Binomial Model

- Consider flipping n coins, each of which has (independent) probability ***p*** of coming up heads, and probability ***1 –p*** of coming up tails. (Again 0 < ***p*** < 1.)
  [two possible outcomes – success or failure; each flip follows Bernoulli Distribution]

- Let X(random variable) be the total number of heads showing. We see that X can take values 0, 1, 2,…, n, where X = 0 means getting no head out of n flips, X = 1 means getting 1 head out of n flips.

- In general, X = number of successes in n trials. And the random variable X is said to have the Binomial(n, p). We write this as X ~ Binomial (n, p).

Approximation of Binomial with Normal Model

- Remarkably, when n, np and nq are large, or p is close to ½, the Binomial distribution is well approximated by Normal distribution.

- A Normal model works pretty well if we expect to see at least 10 successes and 10 failures. Use Success/Failure Condition(np ≥ 10 and n(1 – p) ≥ 10).

- Recall that X_i is a Bernoulli random variable with mean:
  $$\mu = E(X) = (0)(1-p) + (1)(p) = p$$
  and variance:
  $$\sigma^2=Var(X)=E[(X-p)^2]= (0-p)^2(1-p)+(1-p)^2(p)=p(1-p)[p+1-p]=p(1-p)$$
  With a sample of n Bernoulli distribution, which is a Binomial (n, p),
  Mean = np
  Sd = $\sqrt{npq}$

Question 1:
A particular tennis player makes a successful first serve 40% of the time. Assume that each serve is independent of the others. If she serves 6 times, what is the probability that she gets

a) All six serves in?
b) Exactly four serves in?
c) At least 4 serves in?
d) No more than 4 serves in?

Question 2 (Seatbelts 2, Chapter 13 Page 485):
Police estimate that 80% of drivers now wear their seatbelts. They set up a safety roadblock, stopping cars to check for seatbelt use.

a) What is the probability that the first 10 drivers are all wearing their seatbelts?
b) If police stop 30 cars during the first hour, find the mean and standard deviation of the number of drivers who will be wearing seatbelts.
c) If they stop 120 cars during this safety check, what is the probability they find at least 20 drivers not wearing their seatbelts?