Exact Maximum Likelihood Estimation Using Masked System Data

Dennis K. J. Lin
University of Tennessee, Knoxville
John S. Usher, Member IEEE
University of Louisville, Louisville
Frank M. Guess, Associate Member IEEE
University of Tennessee, Knoxville

Key Words — **Masked data, Maximum likelihood estimation, Competing risk**

Reader Aids —
General purpose: Advance state of the art
Special math needed for explanations: Elementary probability
Special math needed to use results: Same
Results useful to: Reliability analysts & researchers

Summary & Conclusions — **This paper estimates component reliability from masked *series*-system life data, *viz*, data where the exact component causing system failure might be unknown. We extend the results of Usher & Hodgson (1988) by deriving exact maximum likelihood estimators (MLE) for the general case of a *series* system of 3 exponential components with independent masking. Their previous work shows that closed-form MLE are intractable, and they propose an iterative method for the solution of a system of 3 non-linear likelihood equations. They do not, however, prove convergence for their iterative method. As such, we show how this system of non-linear equations can be replaced by a single quartic equation, whose solution is straight-forward. Since it does not depend upon the convergence of numerical solution algorithms, the results are exact. Though the resulting estimators are somewhat lengthy & cumbersome to find manually, they can be written as a straightforward computer code. The calculations can then be easily performed on a personal computer. This method for reducing the likelihood equations to simpler-to-solve forms can be extended readily to a higher number of components. In many cases for more than 3 components it is easier while for others it is more complicated; even in the more complicated cases, this simplification makes the problems much more tractable.**

1. INTRODUCTION

The reliability of components from system life data is often estimated by assuming a 1-out-of-n:F competing-risk system. The observable quantities of interest are the lifelength of the system (failure or censoring time) and the exact component causing failure. Finding maximum likelihood estimates (MLE) for component life distribution parameters is widely addressed in the literature. For numerous references & results, see [4,7].

The component-reliability estimates from analysis of system life data are extremely useful because they reflect the reliability of components after their assembly into an operational system. As such, the estimates account for the many degrading effects introduced by the system manufacturing, assembly, distribution, and installation processes. Because of these advantages, companies are beginning to implement computer plans designed to track such system life data and generate component reliability estimates [9].

In practice, however, this type of analysis is often confounded by the problem of masking, *viz*, the exact cause of system failure is unknown. This occurs frequently in complex systems and in field data where the failure cause might be isolated only to some subset of components, such as a circuit card containing many individual components. The quantities observed are then: 1) the lifelength of the system, and 2) partial information on the cause of failure.

Miyakawa [6] considers a 1-out-of-2:F system of exponential components and derives closed-form expressions for the MLE. Under the same exponential assumption, Usher & Hodgson [10] extend Miyakawa's results to a 1-out-of-3:F system; in all but several special cases, closed-form MLE are intractable. Thus a simple solution using Picard iteration was proposed. Ref [3] extended & clarified the derivation of the general likelihood in the masked-data case, and examined the effect of masking on the bias and mean square error of the MLE for a special-case: 1-out-of-3:F system of exponential components. These results are based upon an assumption of s-independent masking, *ie*, masking occurs s-independently of the cause of failure. A recent extension of this work [5] investigates the effects of varying degrees of proportional s-dependent masking on the MLE for a 1-out-of-2:F system.

Ebrahimi [1] develops helpful methods for s-independent masked data that broadly allow for mis-specification of the cause of failure. See Gross [2] and his work on non-masked data. For a recent survey of masked data see [8].

This paper extends the results of [10] by deriving exact MLE for the general case of a 1-out-of-3:F system of exponential components with s-independent masking. In particular we show how the 3 non-linear likelihood equations can be replaced by a single quartic equation, whose solution leads to the required MLE. Our results are illustrated with a numerical example.

Acronym[1]

MLE maximum likelihood estimator.

