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Estimating system and component reliabilities under partial information on cause of failure

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Abstract: Estimating component reliabilities along with system reliability frequently requires using lifetimes from the system level. Due to cost and time constraints, however, the exact cause of system failure may be unknown. Instead it may only be ascertained that the cause of system failure is due to one component in a subset of components, e.g. the subset forms a subsystem. Confronted with such data, this article discusses how to fully exploit all of the available information using a maximum likelihood approach. We extend and clarify the useful work of Miyakawa (1984). A small Monte Carlo simulation study indicates the helpfulness of this approach.

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1. Introduction

Engineers are often interested in estimating the reliability of components through the analysis of system lifetimes. This involves observing the lifelengths of systems placed onto test, along with the cause of any system failures. The resulting data can

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then be used to estimate the life distribution of the system and each of its components.

Typically this type of analysis is handled by applying a competing risks model. See, for example, Basu and Klein (1982), Basu (1981), Basu and Ghosh (1980), and David and Moeshberger (1978) for helpful material on competing risks. In this model one implicitly assumes that the lifelength and cause of system failures are known. However, Miyakawa (1984) remarked that, "... investigation of the cause of failure is expensive and requires time, and hence sometimes the cause of failure is not observed, even if the failure time is observed." Usher (1987) noted, "... when large computer systems fail in the field, analysis is usually performed such that a small subset of components, perhaps a circuit card, is identified as the cause of failure. In an attempt to repair the system as quickly as possible, the entire subset of components is replaced and the exact failing component may not be investigated further." See also Gross (1970). For related biological data compare Dinse (1986) and Dinse (1982).

In this paper, we develop a likelihood approach for estimating component reliabilities from system life data when the cause of failure may be unknown. We use a parametric approach because of the availability of parametric models in engineering settings. For details on various types of parametric models, see Lawless (1982), Kalbfleisch and Prentice (1980), Gross and Clark (1975), etc. Also, parametric estimators of reliability can be much more efficient than nonparametric ones. Compare Efron (1988) and Miller (1983).

Data for which the cause of failure is narrowed to a subset of components, we call *masked* because the true cause of failure is masked from our knowledge. Note that this can be viewed as a type of censored data. Here, the cause of failure is censored, but the system lifelength may be complete (i.e., uncensored) or censored. We handle both cases related to time and masked data.

For the first case of masked but time complete data, we derive the full likelihood in Section 2. This full likelihood extends and clarifies Miyakawa's (1984) useful likelihood. From our full likelihood, we give a partial likelihood and conditions for its proper statistical use. This provides further insight and understanding of Miyakawa's likelihood. In fact, if the conditions we give are not met, then the estimators based upon Miyakawa's likelihood can actually be inconsistent (i.e., the estimators will not converge in probability to the desired, true parameters). For an example of this, see the Appendix. In Section 3, we develop a likelihood expression for the case where systems are time censored and the cause of system failure is masked.

The techniques presented here were developed through the analysis of actual data on complex electronic systems, such as monitors, graphics display terminals, modems, point-of-sale terminals, and network controllers. However, due to proprietary restrictions, the actual data is unavailable for this article. As such, in Section 4, we perform a small Monte Carlo study to investigate the effects of masking on our proposed estimators. As expected, the mean square error and the bias get worse as masking increases, and they improve with increased sample size. Our concluding remarks about analysis of masked data are presented in Section 5.

2. The likelihood with system time complete data

In this section we deal with the case of system time complete but masked data. We do this first and separately to keep clear the differences between censoring of the cause of failure (masked data) and censoring of the time of failure. We develop a full likelihood for a series system of J components. Note that a similar development is possible if the system is a parallel, I-out-of-J for $1 \leq I \leq J$, or arbitrary configuration of components. The components could also be 'modules' in series. See, for example, Barlow and Proschan (1981) for more on modules.

Consider a sample of n systems each consisting of J components in series. Let T_i be the random life for the i-th system, $i=1,\dots,n$. Let T_{ij} be the random life of the j-th component in the i-th system, $j=1,\dots,J$. Note that

$$T_i = \min(T_{i1},\dots,T_{iJ})$$

for $i=1,\dots,n$. We assume that the T_{ij}'s are independent. For each fixed j the $T_{1j},\dots,T_{nj}$ would represent a random sample from component j's life distribution F_j. We require the mild condition that F_j has a density, f_j indexed by the parameter vector $\boldsymbol{\theta}_j$. For each j, a different number of parameters in $\boldsymbol{\theta}_j$ is allowed if needed. Let $\bar{F}_j(t) = 1 - F_j(t)$ be the reliability of component j at time t.

