Component Cause of Failure in a Series System with Masked Component Cause of Failure

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0.1 Probability of Component Cause of System Failure

This subsection is not necessary in our likelihood model, but it derives a useful result for making predictions about the component cause of failure.

Suppose we have observed a candidate set and a series system failure and we are interested in the probability that a particular component is the cause of failure.

Theorem 1. *Assuming Conditions* **??** *and* **??***, the conditional probability of the component cause of failure is component j ($K_i = j$) given a masked component cause of failure ($\mathcal{C}_i = c_i$) and system lifetime ($T_i = t_i$) is given by*

$$\Pr\{K_i = j | T_i = t_i, \mathcal{C}_i = c_i\} = \frac{h_j(t_i | \boldsymbol{\theta_j})}{\sum_{l \in c_i} h_l(t_i | \boldsymbol{\theta_l})} 1_{\{j \in c_i\}}. \tag{0.1}$$

Proof. The conditional probability $\Pr\{K_i = j | T_i = t_i, \mathcal{C}_i = c_i\}$ may be written as

$$\Pr\{K_i = j | T_i = t_i, \mathcal{C}_i = c_i\} = \frac{\Pr_{\boldsymbol{\theta}}\{\mathcal{C}_i = c_i | K_i = j, T_i = t_i\} f_{K_i,T_i}(j, t_i; \boldsymbol{\theta})}{\sum_{j=1}^m \Pr_{\boldsymbol{\theta}}\{\mathcal{C}_i = c_i | K_i = j, T_i = t_i\} f_{K_i,T_i}(j, t_i; \boldsymbol{\theta})}.$$

By Theorem **??**, $f_{K_i,T_i}(j, t_i; \boldsymbol{\theta}) = h_j(t_i; \boldsymbol{\theta}) R_{T_i}(t_i; \boldsymbol{\theta})$. We may make this substitution and simplify:

$$\Pr\{K_i = j | T_i = t_i, \mathcal{C}_i = c_i\} = \frac{\Pr_{\boldsymbol{\theta}}\{\mathcal{C}_i = c_i | K_i = j, T_i = t_i\} h_j(t_i; \boldsymbol{\theta_j})}{\sum_{j'=1}^m \Pr_{\boldsymbol{\theta}}\{\mathcal{C}_i = c_i | K_i = j', T_i = t_i\} h_{j'}(t_i; \boldsymbol{\theta_{j'}})}.$$

Assuming Conditions **??** and **??**, we may rewrite the above as

$$\Pr\{K_i = j | T_i = t_i, \mathcal{C}_i = c_i\} = \frac{\Pr_{\boldsymbol{\theta}}\{\mathcal{C}_i = c_i | K_i = j, T_i = t_i\} h_j(t_i; \boldsymbol{\theta_j})}{\Pr_{\boldsymbol{\theta}}\{\mathcal{C}_i = c_i | K_i = j, T_i = t_i\} \sum_{l \in c_i} h_l(t_i; \boldsymbol{\theta_l})} = \frac{h_j(t_i; \boldsymbol{\theta_j})}{\sum_{l \in c_i} h_l(t_i; \boldsymbol{\theta_l})}.$$

□

Frequently, we may not have any information at all about the component cause of failure. In this case, $c_i = \{1, \ldots, m\}$, and we obtain the following corollary.

Corollary 1. *The probability that the j^{th} component is the cause of system failure given only that we know a system failure occured at time t_i is given by*

$$\Pr\{K_i = j | T_i = t_i\} = \frac{h_j(t_i; \boldsymbol{\theta_j})}{\sum_{l=1}^m h_l(t_i; \boldsymbol{\theta_l})}.$$