Stat 581, Problem Set #5 Solutions

1. Factor A = gap ("0.8", "1.2"), B = RFp setting ("275", "325"), response = etch rate

(a) An interaction effect occurs when the effect of one factor depends on the level of the other factor.

(b) model: $Y_{ijk} = \mu + \tau_i + \beta_j + (\tau\beta)_{ij} + \varepsilon_{ijk}$ $\begin{cases} i=1,\dots,a \\ j=1,\dots,b \\ k=1,\dots,n \end{cases}$

estimates: $\hat{\mu} = 776.0625$, $\hat{\tau}_1 = 50.8$, $\hat{\tau}_2 = -50.8$

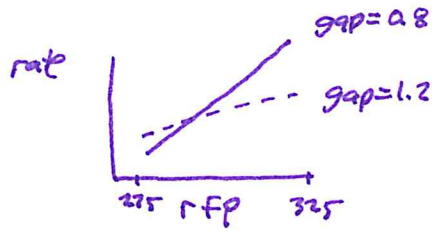
$\hat{\beta}_1 = -153$, $\hat{\beta}_2 = 153$

$\hat{\tau\beta}_{11} = -76.8$, $\hat{\tau\beta}_{12} = 76.8$, $\hat{\tau\beta}_{21} = 76.8$, $\hat{\tau\beta}_{22} = -76.8$

(c) $F_{AB} = 54.31$, $p = .000$

The experiment finds an interaction effect (between gap and RF_p) on etch rate.

(d) see interaction plot



(e) see output for pairwise comparison p-values

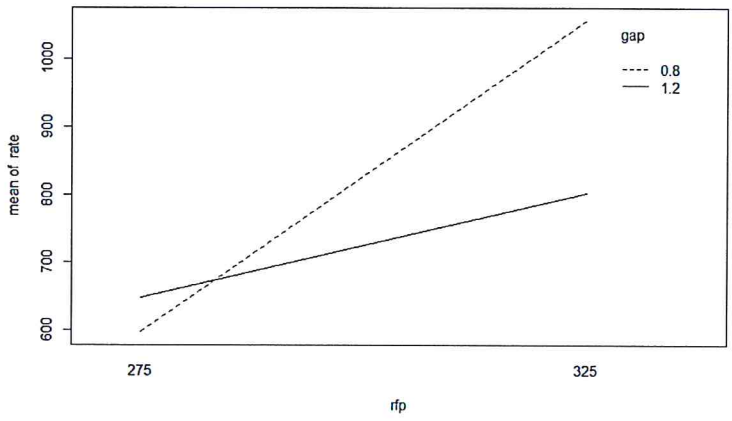
grouping information

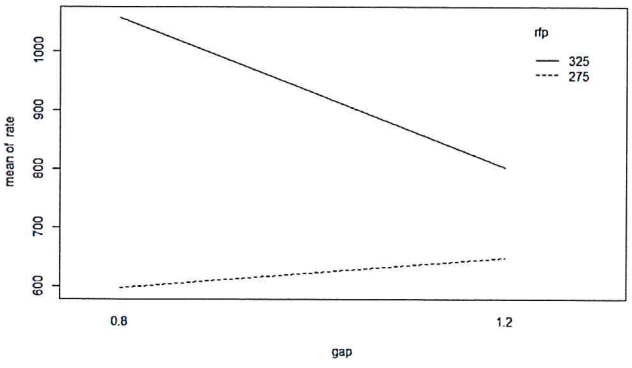
rfp=275, gap=0.8, 1.2	A	
rfp=325, gap=1.2		B
rfp=325, gap=0.8		C