To precisely derive the likelihood involving masking, we need the following notation. Let K_i be the index of the component causing the failure of system i. (We, of course, assume that the cause of failure K_i is unique.) Note that K_i is a random variable. Also note that K_i may or may not be observed. That is, the component causing system failure may be masked with other components in the system.

Before the sample we are led to the *minimum random subset*, M_i, of components known to contain the true cause of failure of system i. In short, $K_i \in M_i$ and M_i is minimum. After the sample data is obtained, we observe

$$M_i = S_i \subset \{1,2,\dots,J\}, \qquad T_i = t_i, \tag{2.1}$$

where $i=1,\dots,n$. If $S_i = \{j\}$ then we know that $K_i = j$, and hence, the cause of failure is not masked. If, for example, $S_i = \{1,2\}$, we have that $K_i \in S_i$ but the true value of K_i is masked.

From (2.1) the observed data can be expressed as $(t_1,S_1),\dots,(t_n,S_n)$. We now derive the full likelihood for this data. Consider (t_i,S_i) and its contribution c_i to the full likelihood L,

$$c_i = \sum_{j \in S_i} \left(f_j(t_i) \prod_{\substack{s=1 \\ s \neq j}}^{J} \bar{F}_s(t_i) \cdot P(M_i = S_i \mid T_i = t_i,\ K_i = j) \right).$$

The term, $f_j(t_i)\prod_{s=1,s\neq j}^{J}\bar{F}_s(t_i)$ is derived from system i failing at time t_i due to cause (i.e., component) j. The expression $P(M_i=S_i \mid T_i=t_i, K_i=j)$ represents the conditional probability that the observed minimum random subset is S_i *given* that system i failed at time t_i and the true cause was component j. For $S_i=\{j\}$, this expression is the conditional probability that the cause of failure is known. For S_i containing more than j, it yields the conditional probability of masking with the set S_i.

In our industrial problems, we found that masking generally occurred due to constraints of time and the expense of failure analysis. Schedules often dictated that complete failure analysis (to determine the true cause of failure) be curtailed. In this setting we found that for j' fixed and in S_i that

$$P(M_i=S_i \mid T_i=t_i, K_i=j')=P(M_i=S_i \mid T_i=t_i, K_i=j) \quad \text{for all } j\in S_i. \tag{2.2}$$

As a result, this term can be factored out of the summation to yield

$$c_i=P(M_i=S_i \mid T_i=t_i, K_i=j')\cdot \sum_{j\in S_i}\left(f_j(t_i)\prod_{\substack{s=1\\ s\neq j}}^{J}\bar{F}_s(t_i)\right).$$

The full likelihood under (2.2) is then

$$L=\prod_{i=1}^{n} c_i.$$

Note that the masking probabilities can be a function of time. Also, we allow for $P(M_i=S_i \mid T_i=t_i, K_i=j')\neq P(M_i=S_i \mid T_i=t_i, K_i=j)$ for all $j\notin S_i$. We assume only that the masking probabilities conditional on time are not functions of the life distribution parameters. We state this for future reference as:

$$P(M_i=S_i \mid T_i=t_i, K_i=j') \text{ does not depend on the life distribution parameters.} \tag{2.3}$$

This is analogous to a censoring distribution not depending on the life distribution parameters, cf. Miller (1981).

Using (2.3) we write a reduced or partial likelihood as

$$L_{\mathrm{R}}=\prod_{i=1}^{n}\left\{\sum_{j\in S_i}\left(f_j(t_i)\prod_{\substack{s=1\\ s\neq j}}^{J}\bar{F}_s(t_i)\right)\right\}. \tag{2.4}$$

Under (2.2) and (2.3), maximizing L_{R} with respect to the life parameters is equivalent to using L. This is similar to the usual derivation of a time censored (and not masked) data partial likelihood.

