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Estimation of exponential component reliability from uncertain life data in series and parallel systems

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Abstract

Estimating reliability of components in series and parallel systems from masking system testing data has been studied. In this paper we take into account a second type of uncertainty: censored lifetime, when system components have constant failure rates. To efficiently estimate failure rates of system components in presence of combined uncertainty, we propose a useful concept for components: equivalent failure and equivalent lifetime. For a component in a system with known status and lifetime, its equivalent failure is defined as its conditional failure probability and its equivalent lifetime is its expectation of lifetime. For various uncertainty scenarios, we derive equivalent failures and test times for individual components in both series and parallel systems. An efficient EM algorithm is formulated to estimate component failure rates. Two numerical examples are presented to illustrate the application of the algorithm.

Keywords: Reliability; Masked data; Equivalent failure and life time; EM theorem; Bayes theorem

1. Introduction

Estimating component reliability from system life data is very useful because the estimates account for many degrading effects introduced by system manufacturing, assembly, distribution, and installation processes, thus reflect component operational reliability after their assembly into an operational system [1]. In practice, however, this type of estimation is often confounded by data uncertainty: component status (failure or survival) and/or lifetime (actual failure or survival time) is unknown in the system test data.

Uncertainty in component status is also called data masking in literature. This occurs frequently in complex systems and in field data where failure causes might be isolated only to some subset of components. For the analysis of masked data a substantial literature has developed under various parametric and nonparametric settings, using both frequentist and Bayesian methodology, and for exponential, Weibull and other distributions. For example, Miyakawa considers a two-component series system of "exponential" components and derives closed-form expressions for the MLE [2]. Under the same exponential assumption, Ref. [3] extended the Miyakawa results to a three-component system. It is shown that, in all but a few special cases, close-form MLE is intractable, and a simple iterative solution was proposed. Ref. [1] finds the exact MLE in the three-component case by deriving a single quadratic equation. Recently a Bayes methodology for estimating component reliability from masked system-life data was proposed [4]. It allows analysts to directly quantify prior engineering judgment in the development of component reliability estimates. In addition to exponential distributions, reliability estimation for Weibull components from masked system life data was also studied [5,6]. Other different distributions such as Pareto distribution and linear failure rate distribution were considered in [7,8]. To address strong symmetric assumption, a two-stage experimental procedure was recommended and studied [9–11]. In the procedure a sample of the masked cases obtained in stage-1 are subjected to sufficient failure analysis to resolve the causes of each of the sampled masked cases. Generally close-form MLE of component reliability from masked system testing data is intractable in most cases and numerical iterative procedures were developed. However, convergence of the proposed iteration

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processes is not available and in most cases only series systems are taken into account. Though estimating reliability of components in series systems in presence of uncertainty has been intensively studied, few studies have been seen for parallel systems [12].

In this paper, we estimate component reliability for both series and parallel systems. Exponential distributions are assumed, thus the goal is to estimate component failure rates from uncertain system data. In addition to data masking, we also consider censoring and their combinations as well. The combinations of data masking and censoring then include:

- uncertain status and certain test time;
- certain status and uncertain test time;
- uncertain status and uncertain test time.

To efficiently estimate component failure rates for both series and parallel systems in presence of data masking and censored lifetime, we propose the concept of equivalent failure and test time to decouple component interactions and derive component inferences directly. The similar idea of equivalent failures has been successfully applied to estimate component failure probability from masked binomial system test data [13]. For a given series or parallel system with known status and censored or uncensored test time, we define equivalent failure and test time for individual components which are associated with uncertainty. The definitions are based on Bayes theorem and probability theory. They are simple and intuitive. From defined equivalent values, we develop an algorithm to estimate component failure rates. Since the algorithm is an EM algorithm, it always converges. The EM theorem, first proposed by Dempster [14], is an elaborate technique for solving the MLE problem when the observed data is incomplete or has missing values. EM algorithm is remarkable because of its simplicity, generality and convergence. A good tutorial on the EM algorithm and its application can be found in [15].