(f) The experiment finds that gap has no effect on etch rate when RFp setting is 275, but that gap=0.8 leads to greater etch rates than gap=1.2 when RFp setting is 325.

```
> library("multcomp")
> library("readxl")

> setwd("C:/Users/aneath/iCloudDrive/Lexar/stat581 fall2021")
>
> hw5.data = read_excel("handout5data.xlsx")
> str(hw5.data)
Classes ‘tbl_df’, ‘tbl’ and 'data.frame':    36 obs. of  15 variables:
 $ plate      : num  1 1 1 1 2 2 2 2 3 3 ...
 $ temp       : num  15 15 15 15 15 15 15 15 15 15 ...
 $ life       : num  130 155 74 180 150 188 159 126 138 110 ...
 $ temperature: num  150 150 160 160 170 170 150 150 160 160 ...
 $ pressure   : num  200 200 200 200 200 200 215 215 215 215 ...
 $ yield      : num  90.4 90.2 90.1 90.3 90.5 90.7 90.7 90.6 90.5 90.6 ...
 $ pour.temp  : chr  "low" "low" "low" "low" ...
 $ titanium   : chr  "low" "low" "high" "high" ...
 $ brk.strn   : num  1.55 1.67 1.42 1.48 1.35 1.5 1.71 1.91 NA NA ...
 $ operator   : num  1 1 2 2 3 3 1 1 2 2 ...
 $ machine    : num  1 1 1 1 1 1 2 2 2 2 ...
 $ strength   : num  109 110 110 112 116 114 110 115 110 111 ...
 $ gap        : num  0.8 0.8 0.8 0.8 0.8 0.8 0.8 0.8 1.2 1.2 ...
 $ rfp        : num  275 275 275 275 325 325 325 325 275 275 ...
 $ rate       : num  550 633 604 601 1052 ...
>
> gap = as.factor(na.omit(hw5.data$gap))
> rfp = as.factor(na.omit(hw5.data$rfp))
> rate = na.omit(hw5.data$rate)
>
> contrasts(gap)=contr.sum
> contrasts(rfp)=contr.sum
>
> two.way.mod = aov(rate ~ gap + rfp + gap:rfp)
> dummy.coef(two.way.mod)
Full coefficients are

(Intercept):     776.0625
gap:                  0.8       1.2
                   50.8125  -50.8125
rfp:                  275       325
                 -153.0625  153.0625
gap:rfp:           0.8:275   1.2:275  0.8:325  1.2:325
                  -76.8125   76.8125  76.8125 -76.8125
> summary(two.way.mod)
            Df Sum Sq Mean Sq F value   Pr(>F)
gap          1  41311   41311   23.77 0.000382 ***
rfp          1 374850  374850  215.66 4.95e-09 ***
gap:rfp      1  94403   94403   54.31 8.62e-06 ***
Residuals   12  20858    1738
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
>
>
> interaction.plot(x.factor = rfp,trace.factor = gap,response = rate)
> interaction.plot(x.factor = gap,trace.factor = rfp,response = rate)
>
> combined = interaction(gap,rfp)
> levels(combined)
[1] "0.8.275" "1.2.275" "0.8.325" "1.2.325"
>
> comb.mod = aov(rate~combined)
> comparisons = glht(comb.mod, linfct = mcp( combined = "Tukey"))
> summary(comparisons,test=univariate())
```

```
        Simultaneous Tests for General Linear Hypotheses

Multiple Comparisons of Means: Tukey Contrasts


Fit: aov(formula = rate ~ combined)

Linear Hypotheses:
                       Estimate Std. Error t value Pr(>|t|)
1.2.275 - 0.8.275 == 0    52.00      29.48   1.764 0.103166
0.8.325 - 0.8.275 == 0   459.75      29.48  15.595 2.49e-09 ***
1.2.325 - 0.8.275 == 0   204.50      29.48   6.937 1.57e-05 ***
0.8.325 - 1.2.275 == 0   407.75      29.48  13.831 9.79e-09 ***
1.2.325 - 1.2.275 == 0   152.50      29.48   5.173 0.000232 ***
1.2.325 - 0.8.325 == 0  -255.25      29.48  -8.658 1.66e-06 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
(Univariate p values reported)

> cld(summary(comparisons,test=univariate()))
0.8.275 1.2.275 0.8.325 1.2.325
    "a"     "a"     "c"     "b"
```





>

(2.) Factor A = machine ($a=4$), factor B = operator ($b=3$), response = breaking strength

(a) $F_A = 1.095$, $F_B = 21.143$, $F_{AB} = 1.963$

($P_A = .389$) ($P_B = .000$) ($P_{AB} = .15$)

The experiment finds that operator has an effect on strength, but that machine has no effect on strength. Furthermore, the experiment finds that there is no interaction effect.

