Reliability Estimation in Series Systems

Maximum Likelihood Techniques for Right-Censored and Masked Failure Data

Alex Towell

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Introduction

Context & Motivation

- Quantifying reliability in series systems is essential.
- Real-world systems often only provide system-level failure data.
  - Masked and right-censored data obscure reliability metrics.
- Need robust techniques to decipher this data and make accurate estimations.

Core Contributions

- Derivation of likelihood model that accounts for right-censoring and masking.
  - Trivial to add more failure data via a likelihood contribution model.
  - R Library: github.com/queelius/wei.series.md.c1.c2.c3
- Clarification of the assumptions required for the likelihood model.
- Simulation studies with Weibull distributed component lifetimes.
  - Assess performance of MLE and BCa confidence intervals under various scenarios.

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Series System

$$x_1 \rightarrow x_2 \rightarrow x_3 \rightarrow x_4 \rightarrow x_5$$

- **Main Concept**: If one component fails, the entire system fails.

Reliability Function

Definition: Probability a system/component works beyond time t:

$$R_X(x) = \Pr\{X > x\}.$$

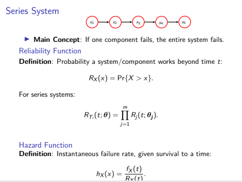
For series systems:

$$R_{T_i}(t; \boldsymbol{\theta}) = \prod_{j=1}^{m} R_j(t; \boldsymbol{\theta_j}).$$

Hazard Function

Definition: Instantaneous failure rate, given survival to a time:

$$h_X(x) = \frac{f_X(t)}{R_X(t)}.$$



Remember the analogy: "A chain is only as strong as its weakest link."High reliability = low failure probability. Used directly in likelihood models for right-censoring events.Core of many reliability analyses. Influences system design and maintenance decisions.Useful for guiding maintenance and interventions based on failure patterns.Critical for understanding masked failures in our likelihood model.A well-designed series system has components with matching MTTFs and failure causes. The simulation study focuses on such systems.

Likelihood Model

We have our system model, but we don't observe component lifetimes. We observe data related to component lifetimes.

Observed Data

- Right censoring: No failure observed.
  - The experiment ended before the system failed.
    - τ is the right-censoring time.
    - $\delta_i = 0$ indicates right-censoring for system i.
- Masked causes
  - The system failed, but we don't know the component cause.
    - S_i is the observed time of system failure.
    - $\delta_i = 1$ indicates system failure for system i.
    - $\mathcal{C}_i$ are a subset of components that could have caused failure.

Observed Data Example

Observed data with a right-censoring time $\tau = 5$ for a series system with 3 components.

System	Right-censored lifetime	Event indicator	Candidate set
1	1.1	1	$\{1, 2\}$

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Series System with Weibull Component Lifetimes

The Weibull distribution has been crucial in reliability analysis due to its versatility. In our study, we model a system's components using Weibull distributed lifetimes.

- Introduced by Waloddi Weibull in 1937.
- Reflecting on its utility, Weibull modestly noted: "[...] may sometimes render good service."

Weibull Distribution Characteristics

The lifetime distribution for the j^{th} component of the i^{th} system is:

$$T_{ij} \sim \text{Weibull}(k_j, \lambda_j)$$

Where:

- $\lambda_j > 0$ is the scale parameter.
- $k_j > 0$ is the shape parameter.

Significance of the Shape Parameter:

- $k_j < 1$: Indicates infant mortality. E.g., defective components weeded out early.

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Simulation Study Overview

We conduct a simulation study based on a series system.

System Description

This study is centered around the following *well-designed series system*:

Component	Shape	Scale	MTTF	$\Pr\{K_i\}$
1	1.26	994.37	924.87	0.17
2	1.16	908.95	862.16	0.21
3	1.13	840.11	803.56	0.23
4	1.18	940.13	888.24	0.20
5	1.20	923.16	867.75	0.20
System	NA	NA	222.88	NA

Performance Metrics

Our main objective is to evaluate the MLE and BCa confidence intervals' performance across various scenarios.

- **MLE Evaluation:**

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Conclusion

Part 1

Key Findings

- Employed maximum likelihood techniques for component reliability estimation in series systems with masked failure data.
- Methods performed robustly despite masking and right-censoring challenges.

Simulation Insights

- Right-censoring and masking introduce positive bias; more reliable components are most affected.
- Shape parameters harder to estimate than scale.
- Large samples can counteract these challenges.

Part 2

Confidence Intervals

- Bootstrapped BCa CIs demonstrated commendable coverage probabilities, even in smaller sample sizes.

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Discussion