HW7

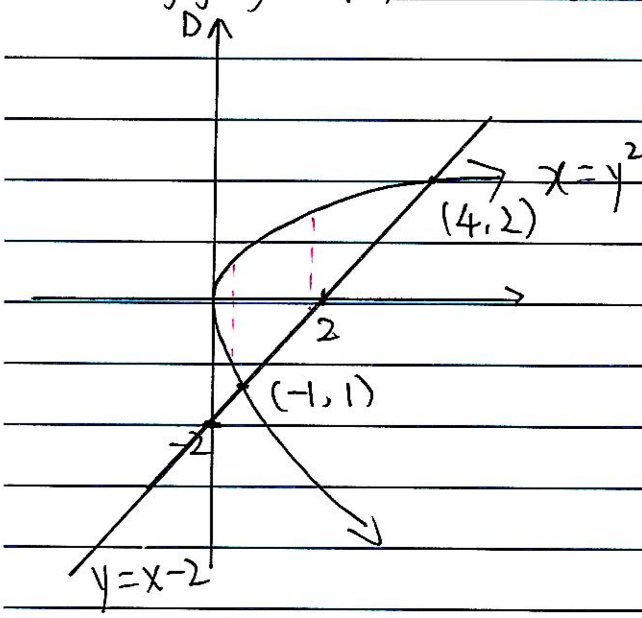
July 3rd

Pg1008 Q1–Q10, Q15 – Q22

Chp 15.2

Set up iterated integrals for both orders of integration. Then evaluate the double integral using the easier order and explain why it's easier.

Q15. $\iint_D y\,dA$, D is bounded by $y = x-2$, $x = y^2$



① $\int_{-1}^{2}\int_{y^2}^{2+y} y\,dx\,dy$

② $\iint y\,dy\,dx$

$= \int_0^1\int_{-\sqrt{x}}^{\sqrt{x}} y\,dy\,dx + \int_1^4\int_{x-2}^{\sqrt{x}} y\,dy\,dx$

① $\int_{-1}^{2}\int_{y^2}^{2+y} y\,dx\,dy$

$= \int_{-1}^{2} xy\Big|_{y^2}^{2+y} dy$

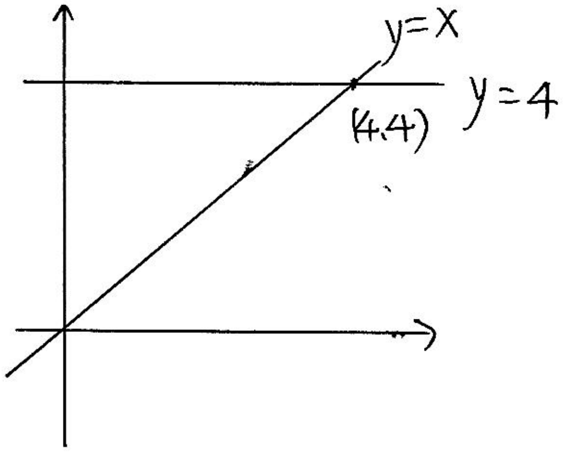
$= \int_{-1}^{2} (2+y)\cdot y - y^3\,dy$

$= \int_{-1}^{2} 2y + y^2 - y^3\,dy$

$= y^2 + \frac{y^3}{3} - \frac{y^4}{4}\Big|_{-1}^{2}$

$= \left(2^2 + \frac{2^3}{3} - \frac{3^4}{4}\right) - \left((-1)^2 + \frac{(-1)^3}{3} - \frac{(-1)^4}{4}\right) = \frac{9}{4}$

Q16 $\iint_D y^2 e^{xy}\, dA$. D is bounded by $y=x$, $y=4$, $x=0$



① $\int_0^4 \int_0^y y^2 e^{xy}\, dx\, dy$

② $\int_0^4 \int_0^x y^2 e^{xy}\, dy\, dx$

① $\int_0^4 \int_0^y y^2 e^{xy}\, dx\, dy$

$= \int_0^4 y e^{xy} \Big|_0^y \, dy$

$= \int_0^4 y e^{y^2} - y \, dy$

$= \left(\frac{1}{2} e^{y^2}\right) - \frac{y^2}{2} \Big|_0^4$

$= \frac{e^{16}}{2} - \frac{17}{2}.$

e^{xy}

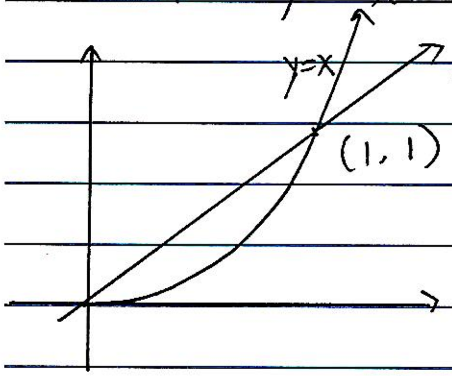
$u = xy$

$\frac{du}{dx} = y$

$\frac{du}{y} = dx$

$e^u \frac{1}{y} du$

Q18 $\iint_D (x^2+2y)\, dA$, D is enclosed by the curves $y = x$, $y = x^3$, $x \geq 0$.



