Bernoulli candidate sets that are functions of the unkown parameters masked data

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[section] [theorem]

0.0.1 Choices for $p_1, \ldots, p_m$ that are functions of θ and other observables

In many cases, the failure rate increases as the lifetime of the system increases. Thus, a reasonable choice for p_j may be of the form

$$p_j(s) = 1 - \exp(-\beta_j s),$$

which is the cdf of an exponential distribution with rate parameter β_j. As $s \to \infty$, $p_j(s) \to 1$.

$$f(x_1, \ldots, x_m, s|\theta) = \frac{f(s|\theta)}{\sum_{j=1}^m h_j(s|\theta_j)} \sum_{k=1}^m \left\{ h_k(s|\theta_k) \prod_{j=1}^m (1 - \exp(-\beta_j s))^{x_j} \exp(-\beta_j s)^{1-x_j} \right\}. \tag{1}$$

A possibly more interesting choice is, say, $p_j(s|\theta_j) = F_j(s|\theta_j)$, the cdf of X_j, so that a random sample $X_1, \ldots, X_m$ stores more information about θ.

A potentially even more interesting choice is given by

$$p_j(s|\theta_j) = f_{K|S}(j|s, \theta).$$

0.0.1.0.1 Estimating p Under the maximum entropy model, p has a straightforward method of moments estimator given by

$$\hat{p} = \frac{\overline{|C|} - 1}{m - 1},$$

where $\overline{|C|} = \sum_{i=1}^n |C_i|/n$.

Proof.

$$\begin{aligned} E(|C|) &= E\left(\sum_{j=1}^m 1_{\{j=k\}} + 1_{\{j \neq k\}} X_j\right) \\ &= 1 + E\left(\sum_{\substack{j=1 \\ j \neq k}} X_j\right). \end{aligned}$$

Since $X_1, \ldots, X_m$ are independent, this simplifies to

$$\begin{aligned} E(|C|) &= 1 + \sum_{\substack{j=1 \\ j \neq k}}^{m} E(X_j) \\ &= 1 + \sum_{\substack{j=1 \\ j \neq k}}^{m} p \\ &= 1 + (m-1)p. \end{aligned}$$

We may estimate $E(|C|)$ with the sample average $\overline{|C|} = \sum_{i=1}^{n} |C_i|/n$ and we may estimate p with $\hat{p}$,

$$\overline{|C|} = 1 + (m-1)\hat{p}.$$

Solving for $\hat{p}$, we obtain the result

$$\hat{p} = \frac{\overline{|C|} - 1}{m-1}.$$

$\square$

Under different models, where $p_1, \ldots, p_m$ are functions or not all the same constant value, the likelihood approach may be used to estimate them.