Reliability Estimation in Series Systems: Maximum Likelihood Techniques for Right-Censored and Masked Failure Data

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Introduction

- Estimating reliability of individual components from system-level data is challenging
- Failure times and causes often unobserved
- Developed likelihood framework to leverage observational data
- Simulation studies assess accuracy under small samples and significant masking

Series System Model

- Components in series configuration
- Lifetimes iid Weibull
- Derived system:
  - Reliability function: Product of components
  - Probability density: Function of component densities
  - Hazard function: Sum of component hazards

Likelihood Model

- Right censoring: Unobserved lifetimes
- Masked causes: Candidate sets
- Assumptions on candidate sets:
  - Condition 1: Contains failed component
  - Condition 2: Equal failure probability within candidate set
  - Condition 3: Independent of parameters
- Likelihood contributions:
  - Right censoring: System reliability $\prod_j R_j(t; \boldsymbol{\theta_j})$
  - Masked causes: $\prod_l R_l(t; \boldsymbol{\theta_j}) \sum_j h_j(t; \boldsymbol{\theta_j})$

Estimation Methodology

- Maximum Likelihood Estimation
  - Asymptotic normality and efficiency
  - Small sample issues
- Bootstrap Confidence Intervals
  - Approximates sampling distribution
  - Bias correction
  - Acceleration
  - Flexibility for small samples

Simulation Studies

- Assessed accuracy and precision of MLE
- Metrics:
  - Bias
  - Coverage probability
  - Confidence interval width
- Key scenarios:
  - Varying sample size
  - Censoring level
  - Masking probability
  - Component parameters

Conclusion

- Provided likelihood framework for limited data
- MLE accurate under small samples and masking
- Bootstrap CIs well-calibrated
- Shape parameters sensitive, scales more robust