The rest of paper is organized as follows. In Section 2 notation and assumptions are given. We identify uncertainty regarding system test results in Section 3. In Section 4, we define equivalent failure and test time for individual components and unique distributions. The EM algorithm is presented in Section 5. Two numerical examples are given in Section 6. Finally we conclude in Section 7.

2. Notation and assumptions

2.1. Notation

n	number of systems observed
m	number of components in the n observed systems
k	number of unique exponential distributions in the m components, $k \leqslant m$
$[F(\bullet), R(\bullet), f(\bullet)]$	[cdf, sf, pdf] of components and subsystems
λ_j	failure rate of component j
$\hat{\lambda}_l$	failure rate of distribution l
θ	parameter set $(\lambda_1, \lambda_2, \ldots, \lambda_k)$
θ'	known parameter value set $(\lambda'_1, \lambda'_2, \ldots, \lambda'_k)$
C^i	set of components in system i
U^i	sub set of components in system i whose status or life time is unknown, $U^i \subseteq C^i$
s^i	subsystem i consisting of components in U^i
δ^i	status of subsystem i, $\delta^i = 1$, failure; $\delta^i = 0$, survival
η^i	$\eta^i = \frac{1}{0}$ indicating failure terminated/left censored failure of subsystem i
z^i_j	status of component j in system i; $z^i_j = 1$, failure; $z^i_j = 0$, survival
ζ^i_j	$\zeta^i_j = \frac{1}{0}$, indicating failure terminated/left censored failure of component j in system i
t^i	failure, survival, or left censored time of system i
t^i_j	failure or survival time of component j in system or sub system i
$1 \cdot \{\,\}$	indicating function defined as $1 \cdot \{\text{True}\} = 1$, $1 \cdot \{\text{False}\} = 0$
n^{*i}_j	equivalent failure of component j in system or subsystem i
N^*_j	total equivalent failure of component j
t^{*i}_j	equivalent test time of component j in system or sub system i
T^*_j	total equivalent test time of component j
$\tilde{N}^*_l$	total equivalent failures of distribution l
$\tilde{T}^*_l$	total equivalent test time of distribution l
$\mathrm{LogL}(\bullet)$	The log likelihood function

In this context, components having same failure rate can appear in the same system or different systems, thus the number of unique distributions (failure rates) may be less than the number of components. For $\lambda_j = \hat{\lambda}_l$, it means that component j has the lth distribution defined by failure rate $\hat{\lambda}_l$.

2.2. Assumptions

1. The n systems are s-independent.
2. The m components are s-independent and follow exponential distributions.
3. Systems, subsystems and components have only two statuses: failure and survival.
4. The status of subseries or parallel systems is known.

5. When subsystems fail, components that do not fail stop functioning.

2.3. Problem statement

The problem is to obtain MLE of failure rates in k different exponential distributions from censored and masked system data described by n, m, C^i, U^i, s^i, δ^i, η^i, z^i_j, ζ^i_j, t^i, t^i_j, $j \leqslant m$ and $i \leqslant n$. The log likelihood function regarding the system testing data can be written as

$$\begin{aligned}\mathrm{LogL}(\bullet) = {} & \sum_{i=1}^{n}[\delta^i[\eta^i \log f_{s^i}(t^i) + (1-\eta^i)\log F_{s^i}(t^i)] \\ & + (1-\delta_i)\log R_{s^i}(t^i)] + \sum_{i=1}^{n}\sum_{\substack{j\in C^i \\ j\notin U^i}}[z^i_j[\zeta^i_j \log f_j(t^i_j) \\ & + (1-\zeta^i_j)\log F_j(t^i_j)] + (1-z^i_j)\log R_j(t^i_j)].\end{aligned}$$

The first part corresponds to the subsystems in presence of uncertainty. The second part is for all components that have certain status and lifetime. When a large number of components exist, it is difficult to solve the MLE problem by obtaining the derivatives of the log likelihood function regarding to the failure rates. We need to develop an efficient approach to estimating failure rates from system life data in presence of uncertainty.

