Homework Set # 2 Solutions

(1.) $\ell(\pi) = \binom{n}{n_1} \pi^{n_1} (1-\pi)^{n_2}$, $n_1 + n_2 = n$

$$L(\pi) = n_1 \log \pi + n_2 \log(1-\pi)$$

$$\frac{dL}{d\pi} = \frac{n_1}{\pi} - \frac{n_2}{1-\pi} \overset{\text{set}}{=} 0 , \quad \frac{\pi}{n_1} = \frac{1-\pi}{n_2} ,$$

$$\pi\left(1 + \frac{n_1}{n_2}\right) = \frac{n_1}{n_2} , \quad \pi\left(\frac{n}{n_2}\right) = \frac{n_1}{n_2} , \quad \hat{\pi} = \frac{n_1}{n} \quad \blacksquare$$

(2.) $\underbrace{P(A|B) > P(A|B')}_{p_1 > p_2 \text{ iff } \frac{p_1}{1-p_1} > \frac{p_2}{1-p_2}}$ iff $\dfrac{P(A|B)\cdot P(B)}{P(A'|B)\cdot P(B)} > \dfrac{P(A|B')\cdot P(B')}{P(A'|B')\cdot P(B')}$

iff

$$\frac{P(A\cap B)/P(A)}{P(A'\cap B)/P(A')} > \frac{P(A\cap B')/P(A)}{P(A'\cap B')/P(A')}$$

iff

$$\frac{P(B|A)}{P(B|A')} > \frac{P(B'|A)}{P(B'|A')}$$

iff

$$\underbrace{\frac{P(B|A)}{P(B'|A)} > \frac{P(B|A')}{P(B'|A')}}_{} \quad \text{iff} \quad P(B|A) > P(B|A')$$

again, now in reverse $\quad \dfrac{p_1}{1-p_1} > \dfrac{p_2}{1-p_2}$ iff $p_1 > p_2$