① $\displaystyle\int_0^1 \int_{x=y}^{x=\sqrt[3]{y}} (x^2+2y)\, dx\, dy$

② $\displaystyle\int_0^1 \int_{x^3}^{x} (x^2+2y)\, dy\, dx$

② $\displaystyle\int_0^1 \int_{x^3}^{x} (x^2+2y)\, dy\, dx$

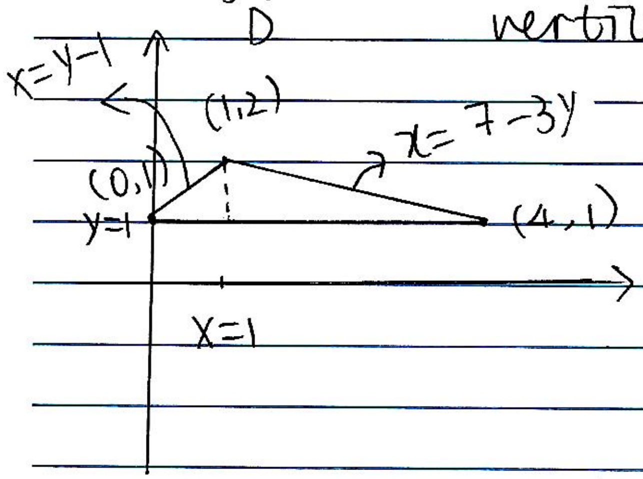
$\displaystyle = \int_0^1 x^2 y + y^2 \Big|_{x^3}^{x} dx$

$\displaystyle = \int_0^1 (x^3+x^2) - (x^5+x^6)\, dx.$

$\displaystyle = \frac{x^4}{4} + \frac{x^3}{3} - \frac{x^6}{6} - \frac{x^7}{7} \Big|_0^1$

$\displaystyle = \frac{1}{4} + \frac{1}{3} - \frac{1}{6} - \frac{1}{7} = \frac{23}{84}.$

Q19 $\iint_D y^2\, dA$, D is the triangler region w/ vertices $(0,1)$, $(1,2)$, $(4,1)$ $\frac{11}{3}$



① $\displaystyle\int_1^2 \int_{y-1}^{7-3y} y^2\, dx\, dy = \int_1^2 x y^2 \Big|_{y-1}^{7-3x} dy \cdots$

② $\displaystyle\int_0^1 \int_1^{x+1} y^2\, dy\, dx + \int_1^4 \int_1^{-\frac{x}{3}+\frac{7}{3}} y^2\, dy\, dx$

$\frac{11}{12}$ $\frac{11}{4}$

July 10th

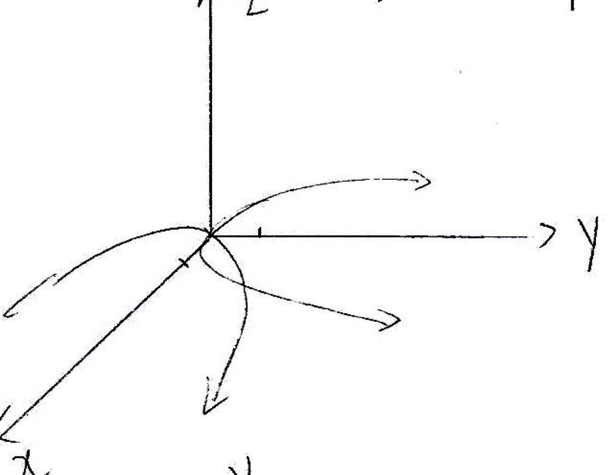
HW8

Q23-Q32, Q35-Q40, Q45-Q58 on Pg 1008.

Q23-Q32

Find the volume of the given solid.

Q23. Under the plane $3x+2y-z=0$ and above the region enclosed by the parabola $y=x^2$ and $x=y^2$.



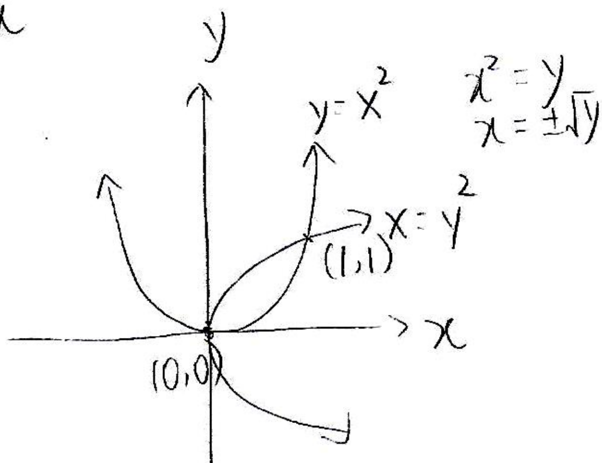
Sub $y=x^2$ into $x=y^2$.

$x=x^4$

$x-x^4=0$

$x(1-x^3)=0$

$x=0,\ x=1$



$$\int_0^1 \int_{x^2}^{\sqrt{x}} 3x+2y \, dy\, dx$$

$$= \int_0^1 3xy + y^2 \Big|_{x^2}^{\sqrt{x}} dx$$

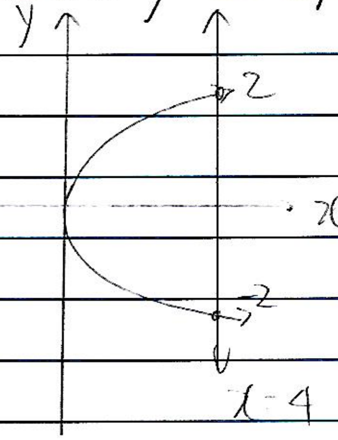
$$= \int_0^1 (3x\sqrt{x} + x) - (3x x^2 + x^4)\, dx$$

$$= \int_0^1 3x^{3/2} + x - 3x^3 - x^4 \, dx$$

$$= \frac{6}{5}x^{5/2} + \frac{x^2}{2} - \frac{3x^4}{4} - \frac{x^5}{5} \Big|_0^1$$

$$= \frac{3}{4}$$

Q24 Under the surface $z = 1 + x^2y^2$ and above the region enclosed by $x = y^2$ and $x = 4$.