3. Uncertainty classification

We place components in each system into two sets: U^i and $C^i - U^i$. Components in U^i are associated with unknown status or test time, and components in $C^i - U^i$ are certain in both status and test time. By removing components in $C^i - U^i$ from the original system, the remaining components in U^i form a subsystem with known structure, status and test time. For example, consider a series–parallel system in Fig. 1. If we know that component A survives, then B and C are in parallel after removing A. If component B fails, then A and C are in series after removing B. In this paper we assume the structure of all subsystems is known to be series or parallel. Notice that the structure of a subsystem is not necessarily the same as the structure of its parent system.

There is variant uncertainty in component status and test time based on subsystem test result. In this paper we consider the following three test results for subsystems and components:

- survival, meaning subsystems or components survive at observed time;
- failure terminated, meaning subsystems or components fail at observed time;
- left censored failure, meaning subsystems or components fail before the observed time.

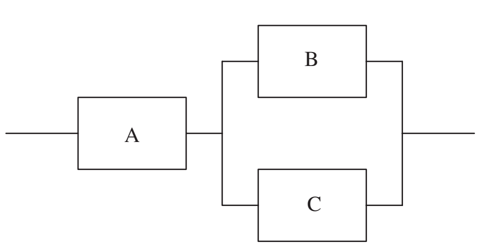

Fig. 1. An example of a series–parallel system.

In the following, we will identify different uncertainty in terms of component status and test time when subsystems have different test results. We consider both series and parallel systems.

3.1. Uncertainty in series systems

3.1.1. Case I–survival

When a series system survives at some time, all its components must survive and have the same test time of the system. In this case, there is no uncertainty regarding component status and test time.

3.1.2. Case II—failure terminated

In this case, we do not know which component causes the system failure. There is uncertainty in component status, which is also called data masking. There is no uncertainty in component test time, which is the same as the system test time.

3.1.3. Case III—left censored

In this case, both component status and test time are unknown. In fact, the actual failure time of the system is also unknown. But we know that the test time for all components is the same as the system test time.

3.2. Uncertainty in parallel systems

3.2.1. Case I—survival

In this case, regarding component status, we only know that at least one component survives, but we do not know exactly which component fails or survives. For failed components, their test time is also unknown. For survived components, their test time is the system test time. In summary, in this case, both component status and test time are uncertain.

3.2.2. Case II—failure terminated

In this case, all components fail. Regarding test time, we only know that at least one component fails at the time when the system fails. Some other components may fail before the system fails. There is uncertainty in component test time.

3.2.3. Case III—left censored

In this case, all components fail by the system censored time. In fact, the actual system failure time is also

Table 1
Uncertainty in component status and life time based on system status. "X" stands for existence of uncertainty

System status	Survival	Failure terminated	Left censored failure
Series components			
Status		X	X
Time			X
Parallel components			
Status	X		
Time	X	X	X

unknown. Thus, there is uncertainty in component test time. It is different from Case II because in Case II there is at least one component that must fail when the system fails.

For the three results of system testing, the possible combinations of data masking and left censored time for individual components are summarized in Table 1.

4. Equivalent failure and lifetime

4.1. Concept

To illustrate the concept of equivalence, let us consider the case in which there is no uncertainty in system life data, meaning the status and test time of components in every system are known. Instead of developing the likelihood function, we may want to take another approach. We can group all system test data by components and then separately estimate reliability for each component without considering other components. This approach is very efficient because it decouples component interactions. It decomposes a big problem into several much smaller problems and then solves the smaller problems separately. The same approach applies when uncertainty is present if we can define appropriate equivalent failure and test time.