The above clarifies and extends Miyakawa's (1984) likelihood. He has m systems with the cause of failure known (not masked) and $n-m$ with only the time of failure known (the cause is masked). If m is random, as found in our actual industrial data, then his likelihood is really a partial likelihood. If m, for some reason, is fixed, then our full and partial likelihood are the same.

We suggest it best to view his likelihood (as well as L_{R}) here as a partial likelihood that under appropriate conditions will yield good, consistent estimators. Without (2.2), and all else true, for example, such a partial likelihood can yield inconsistent estimators. For a simple example of this point, see our Appendix. For proper statistical applications, we stress that it is important to be clearly aware of the effects of masking probabilities and needed conditions.

It should also be noted for $J=2$, the masking probabilities are similar to Dinse's (1986) uncertainty rates. For $J>2$, however, our masking probabilities generalize his uncertainty rates. Also, note that our assumption (2.2) relates to his excellent discussion of equal uncertainty rates in studying diseases.

3. The likelihood with system time censored data

Life testing in general can result in censored life data. That is, to reduce test time and costs, the test may be stopped before all systems in the sample have failed. System time censoring was found to occur in all of the actual industrial data we had. We present the likelihood for that case here. Let Y_i be the random censoring time associated with the i-th system. Let $G_i(t)=P(Y_i\le t)$, $\bar{G}_i(t)=1-G_i(t)$, and $g_i(t)$ be the density of Y_i. Note that we can handle each Y_i having a different censoring distribution. We allow Y_i to be a fixed number if needed. We have

$$\delta_i=I(T_i\le Y_i)=\begin{cases}1 & \text{if } T_i\le Y_i \quad \text{(uncensored)},\\ 0 & \text{if } T_i> Y_i \quad \text{(censored)},\end{cases}$$

and

$$X_i=\min(T_i,\, Y_i).$$

We assume that T_i and Y_i are independent and that $G_i(t)$ does not depend on the life parameters. Note that if $\delta_i=0$ then $M_i=\{1,\dots,J\}$ because we do not observe the cause of failure (as well as not observing the time of failure).

The data is $(X_1,\delta_1,M_1),\dots,(X_n,\delta_n,M_n\}$. By considering the two possibilities of $(X_i=t_i,\delta_i=1,M_i=S_i)$ or $(X_i=t_i,\delta_i=0,M_i=\{1,\dots,J\})$, we combine these two cases and express under (2.2), the likelihood contribution of the i-th observation as

$$c_i=\left\{P(M_i=S_i\mid T_i=t_i,K_i=j')\cdot\left(\sum_{j\in S_i} f_j(t_i)\prod_{\substack{s=1\\ s\ne j}}^{J}\bar{F}_s(t_i)\right)\cdot\bar{G}_i(t_i)\right\}^{\delta_i}$$

$$\cdot\left\{g_i(t_i)\prod_{s=1}^{J}\bar{F}_s(t_i)\right\}^{1-\delta_i}.$$

The full likelihood for the sample is then given as

$$L=\prod_{i=1}^{n} c_i.$$

With the conditions stated above and (2.3), the reduced likelihood is

$$L_{\mathrm{R}}=\prod_{i=1}^{n}\left(\left\{\sum_{j\in S_i}\left(f_j(t_i)\prod_{\substack{s=1\\ s\neq j}}^{J}\bar{F}_s(t_i)\right)\right\}^{\delta_i}\cdot\left\{\prod_{s=1}^{J}\bar{F}_s(t_i)\right\}^{1-\delta_i}\right).$$

Again, maximizing L_{R} with respect to the life parameters under the stated conditions is equivalent to using L.

The likelihood above is for randomly right censored data. For other types of censoring on the system lifetime (e.g., for interval censored), analogous likelihoods can be derived under appropriate conditions.

4. Monte Carlo study

We now investigate, via a small Monte Carlo study, the effects of masking on the bias and mean square error (MSE) of the maximum likelihood (MLEs) derived from L_{R}.

To cover both large and small sample cases, we chose to simulate samples of size $n=10$ and $n=100$. Consider a series system of $J=3$ components. Assume that components 1 and 2 form a subsystem. Further assume that each component has an exponentially distributed lifelength, i.e., the reliability function is given as

$$\bar{F}_j(t)=\mathrm{e}^{-\lambda_j t}\quad \text{for } t\geq 0 \text{ and } j=1,2,3.$$

For simplicity of comparison in the study we assume that $\lambda_1=\lambda_2=\lambda_3=1$. The exponential random variates were generated using the inverse cumulative distribution function method; see Kennedy and Gentle (1980).