$$\int_{-2}^{2}\int_{y^2}^{4} (1+x^2y^2)\, dx\, dy$$

$$= \int_{-2}^{2} x + \frac{x^3}{3}y^2 \Big|_{y^2}^{4} dy$$

$$= \int_{-2}^{2} \left(4 + \frac{64}{3}y^2\right) - \left(y^2 + \frac{1}{3}y^8\right) dy$$

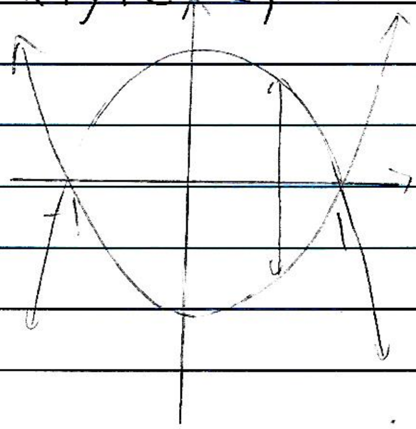
$$= \int_{-2}^{2} 4 + \frac{61}{3}y^2 - \frac{1}{3}y^8 \; dy$$

$$= \frac{2336}{27}.$$

Find the volume of the solid by subtracting 2 volumes.

Q35 The solid enclosed by the parabolic cylinders $y = 1-x^2$, $y = x^2-1$ and the planes $x+y+z=2$, $2x+2y-z+10=0$.

$$\int_{-1}^{1}\int_{x^2-1}^{1-x^2}\int_{x+y-2}^{2x+2y+10} 1 \; dz\, dy\, dx$$



$$= \int_{-1}^{1}\int_{x^2-1}^{1-x^2} x+y+8 \; dy\, dx$$

$$= \int_{-1}^{1} xy + \frac{y^2}{2} + 8y \Big|_{x^2-1}^{1-x^2} dx$$

$1-x^2 = x^2-1$

$x^2 = 1 \quad x = \pm 1$

$$\int_{-1}^{1}\int_{x^2-1}^{1-x^2} 2-x-y \; dy\,dx = \int_{-1}^{1} 2y - xy - \frac{y^2}{2}\Big|_{x^2-1}^{1-x^2} dx$$

$$= \int_{-1}^{1} 4(1-x^2) + 2x(x^2-1) \; dx$$

$$= 2\int_{-1}^{1} x^3 - 2x^2 - x + 2 \; dx = 2\left(\frac{x^4}{4} - \frac{2x^3}{3} - \frac{x^2}{2} + 2x\right)\Big|_{-1}^{1}$$

$$= \frac{16}{3}$$

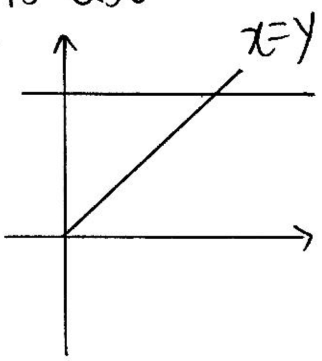
$$\int_{-1}^{1}\int_{x^2-1}^{1-x^2} 2x+2y+10 \; dy\,dx = \frac{80}{3}$$

$$\frac{80}{3} - \frac{16}{3} = \frac{64}{3}$$

Sketch the region of integration and change the order of integration

Q45–Q50

Q45



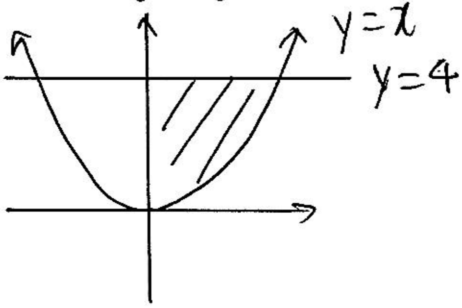
$0 \le x \le y$

$0 \le y \le 1.$

$$\int_0^1\int_0^y f(x,y)\,dx\,dy.$$

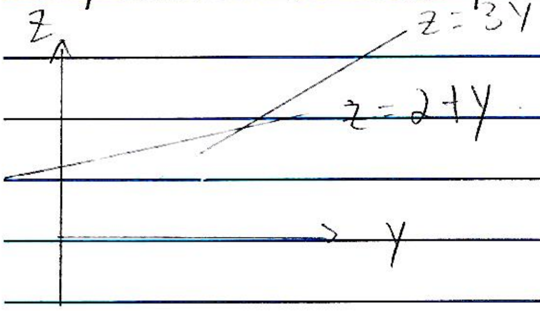
$$\int_0^1\int_x^1 f(x,y)\,dy\,dx$$

Q46 $\displaystyle\int_0^2\int_{x^2}^4 f(x,y)\,dy\,dx$



$$\int_0^4\int_0^{\sqrt{y}} f(x,y)\,dx\,dy$$

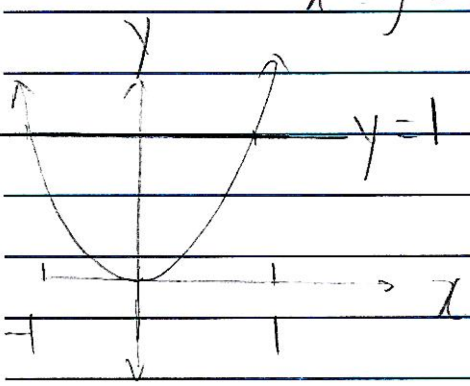
Q36 The solid enclosed by the parabolic cylinder $y=x^2$ and the planes $z=3y$, $z=2+y$.