The concept of equivalence is quite intuitive and straightforward. We say equivalence in the sense that the equivalent value has the same power to estimate concerned interest as real values. For instance, let us consider estimating failure rate of an exponential distribution. Assume we observe five failures in a total of 10 h test time. The MLE of the failure rate would be 0.5/h. Now we test a component with the same distribution in 1 h. If we do not know the status of the component, we can reasonably define the equivalent failure as 0.5 because we will have the same failure rate estimation. The key issue here is to define appropriate equivalent values for concerned interests when situation is complicated.

In this section, we define equivalent failure and test time for components to estimate component and distribution reliability. We derive equivalent failure and test time by applying EM theorem, which is a complex process and will not be presented here. Fortunately, the derived equivalent failures and times can be described in a simple and intuitive manner. Simply put, the equivalent failure of a component is the conditional failure probability and the equivalent test time is the expectation of the test time, given the system status and test time. Given the system status and test time, the distribution of component status at certain time and the distribution of component failure time can be derived by applying Bayes theorem and probability theory. Theoretically, therefore, the equivalent failure and test can always be calculated for any systems in which Bayes theorem and probability theory apply.

In the following, we will present derived component equivalent failure and test time in different cases of uncertainty for series and parallel systems. With limited length, the details in derivation are not included.

4.2. Components with certain status and test time

For generality and completeness only, we also define equivalent status and test time for components without uncertainty involved. For components with certain status and test time in each system, we simply have

$$n_j^{*i} = z_j^i, \tag{1}$$

$$t_j^{*i} = t_j^i. \tag{2}$$

In most case, the component test time is the same as the system test time.

4.3. Uncertain components in series systems

For series systems, we consider three cases regarding the status of systems at considered time.

4.3.1. Case I—survival

As discussed in Section 3.2.1, there is no uncertainty in this case. For any component j in subsystem i, we have

$$n_j^{*i} = 0, \tag{3}$$

$$t_j^{*i} = t^i. \tag{4}$$

4.3.2. Case II—failure terminated

In this case, the failure rate of the subsystem i with assuming known component failure rates is

$$\lambda'_{s^i} = \sum_{q \in U^i} \lambda'_q. \tag{5}$$

Then the equivalent failure and test time for component j can be expressed as

$$n_j^{*i} = \Pr(z_j^i = 1 | \delta^i = 1, t^i; \theta') = \lambda'_j / \lambda'_{s^i}, \tag{6}$$

$$t_j^{*i} = t^i. \tag{7}$$

4.3.3. Case III—left censored failure

Let $R_{s^i}(t)$ and $F_{s^i}(t)$ be the survival and cumulative distribution function of the subsystem i. Given the subsystem fails before t^i, we have the mean time to failure of the subsystem i as

$$E_{f_{s^i}(t|t<t^i;\theta')}[t]=\frac{1}{\lambda'_{s^i}}-\frac{t^i R_{s^i}(t^i)}{F_{s^i}(t^i)}=\frac{1}{\lambda'_{s^i}}-\frac{t^i \mathrm{e}^{-\lambda' s^i t^i}}{1-\mathrm{e}^{-\lambda' s^i t^i}}. \quad (8)$$

The equivalent failure and test time for any component $j \in U^i$ in the subsystem i are defined as

$$n_j^{*i}=\Pr(z_i^j=1|\delta^i=1,t^i;\theta')=\lambda'_j/\lambda'_{s^i}, \quad (9)$$

$$t_j^{*i}=E_{f_{s^i}(t|t<t^i)}[t]=\frac{1}{\lambda'_{s^i}}-\frac{t^i \mathrm{e}^{-\lambda' s^i t^i}}{1-\mathrm{e}^{-\lambda' s^i t^i}}, \quad (10)$$

in which λ'_{s^i} is defined in Eq. (5). Notice that we apply the assumption that when a system fails, components that do not fail stop functioning and do not fail.

4.4. Uncertain components in parallel systems

Similarly, we consider three cases for parallel systems regarding the status of systems at the considered time.