Notation

- j index for component, $j=1,\ldots,J$
- i index for system, $i=1,\ldots,n$
- T_{ij} lifelength of component j in system i, a r.v.
- T_i lifelength of system i, $T_i = \min(T_{i1},\ldots,T_{iJ})$
- $f_j(t), R_j(t)$ [pdf, Sf] of component j lifelength
- K_i index of the component causing failure of system i

[1]The singular & plural of an acronym are always spelled the same.

M_i,S_i [minimum random, observed] subset of components known to contain the true cause of failure of system i
$\hat{}$ implies the MLE
L likelihood function.

Other, standard notation is given in "Information for Readers & Authors" at the rear of each issue.

Assumptions

1. The components & system are 2-state: failed or not-failed.
2. The system is 1-out-of-J:F; n such systems are observed.
3. The T_{ij} are mutually s-independent r.v; they are i.i.d. for a given j. (While this assumption is restrictive, it is a reasonable approximation for a wide variety of systems & components.) ◄

2. THE LIKELIHOOD FUNCTION

2.1 General Formulation

Assumption

4. Each system is observed until failure, *viz*, no censoring. ◄

For each system, the observed quantities are t_i, the realized value of $\mathbf{T_i}$, and $S_i \subset \{1,2,\ldots,J\}$, the realized value of M_i.[2] If, for example, $S_i = \{1,2\}$, we know that $K_i \in S_i$ but the true value of K_i is masked. For a complete (uncensored) sample, the full likelihood is [3] is:

$$L_F = \prod_{i=1}^{n}\left[\sum_{s\in S_i}[f_s(t_i)\cdot\prod_{j\in J_s}R_j(t_i)\cdot\Pr\{M_i=S_i|T_i=t_i,K_i=s\}]\right] \tag{1}$$

$$J_s \equiv \{1,\ldots,s-1,s+1,\ldots,J\}.$$

2.2 Independent Masking

Assumption

5. Masking is s-independent of the cause of failure. (While masking can depend upon the time of failure and/or the cause of failure, we consider only the time-independent case here. See [5] for more on s-dependent masking.) ◄

The reduced (partial) likelihood is:

$$L_r = \prod_{i=1}^{n}\left[\sum_{s\in S_i}[f_s(t_i)\cdot\prod_{j\in J_s}R_j(t_i)]\right] \tag{2}$$

2.3 Exponentially Distributed Component-Life with Independent Masking

Assumption

6. Component life-lengths are exponentially distributed. ◄

$$L_r = \prod_{i=1}^{n}\left[\sum_{s\in S_i}[\lambda_s\cdot\exp(\lambda_s\cdot t_i)\cdot\prod_{j\in J_s}\exp(\lambda_s\cdot t_i)]\right], \tag{3}$$

$$\ln(L_r) = -\left(\sum_{j=1}^{J}\lambda_j\right)\cdot\left(\sum_{i=1}^{n}t_i\right) + \sum_{i=1}^{n}\ln\left(\sum_{j\in S_i}\lambda_j\right) \tag{4}$$

To find the values of λ_j that maximize (4), take partial derivatives and equate them to 0:

$$\partial\ln(L)/\partial\lambda_j = 0 = \sum_{i=1}^{n}\left[-t_i + \left(1\Big/\sum_{j\in S}\lambda_j\right)\right]. \tag{5}$$

The $\hat{\lambda}_j$ are the values of λ_j that satisfy (5).

Miyakawa [6] showed that the MLE for $J=2$ are available in closed form.

3. 3-COMPONENT EXPONENTIAL CASE

Assumptions

1. - 6. (Same as in the special case of section 2.3)
7. $J=3$.