$2+y=3y$

$y=1$

$x^2=y=1 \Rightarrow x=\pm 1$



$$\int_{-1}^{1}\int_{x^2}^{1}\int_{3y}^{2+y} dz\,dy\,dx$$

$$=\int_{-1}^{1}\int_{x^2}^{1} 2+y-3y \; dy\,dx$$

$$=\int_{-1}^{1}\int_{x^2}^{1} 2-2y \; dy\,dx$$

$$=\int_{-1}^{1} 2y-y^2 \Big|_{y=x^2}^{y=1} dx$$

$$=\int_{-1}^{1} (2-1)-(2x^2-x^4)\; dx$$

$$=\int_{-1}^{1} 1-2x^2+x^4 \; dx$$

$$= x-\frac{2x^3}{3}+\frac{x^5}{5}\Big|_{-1}^{1}$$

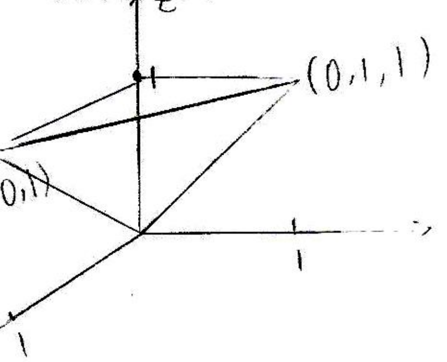
$$= 1-\frac{2}{3}+\frac{1}{5}-\left(-1+\frac{2}{3}-\frac{1}{5}\right)$$

$$=\frac{30}{15}-\frac{20}{15}+\frac{6}{15}=\frac{16}{15}$$

HW 9 July 17th, 2019 ①

Pg1038 Q2–Q22, Q27–Q36, Q45–Q58

Q16 $\iiint_T xz\,dV$, where T is the solid tetrahedron with vertices $(0,0,0)$, $(1,0,1)$, $(0,1,1)$, and $(0,0,1)$.



$$a \times b = \begin{vmatrix} i & j & k \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{vmatrix} = (-1,-1,1)$$

$$(-1,-1,1)\cdot(x-0,\ y-1,\ z-1) = 0$$

$$-x + (-y) + 1 + z + 1 = 0$$

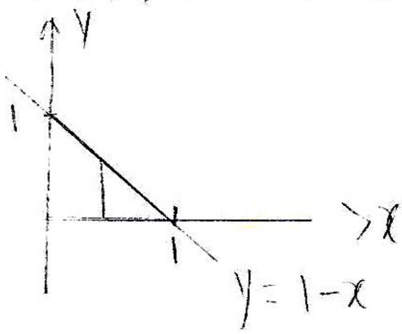
$$-x - y + z = 0$$

$$z = x + y$$

$$\iint \int_{z=x+y}^{1} xz\,dz\,dA$$

cross section

$(0,0)$, $(1,0)$, $(0,1)$, $(0,0)$



$$\int_0^1 \int_0^{1-y} \int_{x+y}^{1} xz\,dz\,dx\,dy$$

$$\int_0^1 \int_0^{1-x} \int_{x+y}^{1} xz\,dz\,dy\,dx.$$

$$= \int_0^1 \int_0^{1-x} \frac{1}{2}xz^2 \Big|_{x+y}^{1} dy\,dx$$

$$= \int_0^1 \int_0^{1-x} \frac{1}{2}x - \frac{1}{2}x(x+y)^2\,dy\,dx = \int_0^1 \int_0^{1-x} \frac{1}{2}x - \frac{1}{2}x^3 - x^2y - \frac{1}{2}xy^2$$

$\vdots$

$$= \frac{1}{30}$$

You can also do

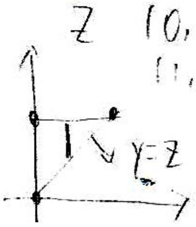
$$\int_0^1 \int_0^{z} \int_0^{z-y} xz\,dx\,dy\,dz$$

$$\int_0^1 \int_0^{z} \int_0^{z-x} xz\,dy\,dx\,dz$$

$(1,0,0)$

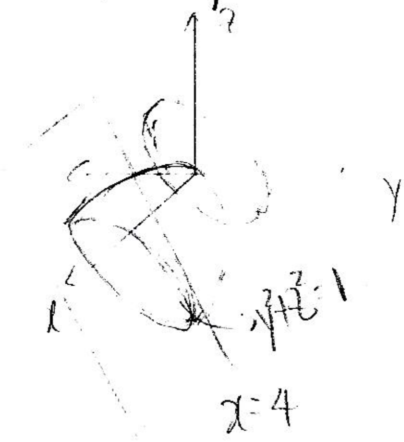
z $(0,1)$

$(1,1)$



$(0,0)$, $(1,1)$, $(0,1)$

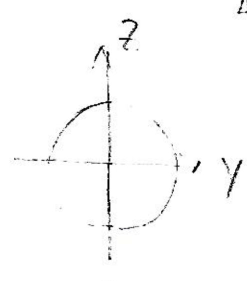
Q17 $\iiint_E x\,dV$, where E is bounded by the paraboloid $x = 4y^2 + 4z^2$ and the plane $x = 4$.