4.4.1. Case I—survival

In this case, at least one component in the subsystem survives. Based on Bayes theorem, the survival probability of component j at t^i is

$$\Pr(z_j^i=0|\delta^i=0,t^i;\theta')=\frac{\mathrm{e}^{-\lambda'_j t^i}}{1-\prod_{q\in U^i}(1-\mathrm{e}^{-\lambda_q t^i})}. \quad (11)$$

Then the equivalent failure of component j is

$$\begin{aligned} n_j^{*i}&=1-\Pr(z_j^i=0|\delta^i=0,t^i;\theta')\\ &=1-\frac{\mathrm{e}^{-\lambda'_j t^i}}{1-\prod_{q\in U^i}(1-\mathrm{e}^{-\lambda'_q t^i})}. \end{aligned} \quad (12)$$

To calculate equivalent test time, two conditions are considered. In the first condition, the component fails at t^i and the test time is t^i. In the second condition, the component fails before t^i and the test time is the mean time to failure given that the component fails before t^i, which is

$$E_{f_j(t|t\leqslant t^i;\theta')}[t]=\frac{1}{\lambda'_j}-\frac{t^i \mathrm{e}^{-\lambda'_j t^i}}{1-\mathrm{e}^{-\lambda'_j t^i}}. \quad (13)$$

To combine the two conditions together, we have the equivalent test time for component j as

$$t_j^{*i}=(1-n_j^{*i})t^i+n_j^{*i}\left(\frac{1}{\lambda'_j}-\frac{t^i \mathrm{e}^{-\lambda'_j t^i}}{1-\mathrm{e}^{-\lambda'_j t^i}}\right), \quad (14)$$

in which n_j^{*i} is defined in Eq. (12).

4.4.2. Case II—failure terminated

In this case, all components in the subsystem fail by t^i. The equivalent failure for all components in the subsystem is 1. That is,

$$n_j^{*i}=1.$$

Similar to Case I, two conditions are considered to calculate equivalent test time. In the first condition, component j fails at t^i, and in the second condition, the component fails before t^i. Based on the Bayes theorem, the conditional probability for component j to fail at t^i given that the subsystem fails at t^i is

$$\begin{aligned} &\Pr(z_j^i=1,t_j^i=t^i|\delta^i=1,\eta^i=1,t^i;\theta')\\ &=\left(\frac{f_j(t^i)}{F_j(t^i)}\right)\Bigg/\left(\sum_{q\in U^i}\frac{f_q(t^i)}{F_q(t^i)}\right)\\ &=\left(\frac{\lambda'_j \mathrm{e}^{-\lambda'_j t^i}}{1-\mathrm{e}^{-\lambda'_j t^i}}\right)\Bigg/\left(\sum_{q\in U^i}\frac{\lambda'_q \mathrm{e}^{-\lambda'_q t^i}}{1-\mathrm{e}^{-\lambda'_q t^i}}\right). \end{aligned} \quad (15)$$

Thus, the equivalent test time is

$$\begin{aligned} t_j^{*i}=&\left(\frac{1}{\lambda'_j}-\frac{t^i \mathrm{e}^{-\lambda'_j t^i}}{1-\mathrm{e}^{-\lambda'_j t^i}}\right)\\ &\times\left(1-\left(\frac{\lambda'_j \mathrm{e}^{-\lambda'_j t^i}}{1-\mathrm{e}^{-\lambda'_j t^i}}\right)\Bigg/\left(\sum_{q\in U^i}\frac{\lambda'_q \mathrm{e}^{-\lambda'_q t^i}}{1-\mathrm{e}^{-\lambda'_q t^i}}\right)\right)\\ &+t^i\left(\frac{\lambda'_j \mathrm{e}^{-\lambda'_j t^i}}{1-\mathrm{e}^{-\lambda'_j t^i}}\right)\Bigg/\left(\sum_{q\in U^i}\frac{\lambda'_q \mathrm{e}^{-\lambda'_q t^i}}{1-\mathrm{e}^{-\lambda'_q t^i}}\right). \end{aligned} \quad (16)$$

The first part corresponds to the second condition and the second part corresponds to the first condition.

