Reliability Estimation in Series Systems

Maximum Likelihood Techniques for Right-Censored and Masked Failure Data

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Section 1

Introduction

Opening Statement

Series systems are integral to various industries, from power grids to data centers. The functionality of the entire system is contingent upon the reliability of each individual component.

Problem Statement

The challenge lies in estimating the reliability of individual components when the available data is primarily at the system level. This task is complicated by the presence of masked and right-censored data, where the precise cause or timing of failures might be ambiguous.

Significance of the Problem

Inaccurate reliability estimation can have tangible consequences. For instance, in a data center, the failure of a single component in a series system might lead to substantial downtime, with both financial and reputational implications. This underscores the critical importance of precise and accurate reliability estimation across a variety of sectors.

Overview of Key Contributions

This study introduces a likelihood model designed to address these challenges. Through comprehensive simulations, the efficacy of the model is validated under a range of conditions. Additionally, the evaluation of the Bootstrapped confidence intervals enhances the reliability of the proposed methodology.

Transition to Main Content

We will now delve into the mathematical foundations of the proposed model, its practical applications, and the insights derived from extensive simulations.

Section 2

Series System Model

Basic Definition

In the context of our study, a series system consists of multiple components aligned in a sequence. The key characteristic of such a system is that the failure of any single component results in the failure of the entire system.

Component and System Lifetimes

- Each system comprises *m* components.
- Individual lifetimes of components in a system are independent and have varied distributions.
- The system's lifetime is determined by the component with the shortest lifespan.

Core Concepts in Reliability Analysis

Reliability analysis rests on three foundational distribution functions: 1. **Reliability Function**: Represents the probability that a system or component lasts longer than a specific time duration. 2. **Probability Density Function (pdf)**: Expresses the rate of change of the reliability function. 3. **Hazard Function**: Illustrates the instantaneous failure rate at a specific time, given survival up to that point.

Mathematical Framework

- Component lifetimes are modeled using a parametric distribution with a parameter vector $\boldsymbol{\theta_j}$.
- The system's parameter vector, $\boldsymbol{\theta}$, encapsulates the parameter vectors of all its components.
- Using the joint pdf, one can derive the marginal pdf and the conditional pdf, which are essential for understanding component interdependencies and their influence on system reliability.

Reliability Function of the Series System

A significant finding is the reliability function of the series system, mathematically represented as:

$$R_{T_i}(t; \boldsymbol{\theta}) = \prod_{j=1}^{m} R_j(t; \boldsymbol{\theta_j})$$

Where $R_j(t; \boldsymbol{\theta_j})$ denotes the reliability function of the j^{th} component.

Concluding Thoughts

The series system model establishes the groundwork for our study. By understanding the intricacies of component and system lifetimes and the interplay between them, we can develop robust methods to estimate reliability, even in the face of data challenges.