$4y^2 + 4z^2 = 4$

$y^2 + z^2 = 1$

$\iiint_{4y^2+4z^2}^{4} x \; dx \, dA$ → polar coordinate b/c of the circle on the yz-plane



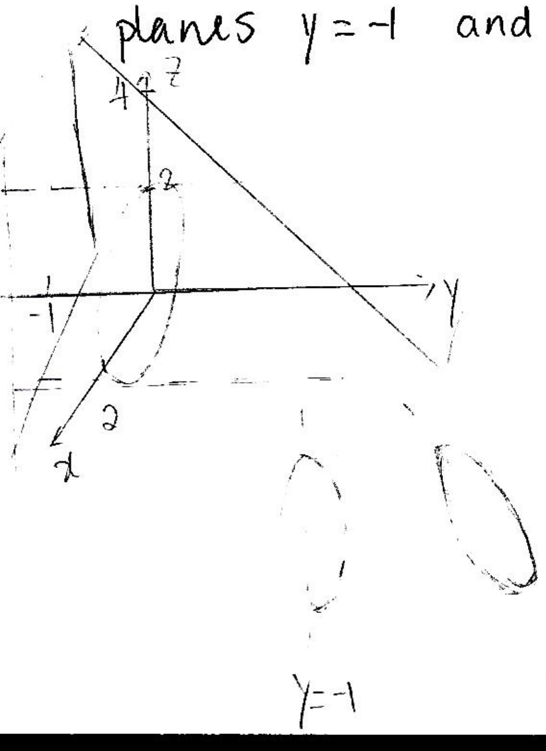
$\int_0^{2\pi}\int_0^1\int_{4r^2}^{4} x \; dx\, r\, dr\, d\theta$

$= \int_0^{2\pi}\int_0^1 \frac{x^2}{2} r \Big|_{4r^2}^{4} dr\, d\theta$

$= \int_0^{2\pi}\int_0^1 8r - 8r^5 \; dr\, d\theta$

$= \int_0^{2\pi} 4r^2 - \frac{4r^6}{3} \Big|_0^1 d\theta = \int_0^{2\pi} \frac{8}{3} \, d\theta = \frac{16}{3}\pi$

Q22 The solid enclosed by the cylinder $x^2+z^2=4$ and the planes $y=-1$ and $y+z=4$.



$\iiint_{-1}^{4-z} 1 \; dy \, dA$ circle on the x-z-plane

$x = r\cos\theta$

$z = r\sin\theta$

$y = 4 - r\sin\theta$

$= \int_0^{2\pi}\int_0^2\int_{-1}^{4-r\sin\theta} 1 \; dy \; r\,dr\,d\theta$

$= \int_0^{2\pi}\int_0^2 r y \Big|_{-1}^{4-r\sin\theta} dr\,d\theta$

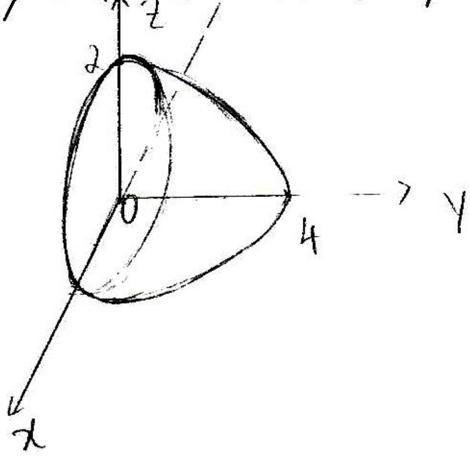
$= \int_0^{2\pi}\int_0^2 4r - r^2\sin\theta + r \, dr\, d\theta$

$$= \int_0^{2\pi}\int_0^{2} 5r - r^2\sin\theta \, dr\, d\theta = \int_0^{2\pi} \frac{5r^2}{2} - \frac{r^3}{3}\sin\theta \Big|_0^{2} d\theta \qquad ③$$

$$= \int_0^{2\pi} 10 - \frac{8}{3}\sin\theta \, d\theta = 10\theta + \frac{8}{3}\cos\theta \Big|_0^{2\pi} = 20\pi + \frac{8}{3} - \frac{8}{3} = 20\pi$$

Express the integral $\iiint_E f(x,y,z)\, dV$ as an iterated integral in six different ways, where E is a solid bounded by the given surfaces.

29. $y = 4 - x^2 - 4z^2, \quad y = 0.$



$$4 - x^2 - 4z^2 = 0$$
$$x^2 + 4z^2 = 4$$
$$x^2 + 2z^2 = 4$$
$$\frac{x^2}{(2)^2} + \frac{z^2}{1} = 1$$

$$x = \pm\sqrt{4 - 2z^2}$$

$$\int_{-1}^{1}\int_{-\sqrt{4-4z^2}}^{+\sqrt{4-4z^2}}\int_0^{4-x^2-4z^2} f(x,y,z)\, dy\, dx\, dz$$

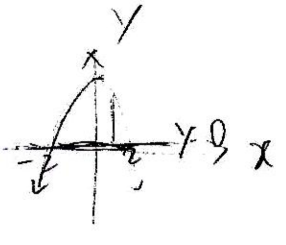
$$\int_{-2}^{2}\int_{-\sqrt{1-\frac{x^2}{4}}}^{\sqrt{1-\frac{x^2}{4}}}\int_0^{4-x^2-4z^2} f(x,y,z)\, dy\, dz\, dx$$

Consider z as the height

$$4z^2 = 4 - x^2 - y$$
$$z^2 = 1 - \frac{x^2}{4} - \frac{y}{4}$$
$$z = \pm\sqrt{1 - \frac{x^2}{4} - \frac{y}{4}}$$

$$\int_0^{4}\int_{-\sqrt{4-y}}^{\sqrt{4-y}}\int_{-\sqrt{1-\frac{x^2}{4}-\frac{y}{4}}}^{\sqrt{1-\frac{x^2}{4}-\frac{y}{4}}} f(x,y,z)\, dz\, dx\, dy$$

$$z = 0 : y = 4 - x^2$$
$$x^2 = 4 - y$$
$$x = \pm\sqrt{4-y}$$



$$\int_{-2}^{2}\int_0^{4-x^2}\int_{-\sqrt{1-\frac{x^2}{4}-\frac{y}{4}}}^{\sqrt{1-\frac{x^2}{4}-\frac{y}{4}}} f(x,y,z)\, dz\, dy\, dx$$

$$\int_{-1}^{1}\int_{0}^{4-4z^2}\int_{-\sqrt{4-y-4z^2}}^{\sqrt{4-y-4z^2}} f(x,y,z)\ dx\,dy\,dz$$

$$\int_{0}^{4}\int_{-\sqrt{1-\frac{y}{4}}}^{\sqrt{1-\frac{y}{4}}}\int_{-\sqrt{4-y-4z^2}}^{\sqrt{4-y^2-4z^2}} f(x,y,z)\ dx\ dz\,dy.$$