4.4.3. Case III—left censored failure

In this case, all components fail by t^i. Thus, the equivalent failure for every component in the subsystem is

$$n_j^{*i}=1.$$

Since every component may fail any time before t^i, its equivalent test time is the mean time to failure given that it fails before t^i, that is,

$$t_j^{*i}=E_{f_j(t|t\leqslant t^i;\theta')}[t]=\frac{1}{\lambda'_j}-\frac{t^i \mathrm{e}^{-\lambda'_j t^i}}{1-\mathrm{e}^{-\lambda'_j t^i}}. \quad (17)$$

4.5. Total equivalent failure and test time for components and distributions

With assumption of independence between systems, the total equivalent failure and test time for components is the sum of equivalent failure and test time in each system. That is, for any $j=1,2,\ldots,m$, we have

$$N_j^*=\sum_{i=1}^{n}1\cdot\{j\in C^i\}n_j^{*i}, \quad (18)$$

$$T_j^*=\sum_{i=1}^{n}1\cdot\{j\in C^i\}t_j^{*i}. \quad (19)$$

When unique distributions rather than individual components are considered, the total equivalent failure and test time for any distribution $l = 1, 2, \ldots, k$ are

$$\tilde{N}_l^* = \sum_{j=1}^{m} 1 \cdot \{\lambda_j = \tilde{\lambda}_l\} N_j^*, \tag{20}$$

$$\tilde{T}_l^* = \sum_{j=1}^{m} 1 \cdot \{\lambda_j = \tilde{\lambda}_l\} T_j^*. \tag{21}$$

5. MLE of component/distribution failure rates

Based on the EM theory, we have the following theorem:

Theorem: *The failure rate $\tilde{\lambda}_l$ in the lth distribution can be updated as $\tilde{\lambda}_l = \tilde{N}_l^*/\tilde{T}_l^*$, for $l = 1, 2, \ldots, k$.*

The theorem is intuitive for exponential distributions. As we know, to estimate failure rate in exponential distribution, we only need to know the total number of failures and total test time. When uncertainty is present, we use total equivalent failures and equivalent test time.

By summary, we have the following algorithm to estimate failure rate in exponential distributions from uncertain system life data.

Algorithm—estimation of component failure rates from uncertain system life data
Input: Censored and masked system life data
Output: MLE of failure rates in unique exponential distributions
Process steps:
1. Initialize unique distribution and component failure rates.
2. For each system $i = 1,2,\ldots,n$, calculate equivalent failure and test time for any component $j \in C^i$ by using equations in Sections 4.2–4.4.
3. Calculate total equivalent failures $\tilde{N}_l$ and total test time $\tilde{T}_l$ for distributions $l = 1,2,\ldots,k$ by using equations in Section 4.5.
4. Update the distribution failure rates by $\tilde{\lambda}_l = \tilde{N}_l^*/\tilde{T}_l^*$, for $l = 1,2,\ldots,k$. Update the component failure rates by letting $\lambda_j = \tilde{\lambda}_l$ if component j follows l distribution, for $j = 1,2,3,\ldots,m$.
5. Repeat steps 2 and 4 until the estimates converge.

In the first step, we can initialize unique distribution failure rates by using total maximal failures divided by total maximal test time. The total maximal failures are obtained by letting all unknown status be certain failures in all systems. Similarly, the total maximal test time is derived by letting component test time be the system test time.

6. Numerical examples

6.1. Example 1

To illustrate the application of the algorithm, we present two examples in this section. In the first example, we apply the algorithm to the systems and data used in [1]. The data is listed in Table 2. The data set includes 30 series systems consisting of three unique components. Uncertainty is in the form of data masking only. The obtained estimates and the one given in [1] are listed in Table 3. To make a comparison, the log likelihood function is calculated for both estimates. Our estimates have higher likelihood function than the ones obtained in [1], an indication of better accuracy in estimation.

