu_j \text{ for at least one pair } (i, j),\ i \neq j,
\end{aligned}
$$

where μ_k is the expected tensile strength at the k-th level of cotton.

If H_0 is true, the percentage of cotton has no effect on tensile strength.

Part 2

Briefly explain how the form of the alternative hypothesis requires a need for further investigation.

If there are differences in the cotton level means, further investigation is required to determine where the differences occur.

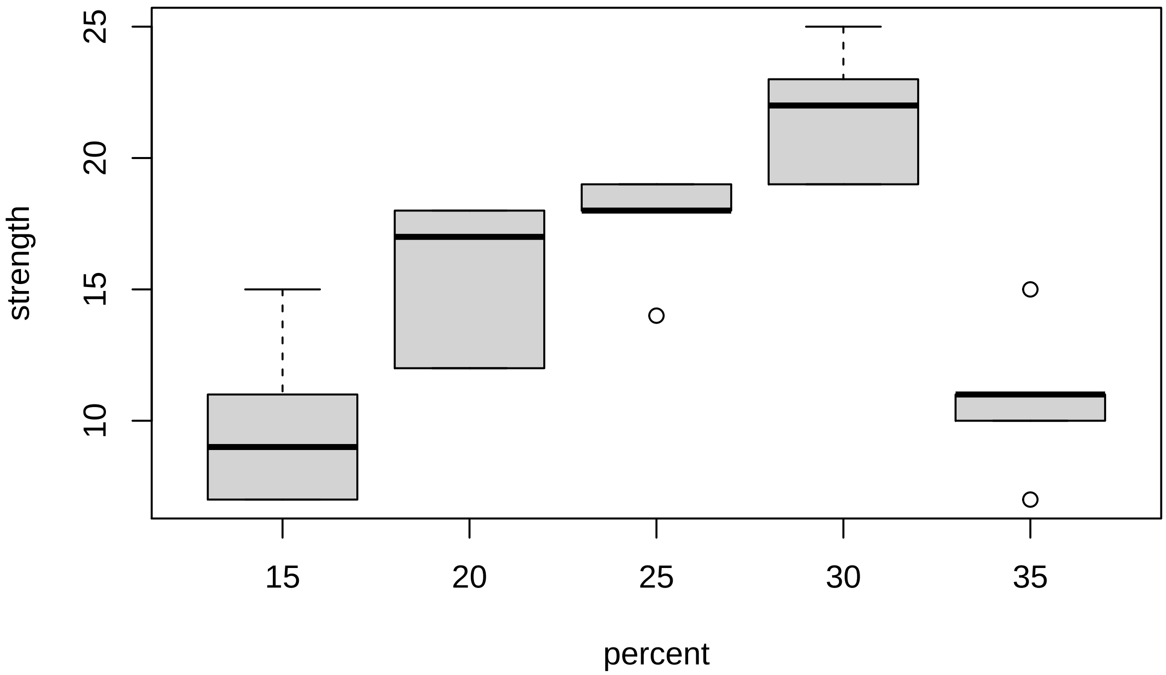
Part 3

Create a Boxplot as a graphical display of the data.

```
library(printr)
library("readxl")

h2.data = read_excel("./handout2data.xlsx")
strength = na.omit(h2.data$strength)
percent = na.omit(as.factor(h2.data$percent))

boxplot(strength~percent)
```



Part 4

Compute the sample mean and sample variance of tensile strength for each level of cotton percentage.

```
means = by(strength,percent,mean)
variances = by(strength,percent,var)
cbind(means,variances)
```

	means	variances
15	9.8	11.2
20	15.4	9.8
25	17.6	4.3
30	21.6	6.8
35	10.8	8.2

Part 5

State the ANOVA model using treatment level effects. Compute estimates of the model parameters.

In this CRD experiment, we observe $n = 5$ responses at each of $a = 5$ levels of cotton (we treat the cotton percentage level as categorical, even though if we show that the cotton level has a practical effect on tensile strength, we may treat it as a quantitative input in, say, a regression model).

The data is given by

$$
\begin{gathered}
Y_{11}, \dots, Y_{15} \sim \mathcal{N}(\mu + \tau_1, \sigma^2) \\
\vdots \\
Y_{51}, \dots, Y_{55} \sim \mathcal{N}(\mu + \tau_5, \sigma^2).
\end{gathered}
$$

The model is given by

$$
Y_{ij} = \mu + \tau_i + \epsilon_{ij} \begin{cases} i = 1, \dots, 5 \\ j = 1, \dots, 5, \end{cases}
$$

where

$$
\begin{aligned}
Y_{ij} &\quad \text{is the } j\text{-th response for tensile strength for the } i\text{-th cotton level,}\\
\mu &\quad \text{is the overall mean of the tensile strength,}\\
\tau_i &\quad \text{is the effect that the } i\text{-th cotton level has on the tensile strength,}\\
\epsilon_{ij} &\quad \overset{\text{iid}}{\sim} \mathcal{N}(0, \sigma^2),\\
\sum_{i=1}^{5} \tau_i &\quad = 0.
\end{aligned}
$$