July 24th, 2019

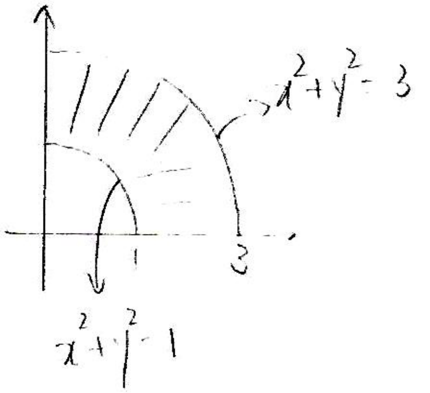
HW10

Q7 – Q27, Q29 – Q32 on Pg1005

Q9 – Q21, Q23 – Q36 on Pg673

Chp 15-3

Q9. $\iint_R \sin(x^2+y^2)\, dA$ (Polar coordinate), where R is the region in the 1st quadrant b/w the circles w/ center the origin and radii 1 and 3.



$$\begin{cases} x = r\cos\theta \\ y = r\sin\theta \end{cases}$$

$$\int_0^{\frac{\pi}{2}} \int_1^3 \sin(r^2)\, r\, dr\, d\theta$$

u-sub $r^2 = t$

$2r\,dr = dt$

$r\,dr = \frac{dt}{2}$

$1 \le r^2 \le 9$ b/c $1 \le r \le 3$

$$\int_0^{\frac{\pi}{2}} \int_1^9 \sin t \, \frac{dt}{2}\, d\theta$$

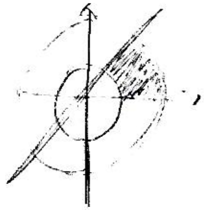
$$= \frac{1}{2}\int_0^{\frac{\pi}{2}} -\cos t \Big|_1^9 \, d\theta$$

$$= \frac{1}{2}\int_0^{2\pi} -\cos 9 + \cos 1 \, d\theta$$

$$= \frac{1}{2}(-\cos 9 + \cos 1)\cdot\frac{\pi}{2} = \frac{\pi}{4}(-\cos 9 + \cos 1)$$

Q13 $\iint_R \arctan(y/x)\, dA$, where $R = \{(x,y) \mid 1 \le x^2+y^2 \le 4, 0 \le y \le x\}$

$$\begin{cases} x = r\cos\theta \\ y = r\sin\theta \end{cases}$$



When $y = x$

$\cancel{r}\sin\theta = \cancel{r}\cos\theta$

$\frac{\sin\theta}{\cos\theta} = 1$

$\tan\theta = 1 \quad \theta = \frac{\pi}{4}.$

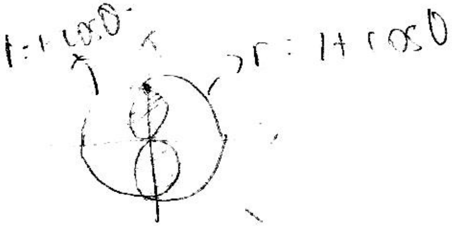
$$\int_0^{\frac{\pi}{4}} \int_1^2 \arctan(y/x)\, dA$$

$$= \int_0^{\frac{\pi}{4}} \int_1^2 \arctan\left(\frac{r\sin\theta}{r\cos\theta}\right) r\, dr\, d\theta$$

$$= \int_0^{\frac{\pi}{4}} \int_1^2 \tan^{-1}(\tan\theta)\, r\, dr\, d\theta$$

$$= \int_0^{\frac{\pi}{4}} \int_1^2 \theta\, r\, dr\, d\theta \quad \cdots = \frac{3}{64}\pi^2$$

Q16 The region enclosed by both of the cardiods $r = 1+\cos\theta$ and $r = 1-\cos\theta$.



area of the overlapping cardiods

$$= 4\int_0^{\frac{\pi}{2}} \int_0^{1-\cos\theta} r\, dr\, d\theta$$

$$= \frac{3\pi}{2} - 4$$

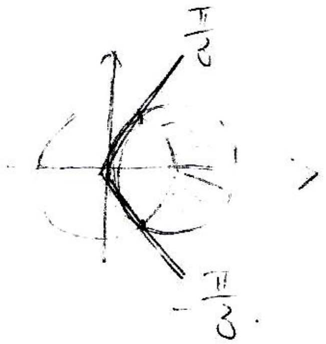
$1 - \cos\theta = 1 + \cos\theta$

$\cos\theta = 0$

$\theta = \frac{\pi}{2}$ or $\frac{3\pi}{2}$

$$A = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_0^{1-\cos\theta} r\, dr\, d\theta + \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \int_0^{1+\cos\theta} r\, dr\, d\theta.$$

Q17 The region inside the circle $(x-1)^2 + y^2 = 1$ and outside the circle $x^2 + y^2 = 1$.