To simulate the effect of masking, we randomly masked the true cause of system failure based upon an assumed total masking probability (i.e., a proportion). This total masking proportion was derived from the probabilities of $M_i=\{1,2,3\}$, and $M_i=\{1,2\}$. This meant that we allowed partial masking where the cause of failure was known to be in the subsystem, $\{1,2\}$, or total masking occurred, $\{1,2,3\}$. No other masking was allowed. To satisfy (2.2) we had $P(M_i=\{1,2,3\}\mid T_i=t_i, K_i=j)$ was constant for $j=1,2,3$ and $P(M_i=\{1,2\}\mid T_i=t_i, K_i=j)$ was constant for $j=1,2$. Condition (2.3) was also easily met by assigning the conditional probabilities without a functional dependence on the life parameters, λ_1, λ_2, λ_3.

Let n_1, n_2, and n_3 denote the number failures for $M_i=\{1\}$, $M_i=\{2\}$, and $M_i=\{3\}$ respectively. Let n_{12} denote the count of $M_i=\{1,2\}$, i.e., n_{12} is the number of partially masked failures here. Let n_{123} represent the number of totally masked, $M_i=\{1,2,3\}$, systems. Using (2.4), the MLEs are found to be

$$\hat{\lambda}_1=\frac{n_1+n_{12}\left(\dfrac{n_1}{n_1+n_2}\right)+n_{123}\left(\dfrac{n_1k}{n_1k+n_2k+n_3}\right)}{\sum_{i=1}^{n} t_i},$$

$$\hat{\lambda}_2 = \frac{n_2 + n_{12}\left(\dfrac{n_2}{n_1+n_2}\right) + n_{123}\left(\dfrac{n_2 k}{n_1 k + n_2 k + n_3}\right)}{\sum_{i=1}^{n} t_i},$$

$$\hat{\lambda}_3 = \frac{n_3 + n_{123}\left(\dfrac{n_3}{n_1 k + n_2 k + n_3}\right)}{\sum_{i=1}^{n} t_i},$$

where $k = 1 + n_{12}/(n_1 + n_2)$. For a more detailed account of exponential modeling and masked data, see Usher and Hodgson (1988).

It is useful to note that when no masking is observed, these estimators reduce to the standard MLEs, i.e., the number of failures divided by the total time on test. Also note that with masking, the numerator can be interpreted as an estimate of the number of failures caused by a particular component. Consider, for example, n_{12}. How many of n_{12} should we allocate as failures due to component 1? It is natural

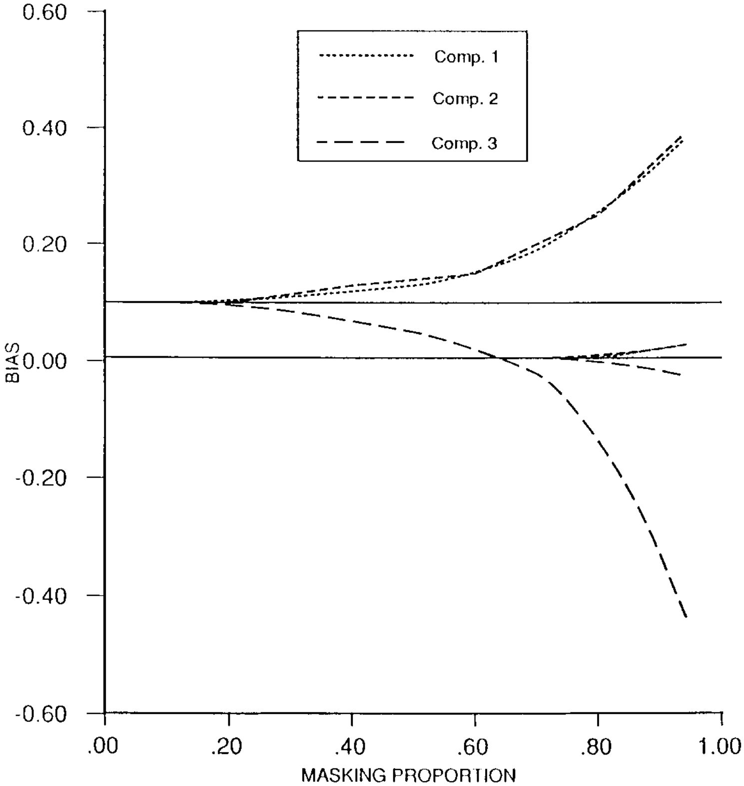

Fig. 1. The effects of masking on bias.