3.1 *General Derivation*

Notation [10]

n_k number of failures where $S_i = \{k\}$, $k=1,2,3$
n_{jk} number of failures where $S_i = \{j,k\}$, $j,k = \{1,2\}, \{1,3\}, \{2,3\}$
n_{123} number of failures where $S_i = \{1,2,3\}$
T $\Sigma_{i=1}^{n} t_i$
$n_{ij}^{(k)}$ n_{ij} if $k=i$ or $k=j$; 0 otherwise.

$$n_1 + n_2 + n_3 + n_{12} + n_{13} + n_{23} + n_{123} = n.$$

The likelihood equations from (5) are [10]:

$$0 = -T + [n_k/\lambda_k] + [n_{12}^{(k)}/(\lambda_1+\lambda_2)] + [n_{13}^{(k)}/(\lambda_1+\lambda_3)]$$
$$+ [n_{23}^{(k)}/(\lambda_2+\lambda_3)] + [n_{123}/(\lambda_1+\lambda_2+\lambda_3)];\ k=1,2,3. \tag{6}$$

Ref [10] concluded that the solutions to (6) are intractable, suggested Picard iteration as a solution technique, and presented several special cases where MLE are available in closed form.

By appropriate algebraic manipulation, (6) can be put into the form:

$$[n_1/\alpha_1] - [n_{23}/(\alpha_2+\alpha_3)] = 1, \tag{7a}$$

$$[n_2/\alpha_2] - [n_{13}/(\alpha_1+\alpha_3)] = 1, \tag{7b}$$

$$[n_3/\alpha_3] - [n_{12}/(\alpha_1+\alpha_2)] = 1, \tag{7c}$$

[2] If $S_i=\{j\}$ then $K_i=j$ and the cause of failure is not masked.

$$\alpha_k \equiv \lambda_k \cdot T/A,$$

$$A \equiv 1 + [n_{12}/(\alpha_1+\alpha_2)] + [n_{13}/(\alpha_1+\alpha_3)]$$
$$+ [n_{23}/(\alpha_2+\alpha_3)] + [n_{123}/(\alpha_1+\alpha_2+\alpha_3)].$$

Then solve (7) for the α_k, which is much easier than solving (6). And, finally,

$$\hat{\lambda}_k = A \cdot \alpha_k / T. \quad (8)$$

3.2 Special Cases

These special cases illustrate the general derivation; in each special case, relabeling can cover all permutations, *eg*, in case 3, relabeling covers the case where $n_{13}=0$ or $n_{12}=0$.

Case 1. $n_{12} = n_{13} = n_{23}=0$

$$\alpha_k=n_k,\ k = 1,2,3$$

$$A = 1 + [n_{123}/(n_1+n_2+n_3)].$$

Use (8) to find the $\hat{\lambda}_j$. These are the same as the case 1 estimators in [10].

Case 2. $n_{13}=n_{23}=0$

$$\alpha_1=n_1,\ \alpha_2=n_2,\ \alpha_3= n_3/[1+ n_{12}/(n_1+n_2)],$$

$$A = 1 + [n_{12}/(n_1+n_2)] + n_{123}\cdot(n_1+n_2+n_{12})/[(n_1+n_2)$$
$$\cdot(n_1+n_2+n_3+n_{12})].$$

Use (8) to find the $\hat{\lambda}_j$. These are equivalent to the case 3 estimators in [3,10].

Case 3. $n_{23}=0$

$$\alpha_1=n_1,$$

$$\alpha_k = \tfrac{1}{2}[\phi_k + n_2\cdot n_3 + n_1\cdot(n_k-n_{k'}) - (n_1+n_{12})$$
$$\cdot(n_1+n_{13})]/(n_1+n_{1k'}+n_{k'}),\ \text{for } k=2,3;\ k' \equiv 5-k;$$

$$\phi^2{}_k \equiv [n^2{}_1 + n_1\cdot n_2 + n_1\cdot n_3 + n_2\cdot n_3+ n_1\cdot(n_{12}+n_{13})]^2$$
$$- 4n_k\cdot n_{12}\cdot n_{13}.$$

Use (8) to find the $\hat{\lambda}_j$.