Represent $(x-1)^2 + y^2 = 1$ in polar system:

$$(r\cos\theta - 1)^2 + r^2\sin^2\theta = 1.$$

$$r^2\cos^2\theta - 2r\cos\theta + 1 + r^2\sin^2\theta = 1$$

$$r^2 - 2r\cos\theta = 0.$$

$$r(r - 2\cos\theta) = 0$$

$$r \neq 0$$

$$r = 2\cos\theta$$

When do 2 circles intersect?

$$2\cos\theta = 1$$

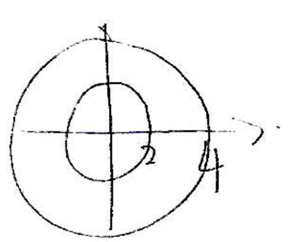
$$\cos\theta = \frac{1}{2}$$

$$\theta = \frac{\pi}{3} \text{ or } -\frac{\pi}{3}$$

$$\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \int_{1}^{2\cos\theta} r \, dr \, d\theta \quad \cdots = \frac{\pi}{3} + \frac{\sqrt{3}}{2}$$

Q22 Inside the sphere $x^2+y^2+z^2 = 16$ and outside the cylinder $x^2+y^2 = 4$.

$$V = \iint \int_{-\sqrt{16-x^2-y^2}}^{\sqrt{16-x^2-y^2}} 1 \, dz \, dA.$$



Switch to polar

$$= \int_0^{2\pi} \int_2^4 \int_{-\sqrt{16-r^2}}^{\sqrt{16-r^2}} dz \, r \, dr \, d\theta \quad \cdots = 32\sqrt{3}\,\pi$$

Q29 - 32 Evaluate the integral by converting to polar coordinate.

Q27 Inside both the cylinder $x^2+y^2=4$ and the ellipsoid

$4x^2+4y^2+z^2=64$

$$4x^2+4y^2+z^2=64$$

$$z^2=64-4x^2-4y^2$$

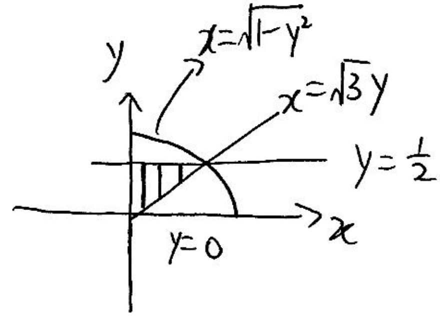
$$z=\pm\sqrt{64-4x^2-4y^2}$$

$$\iint\int_{-\sqrt{64-4x^2-4y^2}}^{\sqrt{64-4x^2-4y^2}} dz\, dA,$$

$$\int_0^{2\pi}\int_0^{2}\int_{-\sqrt{64-4r^2}}^{\sqrt{64-4r^2}} dz\, r\, dr\, d\theta.$$

Q3)

$$\int_0^{1/2}\int_{\sqrt{3}y}^{\sqrt{1-y^2}} x y^2 \, dx \, dy$$



$0 \le y \le \frac{1}{2}$

$\sqrt{3}y \le x \le \sqrt{1-y^2}.$

$$\begin{cases} x = r\cos\theta \\ y = r\sin\theta \end{cases}$$

$$\int\int r\cos\theta \cdot (r\sin^2\theta) \, r \, dr \, d\theta$$

$x = \sqrt{3}y$

$\cancel{r}\cos\theta = \sqrt{3}\cancel{r}\sin\theta$

$\frac{\sin\theta}{\cos\theta} = \frac{1}{\sqrt{3}}$

$\tan\theta = \frac{1}{\sqrt{3}}$

$\theta = \frac{\pi}{6}$

July 31^{st}, 2017.
HW11

Pg 1043 Q15-Q24, Q25a, Q26a, Q29-Q30.
Pg 1050 Q17-Q30, Q35-Q36, Q41-Q43.
Pg 1060 Q1-Q20, Q23-Q27

Q20 Evaluate $\iiint_E (x-y)\, dV$, where E is the solid that lies b/w the cylinders $x^2+y^2=1$ and $x^2+y^2=16$, above the xy-plane, and below the plane $z=y+4$.

Cylinder → cylindrical coordinates.

$$\begin{cases} x = r\cos\theta \\ y = r\sin\theta \\ z = z \end{cases}$$

$0 \leq z \leq y+4$
$0 \leq z \leq r\sin\theta + 4$

$$\int_0^{2\pi}\int_1^4\int_0^{r\sin\theta+4} (r\cos\theta - r\sin\theta)\, r\, dz\, dr\, d\theta.$$

$$= \int_0^{2\pi}\int_1^4 (r^2\cos\theta - r^2\sin\theta)\cdot(r\sin\theta+4)\, dr\, d\theta.$$

$$= \int_0^{2\pi}\int_1^4 r^3 \underbrace{\sin\theta\cos\theta}_{\frac{1}{2}\sin 2\theta} - r^3 \underbrace{\sin^2\theta}_{\frac{1}{2}(1-\cos(2\theta))} + 4r^2\cos\theta - 4r^2\sin\theta \, dr\, d\theta$$

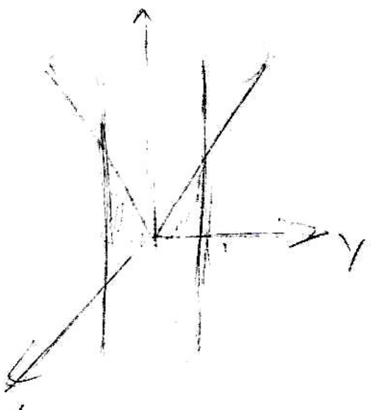
$$= \int_0^{2\pi} \frac{1}{4}r^4\sin\theta\cos\theta - \frac{1}{4}r^4\sin^2\theta + \frac{4r^3}{3}\cos\theta - \frac{4r^3}{3}\sin\theta \Big|_1^4 \, d\theta$$

$$= \int_0^{2\pi} \cdots\cdots = -\frac{255\pi}{4}$$

$$= \int_1^4\int_0^{2\pi} r^3\left(\frac{1}{2}\sin 2\theta\right) - r^3\left(\frac{1-\cos 2\theta}{2}\right) + 4r^2\cos\theta - 4r^2\sin\theta \, dr\ldots$$

