Uncertainties for coincidence counting

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The following convention is used:

- γ = gamma, β = ABC or LD, C = ABCX or LDX, depending on whether doubles or triples data is being used;
- C refers to raw counts, e.g. C_β is the number of beta counts;
- R refers to uncorrected rate, e.g. R_γ is the gamma rate;
- B refers to background rate, e.g. B_C is the background coincidence rate;
- N refers to corrected rate, e.g. N_C is the decay- and background-corrected coincidence count rate.

A is the dilution factor, D is the decay factor, and m is the mass of the source. Negligible uncertainty in A, D, and m is assumed throughout.

β count rate per unit mass

The β count rate per unit mass (here referred to as F) is

$$F = N_\beta \cdot \frac{A}{m}$$

with uncertainty

$$u_F = \sqrt{\frac{1}{C_\beta} + \frac{1 - N_\beta/N_\gamma}{C_C}} \cdot N_\beta \cdot \frac{A}{m}.$$

In [?], the following formula is given:

$$\frac{u_F}{F} = \left(\frac{1 - \epsilon_\beta}{N_C}\right)^{1/2}$$

where ϵ_β is the β detection efficiency.

$\beta\gamma/C$ count rate per unit mass

The $\beta\gamma/C$ count rate per unit mass (here referred to as H) is

$$H = \frac{N_\beta N_\gamma}{N_C} \cdot \frac{A}{m}.$$

The uncertainty is

$$u_H = \sqrt{\frac{1}{C_\beta} + \frac{1 - N_\beta/N_\gamma}{C_C}} \cdot \frac{N_\beta N_\gamma}{N_C} \cdot \frac{A}{m}.$$

In [**?**], the following formula is given:

$$\frac{u_H}{H} = \left(\frac{1}{n_C} (2\epsilon_\beta \epsilon_\gamma - \epsilon_\beta - \epsilon_\gamma + 1) \right)^{1/2}$$

where ϵ_γ is the γ detection efficiency.

$\gamma/C - 1$ "inverse efficiency" parameter

Here $\gamma/C - 1$ is defined as

$$J = \frac{N_\gamma}{N_C} - 1$$

The uncertainty is complicated by the uncertainty in the background counts.

$$u_J^2 = \left(\frac{\partial J}{\partial N_\gamma} \right)^2 u_{N_\gamma}^2 + \left(\frac{\partial J}{\partial N_C} \right)^2 u_{N_C}^2 \quad (1)$$

$$= \frac{1}{N_C^2} u_{N_\gamma}^2 + \frac{N_\gamma^2}{N_C^4} u_{N_C}^2 \quad (2)$$

The γ count rate is given by

$$N_\gamma = \left(\frac{C_\gamma}{\tau} - B_\gamma \right) \cdot D$$

The uncertainty in the background γ count rate is simply

$$u_{B_\gamma} = \sigma_{B_\gamma}.$$

Similarly,

$$u_{B_C} = \sigma_{B_C}.$$

$1 - C/\gamma$ "inefficiency" parameter

Here $1 - C/\gamma$ is defined as

$$K = 1 - \frac{N_C}{N_\gamma}.$$

The uncertainty is

$$u_K^2 = \left(\frac{\partial K}{\partial N_\gamma}\right)^2 u_{N_\gamma}^2 + \left(\frac{\partial K}{\partial N_C}\right)^2 u_{N_C}^2 \tag{3}$$

$$= \frac{N_C^2}{N_\gamma^4} u_{N_\gamma}^2 + \frac{1}{N_\gamma^2} u_{N_C}^2 \tag{4}$$

References

[1] P. J. Campion and J. G. V. Taylor, *Statistical Errors in Disintegration Rate Measurements by the Coincidence Technique*, Int. J. Appl. Radiat. Isot. **10** 131-133 (1961)