Case 4. $n_1=n_2=n_3=0$

No known-cause failures are observed for any of the components. We could solve the equations but they yield inappropriate (non-positive) estimates for some components. It is obviously difficult to obtain estimates when no known-cause failures are observed.

Case 5. $n_1=n_2=0$

$$\alpha_k = \tfrac{1}{4}(\phi - 2n_3 + n_{k'3} - 3n_{k3} - n_{12}),\ \text{for } k=1,2;\ k' \equiv 3-k;$$

$$\alpha_3 = \tfrac{1}{4}(-\phi + 2n_3 - n_{23} - n_{13} + n_{12})$$

$$\phi^2 \equiv (2n_3 - n_{12} + n_{13} + n_{23})^2 + 8n_3\cdot n_{12}.$$

Use (8) to find the $\hat{\lambda}_j$.

3.3 General Case

No simplifying assumptions are made regarding the possible masking sets, except that $n_j > 0$ for some j; label the j so that $j=1$ is one of those j.[3]

$$\alpha_1 = n_1\cdot(\alpha_2+\alpha_3)/(\alpha_2+\alpha_3+n_{23}) \quad (9)$$

$$\alpha_2\cdot\alpha_3\cdot(\alpha_2+\alpha_3) + (n_1+n_{1k'})a^2{}_k - n_k\cdot a^2{}_{k'}$$
$$+ (n_1+n_{23}+n_{1k'}- n_k)\cdot\alpha_k\cdot\alpha_{k'} + (n_{1k'}\cdot n_{23} - n_1\cdot n_k)$$
$$\cdot\alpha_k - n_k\cdot(n_1+n_{23})\cdot\alpha_{k'} = 0,\ k=2,3;\ k' \equiv 5-k. \quad (10)$$

Let —

$$p \equiv \alpha_3/\alpha_2. \quad (11)$$

$$p = [\lambda_3\cdot(T/A)]/[\lambda_2\cdot(T/A)] = \lambda_3/\lambda_2 \geq 0.$$

Then —

$$\alpha_2 = (\theta_{\text{num3}} + \text{p}\cdot\theta_{\text{num2}})/[(1+p)\cdot\theta_{\text{denom}}] \quad (12)$$

$$\theta_{\text{num3}} \equiv n_1\cdot n_3 - n_1\cdot n_2 + n_3\cdot n_{23} + n_{13}\cdot n_{23},$$

$$\theta_{\text{num2}} \equiv n_1\cdot n_2 - n_1\cdot n_3 + n_2\cdot n_{23} + n_{12}\cdot n_{23},$$

$$\theta_{\text{denom}} \equiv p\cdot(n_1+n_2+n_{12}) - (n_1+n_3+n_{13}).$$

$$a\cdot p^4 + b\cdot p^3 + c\cdot p^2 + d\cdot p + e = 0. \quad (13)$$

The a, b, c, d, e are functions of n_1, n_2, n_3, n_{12}, n_{13}, n_{23}; see (14).

The roots of the fourth-order polynomial (13) can be found in various ways. Using the SOLVE routine of MACSYMA, a standard symbolic manipulation program, the roots can be expressed in an extremely lengthy closed form. Alternatively, the roots could be found using common numerical techniques.

If $n_2=0$, then the polynomial is cubic which has known closed-form solutions. After relabeling, this is case 6 in section 2.2.

[3] If all $n_j=0$, for $j=1,2,3$, then see case 4 in section 3.2.

Since $p \geq 0$, then $\alpha_k \geq 0$ for $k=1,2,3$.