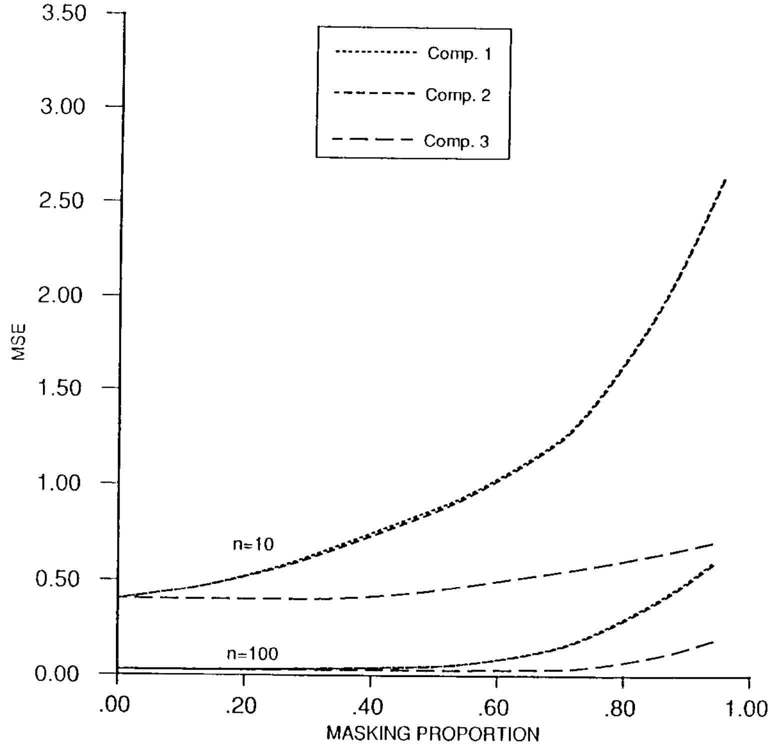

Fig. 2. The effects of masking on mean square error (MSE).

to consider an empirical allocation of $n_{12}(n_1/(n_1+n_2))$. Similar comments apply to the other terms in the numerators.

It is critical to note that these estimators are undefined when $n_1+n_2=0$, i.e., no known causes of failure for components 1 or 2 are observed. We therefore restricted our study to consider only samples where $n_1+n_2>0$. This condition also assured us of not allowing a sample with all the data masked. (Note that this is analogous to censored data Monte Carlo studies where a sample with all censored data is excluded.)

The entire simulation was programmed in FORTRAN and run on a VAX 11/750. The results are based upon 100000 repeated samples. This number replicated was needed for very large masking proportions to assure an adequate number of samples not having every system being masked. This difficulty also led us to simulate masking proportions only up to 95%. The results are shown in Figures 1 and 2.

As expected, the bias and MSE get worse with increased masking. They improve with an increase in the sample size from 10 to 100. For no masking (i.e., the masking proportion is 0) the three estimators are the standard MLEs with bias $1/(n-1)$. In Figure 1 we have graphed the baseline bias for each n as a reference. For Figure 2, the reader could add similar baseline MSEs using the value at proportion 0. Note that the bias and MSE are fairly well behaved in spite of masking effects as high as 50% to 60% (even for $n=10$). Our industrial data showed that common levels of masking are generally found to be much less than that.

Components 1 and 2 have bias and MSE that track together, as would be expected since they are identical components that form a subsystem. Note that for rather large masking proportions, however, they become even more positively biased. To compensate for this, component 3 becomes strongly negatively biased. It seems for very heavy masking, the numerator in $\hat{\lambda}_1$ (and $\hat{\lambda}_2$) may over assign masked failures as due to components 1 (and 2). (Recall our earlier comments about how the numerator is in effect estimating the number of failures due to that component.) These aspects might motivate a search for modified estimators when the masking is very heavy. For $n=100$, however, the bias and MSE do very well even up to 80% or 90% masking.

