Estimating series systems from masked data

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1 Estimating component cause of failure probabilities

Another characteristic we may wish to estimate is the probability that a particular component in an observation caused the system failure. We wish to use as much information as possible to do this estimation. In what follows, we consider three cases.

1.1 Case 1: Both C_i and T_i are observed

We have an observed candidate set c_i and an observed system failure time t_i and we are interested in the probability that a particular component is the cause of the observed system failure.

Theorem 1.1. *Assuming Conditions* **??** *and* **??***, the conditional probability that $K_i = j$ given $C_i = c_i$ and $T_i = t_i$ is given by*

$$\Pr\{K_i = j | T_i = t_i, C_i = c_i\} = \frac{h_j(t_i|\boldsymbol{\theta_j})}{\sum_{l \in c_i} h_l(t_i|\boldsymbol{\theta_l})}. \tag{1.1}$$

Proof. Assuming Conditions 1 and 2, the conditional probability $\Pr\{K_i = j | T_i = t_i, C_i = c_i\}$ may be rewritten as

$$\begin{aligned}
\Pr\{K_i = j | T_i = t_i, C_i = c_i\} &= \frac{\Pr\{K_i = j, T_i = t_i, C_i = c_i\}}{\Pr\{C_i = c_i, T_i = t_i\}} \\
&= \frac{\Pr\{C_= c_i | K_i = j, T_i = t_i\} \Pr\{K_i = j, T_i = t_i\}}{\Pr\{C_i = c_i, T_i = t_i\}} \\
&= \frac{\Pr\{C_= c_i | K_i = j, T_i = t_i\} h_j(t_i; \boldsymbol{\theta_j}) R_{T_i}(t_i; \boldsymbol{\theta})}{\sum_{l=1}^{m} \Pr\{C_= c_i | K_i = j, T_i = t_i\} h_l(t_i; \boldsymbol{\theta_l}) R_{T_i}(t_i; \boldsymbol{\theta})} \\
&= \frac{\Pr\{C_= c_i | K_i = j, T_i = t_i\} h_j(t_i; \boldsymbol{\theta_j})}{\sum_{l \in c_i} \Pr\{C_= c_i | K_i = j, T_i = t_i\} h_l(t_i; \boldsymbol{\theta_l})} \\
&= \frac{\Pr\{C_= c_i | K_i = j', T_i = t_i\} h_j(t_i; \boldsymbol{\theta_j})}{\Pr\{C_= c_i | K_i = j', T_i = t_i\} \sum_{l \in c_i} h_l(t_i; \boldsymbol{\theta_l})} \\
&= \frac{h_j(t_i; \boldsymbol{\theta_j})}{\sum_{l \in c_i} h_l(t_i; \boldsymbol{\theta_l})}.
\end{aligned}$$

□

1.2 Case 2: Only T_i is observed

We have an observed system failure time t_i (but we do not have a candidate set) and we are interested in the probability that a particular component is the cause of the observed system failure.

The probability that $K_i = j$ given $T_i = t_i$ is given by Equation (**??**),

$$\Pr\{K_i = j | T_i = t_i\} = \frac{h_j(t_i|\boldsymbol{\theta_j})}{\sum_{l=1}^{m} h_l(t_i|\boldsymbol{\theta_l})}.$$

1.3 Case 3: Neither T_i nor C_i are observed

We observe nothing but are interested in the probability that a particular component *will* be the cause of the system failure.

The unconditional probability that $K_i = j$ is given by

$$\Pr\{K_i = j\} = \int_0^\infty f_{K_i,T_i}(j, t; \boldsymbol{\theta}) dt. \tag{1.2}$$

1.4 The maximum likelihood estimates of the probabilities

By the invariance property of the MLE, in each of the component failure probabilities described previously, we may substitute $\boldsymbol{\theta}$ with an MLE $\hat{\boldsymbol{\theta}}$ to obtain the MLEs of these probabilities.

Moreover, assuming the regularity conditions, we also have an approximation of the sampling distribution of $\hat{\boldsymbol{\theta}}$, as described in Section **??**, and thus in this case we can estimate the confidence intervals for these probabilities.