After solving for the appropriate root of (13), the value of p is substituted into (12), (11), (9) to find the α_k. Use (8) to find the $\hat{\lambda}_j$. These are computationally straightforward and easy to find using a computer.

$$N_k \equiv n_1 + n_2 + n_{1k},\ k=2,3$$

$$N_{23} \equiv n_1 \cdot (n_2 - n_3) + n_{23} \cdot (n_2 + n_{12})$$

$$N_{32} \equiv n_1 \cdot (n_3 - n_2) + n_{23} \cdot (n_3 + n_{13})$$

$$M_{23:\pm} \equiv n_1 \pm n_3 + n_{23} + n_{12}$$

$$M_{32:\pm} \equiv n_1 \pm n_2 + n_{23} + n_{13}$$

$$Q_2 \equiv n_2 \cdot (n_1 + n_{23})$$

$$Q_{32} \equiv n_{13} \cdot n_{23} - n_1 \cdot n_2$$

$$a \equiv -n_1 \cdot n_2 \cdot N_2 \cdot M_{23:+} \tag{14a}$$

$$e \equiv -n_1 \cdot n_3 \cdot N_3 \cdot M_{32:+} \tag{14e}$$

$$b \equiv N^2{}_{23} - n_2 \cdot (N_{32} \cdot N_2 + N_{23} \cdot N_3) + N_2 \cdot (-M_{32:-} \cdot N_{23} + 2Q_2 \cdot N_3 + (Q_{32} - Q_2) \cdot N_2) \tag{14b}$$

$$d \equiv N^2{}_{32} + (n_1 + n_{13}) \cdot N_{32} \cdot N_2 + N_3 \cdot [M_{32:-} \cdot N_{32} + (n_1 + n_{13}) \cdot N_{23} - 2Q_{32} \cdot N_2 + (Q_{32} - Q_2) \cdot N_3] \tag{14d}$$

$$c \equiv -2N_{23} \cdot N_{32} + n_2 \cdot N_{32} \cdot N_3 + M_{32:-} \cdot (N_{32} \cdot N_2 + N_{23} \cdot N_3) - (n_1 + n_{13}) \cdot N_{23} \cdot N_2 - Q_2 \cdot N^2{}_3 - 2(Q_{32} - Q_2) \cdot N_2 \cdot N_3 + Q_{32} \cdot N^2{}_2 \tag{14c}$$

4. NUMERICAL EXAMPLE

Table 1, from [10], is a set of masked data. These data, simulated using $\lambda_1 = \lambda_2 = \lambda_3 = 1$, represent failure times and masking sets for a 1-out-of-3:F system.

$$n_1 = 6,\ n_2 = 6,\ n_3 = 8$$

$$n_{12} = 3,\ n_{13} = 1,\ n_{23} = 3$$

$$n_{123} = 3$$

$$T = 10.13$$

From (14) —

$$-10800p^4 - 810p^3 + 23580p^2 + 126p - 11520 = 0,$$

with positive roots:

$p = 1.126079$ *and* $p = 0.8981638$.

Substitute these values into (9), (11), (12); only $p = 1.126079$ yields positive values for the MLE[4]: $\hat{\lambda}_1 = 0.8588016$, $\hat{\lambda}_2 = 0.9890027$, $\hat{\lambda}_3 = 1.1136952$. These values closely coincide with those from the iterative procedure in [10].

TABLE 1
Simulated System-Life Data with Masking [10]
[1-out-of-3:F System, $N=30$]

i	t_i	K_i	S_i
1	.0205	{2}	{2}
2	.0379	{2}	{1,2}
3	.0543	{3}	{3}
4	.0663	{3}	{3}
5	.0757	{2}	{1,2}
6	.0780	{2}	{2,3}
7	.1228	{3}	{3}
8	.1302	{3}	{1,3}
9	.1520	{2}	{1,2,3}
10	.1586	{1}	{1}
11	.1989	{3}	{3}
12	.2011	{1}	{1}
13	.2038	{1}	{1}
14	.2150	{3}	{2,3}
15	.2179	{2}	{1,2}
16	.2807	{1}	{1}
17	.2949	{2}	{2}
18	.3097	{3}	{3}
19	.3376	{3}	{3}
20	.3408	{2}	{2}
21	.3543	{1}	{1}
22	.3575	{2}	{2}
23	.4309	{1}	{1,2,3}
24	.4570	{3}	{3}
25	.5445	{1}	{1,2,3}
26	.5686	{2}	{2}
27	.6767	{3}	{3}
28	.8183	{2}	{2}
29	.9460	{2}	{2,3}
30	1.4860	{1}	{1}