(b) see output for pairwise comparison p-values

grouping information

operator	1, 2	3
	A	B

(c) confidence intervals for between operator differences

CI for $\mu_2 - \mu_1 = [-1.1, 3.6]$

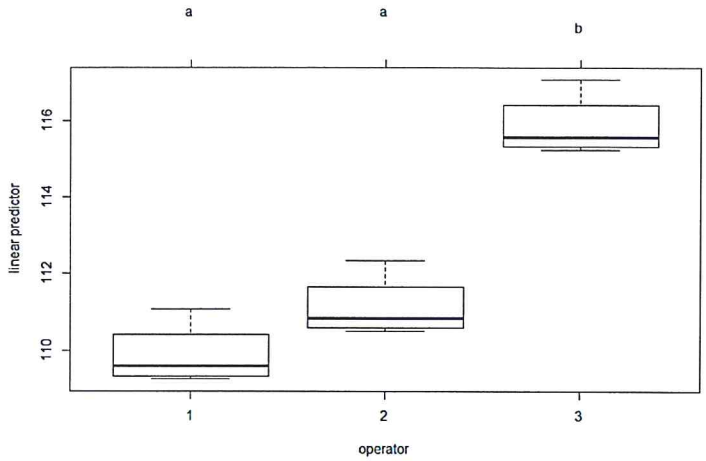
CI for $\mu_3 - \mu_1 = [3.65, 8.35]$

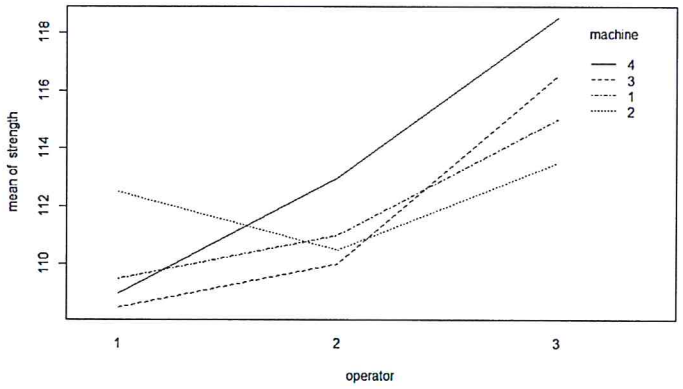
CI for $\mu_3 - \mu_2 = [2.4, 7.1]$

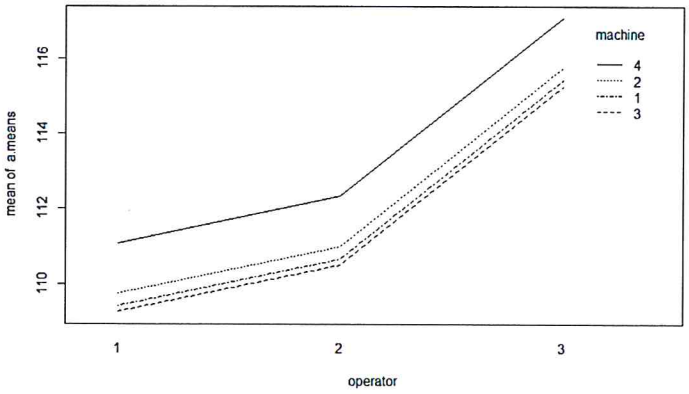
(d) see interaction plots; interaction model, additive model

The model smooths over the randomness in the data, simplifying the analysis.

```
> operator = as.factor(na.omit(hw5.data$operator))
> machine = as.factor(na.omit(hw5.data$machine))
> strength = na.omit(hw5.data$strength)
>
> x.mod = aov(strength ~ operator*machine)
> summary(x.mod)
                 Df Sum Sq Mean Sq F value   Pr(>F)
operator          2 160.33   80.17  21.143 0.000117 ***
machine           3  12.46    4.15   1.095 0.388753
operator:machine  6  44.67    7.44   1.963 0.150681
Residuals        12  45.50    3.79
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
>
> a.mod = aov(strength ~ operator + machine)
> compare = glht(a.mod, linfct = mcp( operator = "Tukey"))
>
> summary(compare,test=univariate())

         Simultaneous Tests for General Linear Hypotheses

Multiple Comparisons of Means: Tukey Contrasts


Fit: aov(formula = strength ~ operator + machine)

Linear Hypotheses:
           Estimate Std. Error t value Pr(>|t|)
2 - 1 == 0    1.250      1.119   1.117 0.278683
3 - 1 == 0    6.000      1.119   5.362 4.27e-05 ***
3 - 2 == 0    4.750      1.119   4.245 0.000487 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
(Univariate p values reported)

> cld(summary(compare,test=univariate()))
  1   2   3
"a" "a" "b"
>
> confint(compare,calpha = univariate_calpha())

         Simultaneous Confidence Intervals

Multiple Comparisons of Means: Tukey Contrasts


Fit: aov(formula = strength ~ operator + machine)

Quantile = 2.1009
95% confidence level


Linear Hypotheses:
           Estimate lwr     upr
2 - 1 == 0  1.2500  -1.1011  3.6011
3 - 1 == 0  6.0000   3.6489  8.3511
3 - 2 == 0  4.7500   2.3989  7.1011

>
> plot(cld(summary(compare,test=univariate())))
>
> a.means = predict(a.mod)
> interaction.plot(operator,machine,strength)
> interaction.plot(operator,machine,a.means)
```


a
a
b
linear predictor
110
112
114
116
1
2
3
operator


mean of strength
110
112
114
116
118
machine
4
3
1
2
1
2
3
operator


mean of a.means
110
112
114
116
machine
4
2
1
3
1
2
3
operator