Probability Axioms

In defining a probability measure, we require that it satisfies the following axioms:

1. $0 \leq P(A) \leq 1$ for all A in event space
2. $P(\phi) = 0$
3. $P(S) = 1$
4. (Countable Additivity) $P(A_1 \text{ or } A_2 \text{ or } A_3 \ldots) = P(A_1) + P(A_2) + P(A_3) + \ldots$where A_n is a sequence of disjoints event.

Random Variable

Definition version 1: A random variable is a function from the sample space S to the real number line R.

Definition version 2: A random variable is a numeric value based on the outcome of a random event.

Random variables provide a condensed way of looking at problems. If we know the probabilistic behaviour of each X, we can solve the problem using less effort than combinations.

Example 1: *Recall from last week, consider flipping a coin once and record its face value.*

1. Sample space is S = {Head, Tail}.

2. A collection of all possible events are {φ, Head, Tail, {Head, Tail}}.

3. The probability measure P assigns a real number that represents "likeness" of each event in the event space.

P(Head) = P(Tail) = ½

P({Head, Tail}) = P(either a head or a tail shows up) = 1

P(φ) = P(neither a head nor a tail shows up) = 0

How can we express the above problem using a random variable?

- Let X model the outcomes of a coin toss experiment.

  $$X = \begin{cases} 1, & if\ head \\ 0, & if\ tail \end{cases}$$

  Probability distribution of X:

  P(X = 1) = ½

  P(X = 0) = ½

P(S) = P(X = 1) + P(X = 0) =1

P(ϕ) = 0

- Another way to model the outcomes of a coin flip:

$$X = \begin{cases} 100, & if\ head \\ 200, & if\ tail \end{cases}$$

Probability distribution of X:

P(X = 100) = ½

P(X = 200) = ½

P(S) = P(X = 100) + P(X = 200) =1

P(ϕ) = 0

Question: Is this probability distribution valid? Check axioms above!

Example 2: Consider flipping an unfair coin for 3 times. The probability of getting a head is 20%.

(a) Calculate the probability that you get no heads.

(b) Use your answer to (a) to find the probability of at least one head is obtained.

Example 3 (without replacement): You are dealt a hand of 3 cards from a standard deck of 52 cards. Find each of the following probabilities:

(a) The first heart you get is the third card dealt

(b) Your cards are all reds (that is, all diamonds or hearts.)

(c) You get no spades

(d) You have least one ace