REFERENCES

[1] N. Ebrahimi, "The theory of competing risks revisited", *Technical Report*, 1992; Div. of Statistics, Northern Illinois Univ, DeKalb, IL 60115 USA.

[2] A.J. Gross, "Minimization of misclassification of component failures in a two-component system", *IEEE Trans. Reliability*, vol R-19, 1970 Aug, pp 120-122.

[4]The number of significant figures is not intended to imply any accuracy in the estimates, but to illustrate the arithmetic. It is likely that the uncertainty in the results is at least ±20%.

[3] F. Guess, J. Usher, T. Hodgson, "Estimating system and component reliabilities under partial information on the cause of failure", *J. Statistical Planning and Inference*, vol 29, 1991 Sep-Oct, pp 75-85.

[4] J.F. Lawless, *Statistical Models and Methods For Lifetime Data*, 1982; John Wiley & Sons.

[5] D. Lin, F. Guess, "The effects of dependent masking on reliability estimation", *Microelectronics and Reliability*, vol 64, 1994 [to appear].

[6] M. Miyakawa, "Analysis of incomplete data in competing risks model", *IEEE Trans. Reliability*, vol R-33, 1984 Oct, pp 293-296.

[7] W. Nelson, *Applied Life Data Analysis*, 1982; John Wiley & Sons.

[8] J. Usher, "On the problem of masked system life data", *Proc. Fourth Int'l Reliability Research Conf*, 1991 Jun; Columbia, Missouri.

[9] J. Usher, S. Alexander, J. Thompson, "System reliability prediction based on historical data", *Quality & Reliability Engineering Int'l*, vol 6, 1990 Jun-Aug, pp 209-218.

[10] J. Usher, T. Hodgson, "Maximum likelihood analysis of component reliability using masked system life-test data", *IEEE Trans. Reliability*, vol 37, 1988 Dec, pp 550-555.

AUTHORS

Dr. Dennis K. J. Lin; Dept. of Statistics; Univ. of Tennessee; Knoxville, Tennessee 37996 USA.

Dennis K. J. Lin is an Associate Professor of Statistics at the University of Tennessee, Knoxville. He is also an instructor for the Experimental Design Institute for Productivity Through Quality at that university. Dr. Lin received his PhD in Statistics (1988) from the University of Wisconsin, Madison. His research interests are design of experiments, quality engineering, and reliability. He has published in *Technometrics*, *J. of Statistical Planning & Inference*, *Computational Statistics and Data Analysis*, *etc*. He is a member of the American Statistical Association, the Institute of Mathematical Statistics, and a Fellow of the Royal Statistical Society.

Dr. John S. Usher; Department of Industrial Engineering; University of Louisville, Louisville, Kentucky 40292 USA.

John S. Usher (M'88) is an Associate Professor of Industrial Engineering at the University of Louisville. He received his BS (1980) and MEng (1981) in Industrial Engineering from the University of Louisville. He received his PhD (1987) in Industrial Engineering from North Carolina State University. While completing this degree he was employed by IBM in Research Triangle Park, NC as a Reliability Engineer. He has published articles in *IEEE Trans. Reliability*, *Quality & Reliability Engineering Int'l*, *IIE Trans*, *J. of Statistical Planning and Inference*, *etc*. His research interests lie in system-reliability and quality-control. He is a member of IIE, ASQC, ORSA, IEEE, and Tau Beta Pi and is a Registered Professional Engineer in the Commonwealth of Kentucky.