5. Conclusions

We have presented an approach for estimating component reliabilities from masked system life data. This approach has actually been applied and found useful in a real-world setting. The problems presented here arose in the case of system life testing. We also feel that the approach could be useful in building reliability databases on components through the analysis of actual field data. Under this scenario, masking is very likely to be present. With careful analysis, this data can yield valuable information for the statistical analysis of industrial devices and for planning future system designs.

In analyzing masked data, it is important to understand the mechanism that is causing the masking. If masking probabilities and conditions such as (2.2) and (2.3) are overlooked, the estimators could be inconsistent. Given (2.2) is true, Usher and Guess (1989) present an iterative estimation approach for a simple two-component system subject to masking. Also see, for related work, Usher (1987).

When, for example condition (2.2) is not true, how could a likelihood approach be developed? We are currently working on a modified method based upon the EM algorithm (see, e.g., Cox and Oakes (1984)) to accomplish that.

Finally, a likelihood development suggests building a Bayesian framework for analyzing masked data and the true cause of failure. We are also exploring this construction.

Appendix

We provide a simple but detailed example to illustrate the possibility of an inconsistent estimator when using Miyakawa's (1984) estimator without our condition

(2.2). Miyakawa (1984) uses a series system of two exponential components with failure rates λ_1 and λ_2. His formula (3.4) for the MLE for λ_1 using his likelihood (3.1) (essentially our partial likelihood, L_R, given in (2.4)) was written as

$$\hat{\lambda}_1 = \frac{r}{m} \frac{n}{\sum_{i=1}^n X_i}.$$

We rewrite his $\hat{\lambda}_1$ in our notation of Sections 2 and 4 as

$$\hat{\lambda}_1 = \frac{n_1}{n_1 + n_2} \frac{n}{\sum_{i=1}^n T_i},$$

where $r = n_1$, $m = n_1 + n_2$, etc. For simplicity, in this example we now assume

$$P(M_i = S_i \mid K_i = j) = P(M_i = S_i \mid T_i = t_i, K_i = j).$$

This implies that the masking effect does not have a time effect.

To show inconsistency of the $\hat{\lambda}_i$ based on L_R, we next consider

$$\hat{\lambda}_1 = \frac{n_1/n}{n_1/n + n_2/n} \frac{n}{T^*}$$

where $T^* = \sum_{i=1}^n T_i$. By using the Law of Large Numbers and continuity (cf. Serfling (1980, pp. 24, 26–27)), note that

$$n/T^* \xrightarrow{\text{p}} (\lambda_1 + \lambda_2) \quad \text{as } n \to \infty$$

while by continuity and taking the limit as $n \to \infty$ again we have

$$\frac{n_1/n}{n_1/n + n_2/n}$$

$$\xrightarrow{\text{p}} \frac{(\lambda_1/(\lambda_1+\lambda_2))P(M_1 = \{1\} \mid K_1 = 1)}{(\lambda_1/(\lambda_1+\lambda_2))P(M_1 = \{1\} \mid K_1 = 1) + (\lambda_2/(\lambda_1+\lambda_2))P(M_2 = \{2\} \mid K_2 = 2)}.$$

If our (2.2) holds, then, $P(M_1 = \{1\} \mid K_1 = 1) = P(M_2 = \{2\} \mid K_2 = 2)$ thus, of course, $\hat{\lambda}_1 \xrightarrow{\text{p}} \lambda_1$. If, however, (2.2) is not true then $\hat{\lambda}_1 \overset{\text{p}}{\not\rightarrow} \lambda_1$ can occur. E.g., for $P(M_1 = \{1\} \mid K_1 = 1) = 0.9$, $P(M_2 = \{2\} \mid K_2 = 2) = 0.1$, and $\lambda_1 = \lambda_2 = 1$, then $\hat{\lambda}_1 \overset{\text{p}}{\not\rightarrow} \lambda_1 = 1$. In fact $\hat{\lambda}_1 \xrightarrow{\text{p}} 0.9 \neq \lambda_1$.

Many other examples are possible. Indeed, it is possible for $\hat{\lambda}_1$ to be grossly off if the masking probabilities are extreme and (2.2) does not hold. Again, we stress the importance of understanding the masking probabilities for planning and accurate reliability estimation.

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