

1. Consider the MA(2) process, where all the $\{e_t\}$ values are independent white noise with variance σ^2 .

$$Y_t = e_t - 0.5e_{t-1} - 0.3e_{t-2}$$

- (a) Find $E(Y_t)$.
- (b) Find $\text{Cov}(Y_t, Y_t) = \text{Var}(Y_t)$.
- (c) Find $\gamma_k = \text{Cov}(Y_t, Y_{t+k})$ and, from this, find the ACF, ρ_k .
- (d) Generate time series datasets of length $n = 200$ according to this MA(2) process. Plot the observed time series and the sample ACF and PACF. Do the plots agree with what you know to be true?

2. Consider the AR(1) process: $Y_t = \phi Y_{t-1} + e_t$, where all the $\{e_t\}$ values are independent white noise with variance σ^2 .

- (a) Show that if $|\phi| = 1$, the process cannot be stationary.
- (b) Take $\phi = -0.6$, calculate and find the ACF, ρ_k .
- (c) Take $\phi = -0.6$. Generate time series datasets of length $n = 200$ according to the AR(1) process. Plot the observed time series and the sample ACF and PACF. Do the plots agree with what you know to be true?
- (d) What happens when we increase the sample size? Repeat part (c) when $n = 1000$. Comment on your findings.

3. A data set of 57 consecutive measurements from a machine tool are in the `deere3` object in the TSA package. Type `library(TSA); data(deere3)` in R to load the data.

- (a) Plot the time series. What basic pattern do you see from the plot? Might a stationary model be appropriate for this plot?
- (b) Plot the sample ACF and PACF. Tentatively specify the type of model (AR, MA, or ARMA) as well as the order(s) of the model. Write up detailed notes that describe how you decided on the model.
- (c) Fit an AR(1) model using `arima` function in R and use it to forecast the next ten values of the series, and provides the forecasted values.

4. A data set of durations until payment for 130 consecutive orders from a Winegrad distributor are in the `days` object in the TSA package. Type `library(TSA); data(days)` in R to load the data set.

- (a) Plot the time series. What basic pattern do you see from the plot? Might a stationary model be appropriate for this plot?
- (b) Plot the sample ACF and PACF. Tentatively specify the type of model (AR, MA, or ARMA) as well as the order(s) of the model. Write up detailed notes that describe how you decided on the model.
- (c) Fit an MA(2) model using `arima` function in R and use it to forecast the next ten values of the series, and list the forecasted values.