Stat 482 Homework Set #6

1. Consider the simple linear regression model with inputs centered so that $\sum x_i = 0$:

$$y_i = \beta_0 + \beta_1 x_i + \varepsilon_i, \quad i = 1, \ldots, n,$$

where

$$\varepsilon_1, \ldots, \varepsilon_n \sim iid\, N\left(0, \sigma^2\right)$$

(a) Write the model using matrix notation.
(b) Use matrix multiplication to derive a simplified expression for each of the following:
(i) b_0, b_1
(ii) $Var\left(b_0\right), Var\left(b_1\right), Cov\left(b_0, b_1\right)$
(iii) $Var\left(\widehat{y}_h\right)$

2. Refer to the data from Exercise 6.15

The goal is to study the relationship between patient satisfaction (y) and patient's age (x_1), severity of illness (x_2), and anxiety level (x_3).

(a) Provide an interpretation of a regression coefficient in a multiple regression model.
(b) Compute b, the estimated regression coefficients for the patient satisfaction data.
(c) Provide a comment on the direction of the input effects, stated in the context of the problem.
(d) Compute $\widehat{Cov}\left(b\right)$, the estimated covariance matrix for the regression coefficients.