ALEX TOWELL

RELIABILITY ESTIMATION IN SERIES SYSTEMS

Contents

1 Introduction

1.1 Presentation Overview

- Reliability estimation in series systems
- Challenges of masked and right-censored failure data
- New maximum likelihood techniques
- Modeling framework and results from simulation studies

1.2 Context & Motivation

- Quantifying reliability in series systems is essential.
- Real-world systems often only provide system-level failure data.
- Masked and right-censored data obscure true reliability metrics.
- Need robust techniques to decipher this data and make accurate estimations.

1.3 Core Contributions

1. New likelihood model that accounts for right-censoring and masking.
2. Extensive simulation studies with Weibull-distributed lifetimes.
3. Evaluations of BCa confidence intervals.
4. Insights into the performance of the maximum likelihood estimator.

1.4 Aim

- Offer a comprehensive understanding of reliability estimation techniques.
- Validate the use of masked reliability data in such analyses.

2

Series System Derivations

2.1 System Reliability Function

- Describes the probability a system functions at a specific time. $R_{T_i}(t'; \boldsymbol{\theta})$ represents the probability the i^{th} system functions at time t'.
- Defined as the product of the reliabilities of its individual components.

$$R_{T_i}(t; \boldsymbol{\theta}) = \prod_{j=1}^{m} R_j(t; \boldsymbol{\theta_j})$$

2.2 System Hazard Function

- Sum of the hazard functions of its components.

$$h_{T_i}(t; \boldsymbol{\theta}) = \sum_{j=1}^{m} h_j(t; \boldsymbol{\theta_j})$$

- Relation to the system's reliability and pdf:

$$f_{T_i}(t; \boldsymbol{\theta}) = \left\{ \sum_{j=1}^{m} h_j(t; \boldsymbol{\theta_j}) \right\} \left\{ \prod_{j=1}^{m} R_j(t; \boldsymbol{\theta_j}) \right\}$$

2.3 System PDF

- Represents how the likelihood of system failure varies over time.

$$f_{T_i}(t; \boldsymbol{\theta}) = h_{T_i}(t; \boldsymbol{\theta}) R_{T_i}(t; \boldsymbol{\theta})$$

2.4 Summary

- Series system models derive mathematical relationships between component and system lifetimes.
- These derivations provide a foundation for understanding system reliability and predicting failures.

3

System and Component Reliabilities

3.1 Mean Time to Failure (MTTF)

- A summary measure of the system's reliability.

$$\text{MTTF} = E_{\boldsymbol{\theta}}[T_i]$$

- Integration of the reliability function over its support given certain assumptions[1].
- MTTF can be misleading, especially for systems with fat-tailed distributions[2].

[1] Assumptions: T_i is non-negative and continuous, $R_{T_i}(t;\boldsymbol{\theta})$ is continuous and differentiable for $t > 0$, and $\int_0^\infty R_{T_i}(t;\boldsymbol{\theta})dt$ converges.

[2] Fat-tailed distributions have tails that decay slower than the exponential family. They can affect MTTF with higher likelihoods of extreme values.

3.2 Component Reliabilities

- System's reliability is determined by its components.
- MTTF for the j^{th} component: MTTF_j.
- Probability j^{th} component causes failure: $\Pr\{K_i = j\}$.
- In a **well-designed** series system:
  - Components have similar MTTFs.
  - Equal probabilities of being the failure cause.

3.2.1 Upcoming

- Use the joint PDF of T and K in the likelihood model derivation.
- Incorporate the reliability function in the likelihood model for right censoring.