Dr. Frank M. Guess, Department of Statistics; University of Tennessee; Knoxville, Tennessee 37996 USA.

Frank M. Guess is an Associate Professor of Statistics at the University of Tennessee, Knoxville. He received his PhD (1984) in Statistics from Florida State University. He won the Ralph A. Bradley Award (1984) for outstanding achievement for a PhD student from Florida State University, Department of Statistics. He has published on reliability, censored data, and non-parametric statistics in *Quality & Reliability Engineering Int'l*, *IEEE Trans. Reliability*, *Annals of Statistics*, *Biometrika*, *Biometrics*, *Reliability Review*, *etc*. He is a member of the American Statistical Association and the Institute of Mathematical Statistics, and is an associate member of IEEE. Dr. Guess was on the Program Committee for the Fourth International Reliability Research Conference in 1991 July. He has had research grants from IBM, the US Air Force Office of Scientific Research, and the US Army Research Office.

Manuscript TR91-142 received 1991 August 9; revised 1992 September 1.

IEEE Log Number 07328 ◄TR►

MANUSCRIPTS RECEIVED *MANUSCRIPTS RECEIVED* *MANUSCRIPTS RECEIVED* *MANUSCRIPTS RECEIVED*

"Factors for prediction interval for an ordered observation from a 2-parameter exponential distribution" (A. Soliman, et al.), Dr. Ahmed A. Soliman S Dept. of Mathematics S Assiut Univ. S Sohag 82516 S EGYPT. (TR93-172)

"Forced-outage rates of generating units, based on expert evaluations" (J. McDonald, et al.), Dr. Jim R. McDonald S Centre for Electrical Power Eng'g S Univ. of Strathclyde S 204 George St. S Glasgow G1 1XW S GREAT BRITAIN. (TR93-173)

"Reliability prediction using nondestructive accelerated-degradation data: Case study on power supplies" (L. Tang, D. Chang), Dr. Loon-Ching Tang S Dept. of Industrial & System Eng'g S National University of Singapore S 10 Kent Ridge Crescent S Singapore 0511 S Republic of SINGAPORE. (TR93-174)

"Parameter estimation for the Birnbaum-Saunders distribution, for symmetrical-censored samples" (J. Rieck), James R. Rieck S Dept. of Exp. Statistics, F-148 Poole Ag Ctr S Clemson University S Box 340367 S Clemson, South Carolina 29634-0367 S USA. (TR93-175)

"Configurations of series-parallel networks with maximum reliability" (G. Pflug, et al.), Dr. Georg Ch. Pflug, o.Prof. S Inst. f. Statisk, Op. Res, u. Computerv. S Univ. Wien S Universitaetsstr 5/1/9 S A-1010 Wien S AUSTRIA. (TR93-176)

"Improved grey-system identification in machinery reliability" (C. Chen, et al.), Dr. Cha'o-Kuang Chen S Dept. of Mechanical Eng'g S National Cheng-Kung Univ. S Tainan 701 S TAIWAN - R.O. CHINA. (TR93-177)

"How to model reliability-growth when times of design modifications are known" (N. Ebrahimi), Dr. Nader B. Ebrahimi S Division of Statistics S Dept. of Mathematical Sciences S Northern Illinois University S DeKalb, Illinois 60115-2888 S USA. (TR93-178)

"The proportional-hazard model for step-stress accelerated life testing" (D. Chen), Di Chen S Mathematics Dept. S Shanghai Teachers' University S 100 Gui Ling Road S Shanghai - 200 234 S Peop. Rep. CHINA. (TR93-179)

"Weibull-parameter estimation for step-stress testing based on the proportional-hazard model" (D. Chen), Di Chen S Mathematics Dept. S Shanghai Teachers' University S 100 Gui Ling Road S Shanghai - 200 234 S Peop. Rep. CHINA. (TR93-180)