$\vdots$

Q21 Evaluate $\iiint_E x^2\, dV$, where E is the solid that lies within the cylinder $x^2+y^2=1$, above the plane $z=0$, and below the cone $z^2=4x^2+4y^2$



$$z^2 = 4x^2+4y^2$$
$$z^2 = 4(r^2)$$
$$z = \pm 2r.$$

$$\int_0^{2\pi}\int_0^1\int_0^{2r} r^2\cos^2\theta \; r\, dz\, dr\, d\theta.$$

$$= \int_0^{2\pi}\int_0^1 2r^4\cos^2\theta \, dr\, d\theta$$

$$= \int_0^{2\pi} 2\cos^2\theta\, d\theta \cdot \int_0^1 r^4\, dr$$

$$= \int_0^{2\pi} 2\cdot\frac{1+\cos 2\theta}{2}\, d\theta \cdot \underbrace{\int_0^1 r^4\, dr}_{\frac{1}{5}}$$

$$= \frac{1}{5}\left(\theta + \frac{\sin 2\theta}{2}\Big|_0^{2\pi}\right)$$

$$= \frac{2\pi}{5}.$$

Q20

The solid go through 3 quarters except for the 1^{st} quadrant.

$\frac{\pi}{2} \le \theta \le 2\pi.$

$1 \le \rho \le 2.$

$0 \le \phi \le \frac{\pi}{2}$ b/c the solid is above xy-plane

$$\int_0^{\frac{\pi}{2}} \int_{\frac{\pi}{2}}^{2\pi} \int_1^2 f(\rho\sin\phi\cos\theta,\ \rho\sin\phi\sin\theta,\ \rho\cos\phi)\, \rho^2 \sin\phi \, d\rho\, d\theta\, d\phi.$$

Q26

$z = \sqrt{x^2+y^2}$

$\rho\cos\phi = \sqrt{\rho^2\sin^2\phi\cos^2\theta + \rho^2\sin^2\phi\sin^2\theta}.$

$= \sqrt{\rho^2\sin^2\phi(\cos^2\theta+\sin^2\theta)} = \sqrt{\rho^2\sin^2\phi} = \rho\sin\phi$

$\cancel{\rho}\cos\phi = \cancel{\rho}\sin\phi$

$\tan\phi = 1, \quad \phi = \frac{\pi}{4}.$

$$\int_0^{\frac{\pi}{4}} \int_0^{2\pi} \int_1^2 \underbrace{\sqrt{x^2+y^2+z^2}}_{\rho}\, \underbrace{dV}_{\rho^2\sin\phi} = \int_0^{\frac{\pi}{4}} \int_0^{2\pi} \int_1^2 \rho^3 \sin\phi \, d\rho d$$

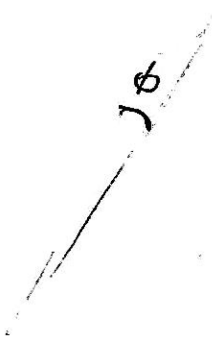
$$= \int_0^{\frac{\pi}{4}} \int_0^{2\pi} \frac{\rho^4}{4} \Big|_1^2 \sin\phi \, d\theta \, d\phi$$

$$= \int_0^{\frac{\pi}{4}} \int_0^{2\pi} \frac{15}{4} \sin\phi \, d\theta \, d\phi .$$

$$= \int_0^{\frac{\pi}{4}} \cancel{2}\pi \cdot \frac{15}{\cancel{4}_2} \sin\phi \, d\phi$$

$$= \frac{15}{2}\pi \left(-\cos\phi\right) \Big|_0^{\frac{\pi}{4}}$$ $\qquad \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}.$

$$= \frac{15}{2}\pi\left(-\frac{1}{\sqrt{2}} + 1\right).$$



$z = \sqrt{x^2+y^2}$

$\rho\cos\phi = \rho\sin\phi$

$$\int_0^{2\pi} \int_0^{\frac{\pi}{4}} \int_0^1 \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta$$

<u>24</u>

$$\int_0^{\frac{\pi}{2}} \int_0^{\frac{\pi}{4}} \int_0^{\sqrt{2}} f(\ ) \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta$$

$$\underbrace{\sqrt{x^2+y^2}}_{\text{cone}} \le z \le \underbrace{\sqrt{2-x^2-y^2}}_{\text{sphere}}$$

$z = \sqrt{2-x^2-y^2}$

$z^2 = 2-x^2-y^2$

$x^2+y^2+z^2 = 2 . \quad \rho = \sqrt{2}.$

region b/w cone and sphere.

Find ϕ: $z = \sqrt{x^2+y^2}$.

$\rho\cos\phi = \rho\sin\phi$

$\phi = \frac{\pi}{4}$

$\theta = \frac{\pi}{2}.$

$\sqrt{x^2+y^2} = \sqrt{2-x^2-y^2}$

$x^2+y^2 = 2-x^2-y^2$

$x^2+y^2 = 1$

$$\boxed{\begin{array}{l} 0 \le y \le \sqrt{1-x^2} . \quad 1^{st} \text{ octant.} \\ 0 \le x \le 1 \end{array}}$$