Reliability Estimation in Series Systems

Maximum Likelihood Techniques for Right-Censored and Masked Failure Data

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Goal

- Estimating reliability of individual components from system-level data can be challenging.
- Failure times and causes often unobserved.
- Developed likelihood model to leverage observed data:
  - Right-censoring
  - Candidate sets indicative of failure causes
- Use maximum likelihood estimation to estimate component parameters.
  - Bootstrap confidence intervals.
- Conduct simulation studies to assess precision and accuracy of estimates.

Section 1

Series System Model

Series System Model

Figure 1: Me, reduced to a 3-component series system

- A lot of complex systems have *critical* components.
  - "A chain is only as strong as its weakest link."
- We call these *series systems*.

Parametric Model

We use a parametric model to describe component lifetimes.

"All models are wrong, but some are useful." - George Box

Assumptions

- Component lifetimes (T_{ij}) mutually independent.
- Distributed according to some parametric distribution: $T_{ij} \sim f_j(\cdot; \boldsymbol{\theta_j})$
- System lifetime: components in series configuration: $T_i = \min_j T_{ij}$
- System lifetimes i.i.d.
- System lifetime: $T_i \sim f(t; \boldsymbol{\theta}) = R(t; \boldsymbol{\theta}) h(t; \boldsymbol{\theta})$
  - Reliability function is product of components: $R(t; \boldsymbol{\theta}) = \prod_j R_j(t; \boldsymbol{\theta_j})$
  - Hazard function is sum of component hazards: $h(t; \boldsymbol{\theta}) = \sum_j h_j(t; \boldsymbol{\theta_j})$

Section 2

Likelihood Model

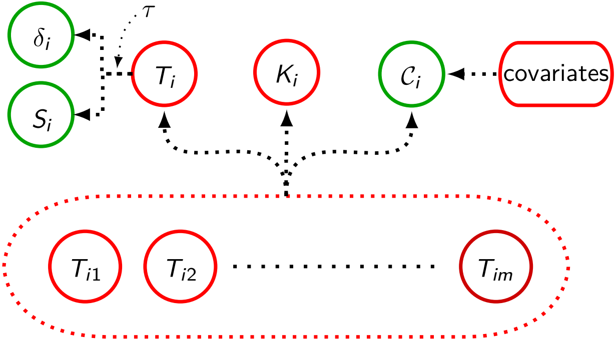
Likelihood Model

We have our system model, but we don't observe component lifetimes. We observe data related to component lifetimes.

Data Generating Process

DGP is underlying process that generates observed data:

- Green elements are observed.
- Red elements are unobserved (latent).
- Candidate sets ($\mathcal{C}_i$) related to component lifetimes (T_{ij}) and other (unknown) covariates.
  - Distribution of candidate sets complex. Seek a simple model valid under certain assumptions.



Observed Data

- Right censoring: No failure observed.
  - The experiment ended before the system failed.
    - τ is the right-censoring time.
    - $\delta_i = 0$ indicates right-censoring for system i.
- Masked causes
  - The system failed, but we don't know the component cause.
    - S_i is the observed time of system failure.
    - $\delta_i = 1$ indicates system failure for system i.
    - $\mathcal{C}_i$ are a subset of components that could have caused failure.

Observed Data Example

Observed data with a right-censoring time $\tau = 5$ for a series system with 3 components.

System	Right-censored lifetime	Event indicator	Candidate set
1	1.1	1	$\{1, 2\}$
2	1.3	1	$\{2\}$
4	2.6	1	$\{2, 3\}$
5	3.7	1	$\{1, 2, 3\}$
6	5	0	$\emptyset$
7	5	0	$\emptyset$

Likelihood Model: Assumptions (Cont.)

- Right-censoring time independent of component lifetimes and parameters:
  - $S_i = \min(\tau, T_i)$
- Candidate sets:
  - Condition 1: Contains failed component (if any).
  - Condition 2: Equal probability within candidate set.
  - Condition 3: Independent of parameters.
- Derived likelihood contribution model:

$$L_i(\boldsymbol{\theta}) = \begin{cases} \prod_{l=1}^{m} R_l(s_i; \boldsymbol{\theta_l}) & \text{if } \delta_i = 0 \\ \beta_i \prod_{l=1}^{m} R_l(s_i; \boldsymbol{\theta_l}) \sum_{j \in c_i} h_j(s_i; \boldsymbol{\theta_j}) & \text{if } \delta_i = 1. \end{cases}$$

- Can add more likelihood contributions.

Estimation Methodology

We seek to estimate component parameters $\boldsymbol{\theta_j}$ from data.

- Maximum Likelihood Estimation
  - Asymptotic normality and efficiency
  - Small sample issues
- Bootstrap Confidence Intervals
  - Approximates sampling distribution
  - Bias correction
  - Acceleration
  - Flexibility for small samples

Simulation Studies

- Assessed accuracy and precision of MLE
- Metrics:
  - Bias
  - Coverage probability
  - Confidence interval width
- Key scenarios:
  - Varying sample size
  - Censoring level
  - Masking probability
  - Component parameters

Conclusion

- Provided likelihood framework for limited data
- MLE accurate under small samples and masking
- Bootstrap CIs well-calibrated
- Shape parameters sensitive, scales more robust