Table 2
Simulated series system life data with masking from [1]

System i	Failure time t^i	Failing components j	System i	Failure time t^i	Failing components j
1	0.021	2	16	0.281	1
2	0.038	1, 2	17	0.295	2
3	0.054	3	18	0.310	3
4	0.066	3	19	0.338	3
5	0.076	1, 2	20	0.341	2
6	0.078	2, 3	21	0.354	1
7	0.123	3	22	0.358	2
8	0.130	1, 3	23	0.431	1, 2, 3
9	0.152	1, 2, 3	24	0.457	3
10	0.159	1	25	0.545	1, 2, 3
11	0.199	3	26	0.569	2
12	0.201	1	27	0.677	3
13	0.204	1	28	0.818	2
14	0.215	2, 3	29	0.946	2, 3
15	0.218	1, 2	30	1.486	1

6.2. Example 2

In this example, we consider a mixture of series and parallel systems with all types of uncertainty discussed in Section 4. Since real data with uncertainty, specifically for parallel systems, is unavailable, we randomly generated a data set as listed in Table 4. The data includes 25 series and 25 parallel systems. Each system consists of two or three components. In total there are three unique components with unique failure rates. The system status at observation time includes survival, failure terminated, and left censored failure. The subsystems with uncertain component status or test time are identified.

We applied the algorithm to the life data in Table 4. By using the maximum failures and test time, we had the initial values of 0.983189, 1.12198, and1.07881 for the three failure rates. The convergence of the iteration process is demonstrated in Fig. 2. It shows that the algorithm converges at about the 11th iteration. The estimated parameters after 100 iterations are listed in Table 5. The corresponding likelihood function is calculated for estimated parameters and settings. The two likelihood function values are close to each other, which may be an indication of goodness of our estimation.

7. Conclusions

Estimating component failure rates from system life data is useful and has seen a lot of research and applications. In addition to data masking which is the most studied uncertainty in research and applications, we also considered uncertainty in lifetime and their combinations as well. More significantly, we studied parallel systems as well as series systems. To efficiently estimate component reliability from system life data in presence of complex uncertainty, we proposed the concept of equivalent failure and test time. The concept is very intuitive and useful because it decouples component interactions, thus inferences for individual components can be separately derived in a very efficient manner. We defined equivalent failure and test time of components for series and parallel systems with consideration of several scenarios of uncertainty. The definition follows Bayes theorem and is very straightforward. Based on the defined equivalents, an algorithm was developed. One of the advantages of the algorithm is that it is an EM algorithm such that its convergence has been proved. We expect that the equivalent concept can be applied to systems with more complex configuration, such as series–parallel systems, networks, etc. Theoretically, for any systems with given status and test time, the equivalent failure and test time of components are derivable by applying Bayes theorem. This will enable us to efficiently

Table 3
Parameter estimates for life data in Table 2

Approaches	λ_1	λ_2	λ_3	Log of likelihood
Our approach	0.8957945	1.2143053	1.0565615	−22.1981062
Algorithm in [1]	0.8588016	0.9890027	1.1136952	−22.2584868

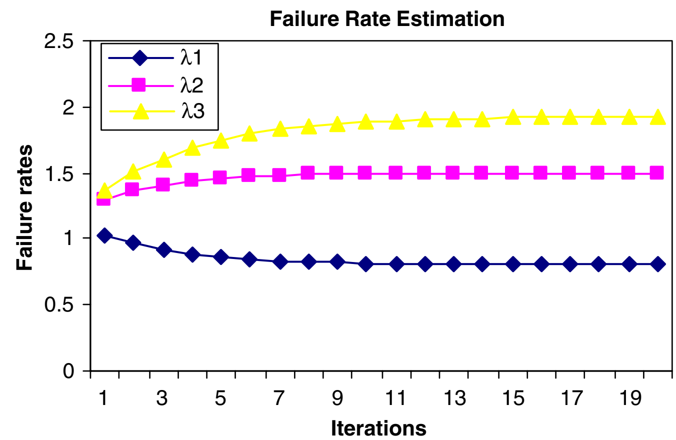

Fig. 2. Failure rate estimation process in Example 2.

