

## Probability Model(or triple)

1. A sample space ( $S$ ) containing all possible outcomes of an experiment
2. A set of all possible events (all possible subsets of  $S$ ; event space)
3. A probability measure defined on the event space

Example: Consider flipping a coin once and record its face value.

1. Sample space is  $S = \{\text{Head, Tail}\}$ .
2. A collection of all possible events are  $\{\phi, \text{Head, Tail, \{Head, Tail\}}\}$ .
3. The probability measure  $P$  assigns a real number that represents “likeness” of each event in the event space.

$$P(\text{Head}) = P(\text{Tail}) = \frac{1}{2}$$

$$P(\{\text{Head, Tail}\}) = P(\text{either a head or a tail shows up}) = 1$$

$$P(\phi) = P(\text{neither a head nor a tail shows up}) = 0$$

## Properties of Probability Models

1. Complement Rule

$$P(\text{not } A) = P(A^c) = 1 - P(A)$$

2. Addition Rule (General Case)

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B) \text{ for any two events, disjoint or not.}$$

Disjoint: the probability of  $A$  and  $B$  happening at the same time is 0

3. Conditional Probability

$$\text{“Probability of } B \text{ given } A\text{”} = P(B|A) = \frac{P(A \text{ and } B)}{P(A)}$$

4. Independence

Events  $A$  and  $B$  are independent iff the probability of  $B$  is the same when we are given that  $A$  has occurred.

$$P(B|A) = P(B)$$

5. General Multiplication Rule

$$P(A \text{ and } B) = P(A) \times P(B|A)$$

If  $A$  and  $B$  are independent,  $P(A \text{ and } B) = P(A) \times P(B)$

Question 1: (exercise 30 in Chapter 11 on Page 417)

In 2014, 52.3% of all immigrants to Canada were females, 18.6% were under 18 years old, and 9.0% were females under 18 years old. Find the probability that a randomly selected person who immigrated to Canada in 2014 was

- a) Females and at least 18 years old
- b) Either female or under 18 years old
- c) Male and at least 18 years old

Question 2: (exercise 21 in Chapter 12 with a little twist)

You roll a fair die for three times. What is the probability that

- a) You roll all 6s?
- b) You roll at least one 5?
- c) You roll a 6 on the third row given that the first two show either 2 or 3?