Stat 581 Exam 2 December 14, 2021

1. A randomized complete block design is used to study the effect of caliper on the measured diameters (in mm) of ball bearings. A sample of $b = 10$ ball bearings is randomly selected, and each of $a = 3$ calipers produces a measurement on each of the selected ball bearings. The data is provided as an attachment.

(a) Write the model and distributional assumptions for a randomized complete block design.
(b) Provide the algebraic formulas for $MStr$, $MSbl$, and MSE. Explain why it makes sense for an interaction effect to serve as a measure of error variance.
(c) Test for systematic differences in the measurements provided by the calipers. Compute the F_o statistic, and the p-value. Provide an interpretation, stated in the context of the problem.

(d) Compute estimates of the variance components. Explain when a block design is better than a completely randomized design.
(e) Compute estimates of the fixed effect parameters. Explain why block effects are modeled differently than treatment effects in this design. Explain what treatment effect is estimable in this design.

2. Now, a mixed effects design is used to study the effect of caliper (fixed effect, factor A) on the measured diameters of ball bearings. There are $a = 2$ calipers under investigation. A random sample of $b = 8$ ball bearings is selected (random effect, factor B), and each caliper produces $n = 3$ measurements on each of the selected ball bearings. The data is provided as an attachment.

(a) Write the model for this mixed effects design, defining the fixed effect parameters, and the random effect parameters.

(b) Create an interaction plot to display the caliper effect on measured diameter. Use a mixed model likelihood approach to test for a systematic difference in the measurements of the two calipers. Compute F_o and the p-value.

(c) State the expected value for each of $MSA, MSB, MSAB$, and MSE. Write the test statistic F_A as a ratio of mean squares. Use the expected values to argue why interaction mean squares is the appropriate error term.

(d) Perform a test for caliper effects. Compute F_A and the p-value.

(e) Compute the unbiased estimates of the random effect parameters.

3. A nested design is used to study the number of cases produced from three bottling machines (factor A, fixed effect). Four operators are randomly selected for each of the machines (nested factor B, random effect). Each operator makes $n = 2$ experimental runs. The data is provided as an attachment.

(a) Explain the difference between crossed factors and nested factors.

(b) Write the model for this nested design, defining the fixed effect parameters, and the random effect parameters.

(c) Provide the algebraic formulas for the estimates $\widehat{\tau}_i, \widehat{\beta}_{j(i)},$ and $\widehat{y}_{ij}$. State the expected value for each of $MSA, MSB\left(A\right),$ and MSE.

(d) Test for differences between bottling machines. Write the test statistic F_A statistic as a ratio of mean squares.

(e) Explain why MSE is the incorrect error term to use when the nested factor is random. In particular, comment on the pertinent sample size.

4. A company wishes to study the effect of promotion type (1,2,3) on the sales of its crackers. A sample of $N = 15$ grocery stores is selected. Response variable y is the number of cases sold during the promotion period. Factor A is the promotion type. Covariate x is the same store sales prior to the promotion. The data is provided as an attachment.

(a) Compute the estimated regression of presales on cases sold for each promotion type. Create a scatterplot of presales versus sales for each promotion type, including the estimated regression lines.

(b) Test for a promotion effect. Write the ANCOVA $F_{A|x}$ statistic using extra sum of squares notation. Compute $F_{A|x}$ and the p-value. Provide an interpretation, stated in the context of the problem. Note the role the covariate is playing in this analysis.

(c) Compute the sample mean sales and the sample mean presales for each promotion type. Compute the least squares means. Explain how the information from the covariate adjusts the determination of promotion effect.