Table 4
System life data

S^i	S-P	C^i	t^i	R	U^i	S^i	S-P	C^i	t^i	R	U^i
1	1	{3 1}	1.185	1	{3 1}	26	2	{3 1}	1.716	1	{3 1}
2	1	{2 3}	0.052	1	{2 3}	27	2	{2 1}	0.963	1	{2 1}
3	1	{1 3}	1.617	2	{1 3}	28	2	{3 2}	3.996	2	{3 2}
4	1	{3 2}	0.060	1	{3 2}	29	2	{1 2}	0.375	1	{1 2}
5	1	{3 2}	0.468	1	{3 2}	30	2	{3 2 1}	1.235	1	{3 2 1}
6	1	{3 1 2}	0.093	1	{3 1 2}	31	2	{3 1}	1.254	1	{3 1}
7	1	{1 3 2}	0.045	0	{ }	32	2	{2 3 1}	0.331	0	{2 3 1}
8	1	{2 1}	0.078	0	{ }	33	2	{1 3 2 }	0.337	0	{1 3 2}
9	1	{2 3 1}	0.015	0	{ }	34	2	{2 1}	0.814	0	{2 1}
10	1	{3 1 2}	0.109	1	{3 1 2}	35	2	{2 3 1}	1.696	0	{2 3 1}
11	1	{3 1}	0.285	1	{3 1}	36	2	{3 1 2}	1.657	1	{3 1 2}
12	1	{1 3 2}	0.016	1	{1 3 2}	37	2	{3 2}	1.234	1	{3 2}
13	1	{1 2}	0.007	0	{ }	38	2	{3 2 1}	1.130	1	{3 2 1}
14	1	{3 1}	0.658	1	{3 1}	39	2	{3 1}	0.940	1	{3 1}
15	1	{3 2}	0.415	1	{3}	40	2	{1 3}	0.839	2	{1 3}
16	1	{2 3}	0.137	1	{2 3 1}	41	2	{3 1 2}	1.472	1	{3 1 2}
17	1	{2 3}	0.826	2	{2 3}	42	2	{2 1}	0.281	0	{2 1}
18	1	{2 1}	0.635	1	{2}	43	2	{2 3 1}	2.599	1	{2 3 1}
19	1	{3 2 1}	0.020	1	{3 2 1}	44	2	{3 1}	3.033	2	{3 1}
20	1	{3 2 }	0.302	2	{3 2}	45	2	{2 1}	2.867	1	{2 1}
21	1	{2 3 1}	0.170	1	{2 3 1}	46	2	{3 1 2}	0.666	0	{3 1 2}
22	1	{3 1 2 }	0.985	2	{3}	47	2	{3 2}	0.437	1	{3 2}
23	1	{2 1 }	0.129	1	{2}	48	2	{2 1}	1.642	1	{2 1}
24	1	{2 1 }	0.303	1	{2 1}	49	2	{1 2}	0.562	1	{1 2}
25	1	{2 1 }	0.254	0	{ }	50	2	{1 3}	0.414	1	{1 3}

In the table, S^i is the system; S-P presents system type: 1 for series and 2 for parallel; C^i is the set of components in the system; t^i is the system observation time; R stands for system observation results: 1 for failure terminated, 2 for left censored failure, and 0 for survival; U^i is the set of components associated with uncertainty.

Table 5
Parameter estimates for life data in Table 5

Approach	λ_1	λ_2	λ_3	Log of likelihood
Our estimates	0.81017238	1.49318810	1.93219218	−19.32104001
Settings	0.8	1.2	2.0	−19.27585104

estimate reliability of components in complex systems when uncertainty is present.

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