The estimates of the model parameters are given by

$$
\begin{aligned}
\hat{\mu} &= \bar{y}_{..},\\
\hat{\tau}_i &= \bar{y}_{i.} - \bar{y}_{..}.
\end{aligned}
$$

We compute these estimates with the following R code:

```
mu.hat = mean(strength)
tau.hat = means - mu.hat
```

We see that $\hat{\mu} = 15.04$ and

cotton level	$\hat{\tau}$
15	-5.24
20	0.36
25	2.56
30	6.56
35	-4.24

Part 6

Part (a)

Compute the F_0 statistic and the p-value. Perform the statistical test at level $\alpha = .05$. Provide an interpretation, stated in the context of the problem.

```
model.2 = aov(strength~percent)
summary(model.2)
```

```
##             Df Sum Sq Mean Sq F value   Pr(>F)
## percent      4  475.8  118.94   14.76 9.13e-06 ***
## Residuals   20  161.2    8.06
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

We see that $F_0 = 14.76$ and the p-value $= .000$. Thus, since $p = .000 < \alpha = .05$, the null model where cotton level has no effect on tensile strength is not compatible with the data.

Part (b)

Compute the t_0 statistic and the p-value for testing the 30% group versus the 25% group. Provide an interpretation, stated in the context of the problem.

All pair-wise hypothesis tests

$$H_0^{(i,j)} : \tau_i = \tau_j$$
$$H_A^{(i,j)} : \tau_i \neq \tau_j.$$

may be computed with the following R code:

```
# to perform multiple comparisons between groups,
# we will use the multcomp package
library("multcomp")
model.2.lsd = glht(model.2, linfct=mcp(percent = "Tukey"))
summary(model.2.lsd, test=univariate())
```

```
##
##   Simultaneous Tests for General Linear Hypotheses
##
## Multiple Comparisons of Means: Tukey Contrasts
##
##
## Fit: aov(formula = strength ~ percent)
##
## Linear Hypotheses:
##              Estimate Std. Error t value Pr(>|t|)
## 20 - 15 == 0    5.600      1.796   3.119 0.005409 **
## 25 - 15 == 0    7.800      1.796   4.344 0.000315 ***
## 30 - 15 == 0   11.800      1.796   6.572 2.11e-06 ***
## 35 - 15 == 0    1.000      1.796   0.557 0.583753
## 25 - 20 == 0    2.200      1.796   1.225 0.234715
## 30 - 20 == 0    6.200      1.796   3.453 0.002514 **
## 35 - 20 == 0   -4.600      1.796  -2.562 0.018595 *
## 30 - 25 == 0    4.000      1.796   2.228 0.037541 *
## 35 - 25 == 0   -6.800      1.796  -3.787 0.001157 **
## 35 - 30 == 0  -10.800      1.796  -6.015 7.01e-06 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## (Univariate p values reported)
```

We are interested in the hypothesis test

$$H_0^{(4,3)} : \tau_4 = \tau_3$$
$$H_A^{(4,3)} : \tau_4 \neq \tau_3,$$

which computes the results

$$t_0^{(4,3)} = 2.228$$

and

$$p^{(4,3)} = .038.$$

At $\alpha = .05$, this result is statistically significant. Since $t_0^{(4,3)} < 0$, we decide $\tau_4 > \tau_3$. The experiment finds that cotton level 4 (30%) has a larger effect on tensile strength than cotton level 3 (25%).

Part (c)

Compute the t_0 statistic and the p-value for testing the 25% group versus the 20% group. Provide an interpretation, stated in the context of the problem.

We are interested in the hypothesis test

$$H_0^{(3,2)} : \tau_3 = \tau_2$$
$$H_A^{(3,2)} : \tau_3 \neq \tau_2,$$

which computes the results

$$t_0^{(3,2)} = 1.225$$

and

$$p^{(3,2)} = .235.$$

At $\alpha = .05$, this result is not statistically significant. The experiment finds that cotton level 3 (25%) and cotton level 2 (25%) have the same effect on tensile strength.

Part (d)

Use the grouping information for Fisher pairwise comparisons to further investigate the nature of the treatment effect. Provide an interpretation, stated in the context of the problem.

```
cld(summary(model.2.lsd, test=univariate()))
```

```
##  15  20  25  30  35
## "a" "b" "b" "c" "a"
```

The experiment finds that 30% cotton has the greatest effect on tensile strength, 25% and 20% have the second greatest effect (but no difference between them), and 15% and 35% have the weakest effect (but no difference between them).

Part 7

Briefly discuss some other issues that may provide additional insight to an experimental result beyond a finding of statistical significance.

Statistical significance is a statement of evidence, not belief. Other issues to consider:

1. The scientific context of the effect being tested. Is there an engineering explanation that may account for a relation between tensile strength and cotton level?

   Are there favorable past experimental effects? For instance, in the Bayesian perspective, we may encode results from prior experiments as a prior.

2. The complexity of the effect being tested, i.e., is multiple testing required to understand the effect?
3. The size of the effect, i.e., is the effect of practical importance? For instance, the data may not be compatible with the no effects model, but if the strength is only marginally effected by the cotton level, it may not be worth acting on this finding of statistical signifance.
4. The quality of the experimental design, i.e., is the type II error considered? Do we adjust for differences in experimental units or other factor effects?