# A LARGE-SIGNAL MODEL FOR CURRENT-PROGRAMMED BOOST CONVERTER

Yan-Fei Liu*, Student member IEEE and Paresh C. Sen, Fellow IEEE
Power Electronics and Drives Lab
Department of Electrical Engineering
Queen's University, Kingston, Canada, K7L 3N6
Tel. 613-545-2942 Fax. 613-545-6615

**Abstract** --- A general and unified large-signal model for current-programmed Boost converter is proposed. In this model, the active switch is modelled by a current source and the diode is modelled by a voltage source. The other parts of the converter remain unchanged. The steady-state relation and small-signal transfer functions can also be derived directly from the proposed large-signal model. The staedy state, small-signal and large-signal characteristics of current-programmed Boost converter are studied by both the phase-plane trajectory and the time domain analysis. Experimental prototypes is breadboarded to verify the analysis.

## I. INTRODUCTION

The current-programmed control is widely used because of its advantages over the conventional duty ratio control, such as fast response, improved damping, automatic over current protection and current sharing, etc., [1,2]. Unfortunately, because there are two feedback loops in the current-programmed control, one is the current loop and the other is the voltage loop, its analysis is very complicated. Only the small-signal models [3-5] are developed for the current-programmed DC-to-DC converters. The large-signal dynamic characteristics have not been presented. There is even no unified model that can describe both the steady-state and small-signal characteristics of the current-programmed converters.

The switching converters under current-programmed control are pulsed and nonlinear dynamic system. Such a system may be stable in the vicinity of the operating point, but may not be stable when the system undergoes a large perturbation. Therefore, the large-signal model for the current-programmed converters is essential to study the global dynamic characteristics of the switching converters under current-programmed control and to design robust and high performance switching power supply.

In this paper, a technique is proposed to establish the large-signal model of the current-programmed converters. The technique utilize the state space average technique [6] and takes the form of the averaged circuit model which has the same topology as Boost converter. The small-signal transfer functions and the steady-state relations can all be derived from the proposed large-signal model. The steady-state, small-signal and large-signal characteristics of the current-programmed Boost converter are investigated in detail. Experimental results are also presented to verify the proposed model.

## II. AVERAGED-CIRCUIT MODEL FOR CURRENT-PROGRAMMED BOOST CONVERTER

The dynamic characteristics of the switching converter can be analyzed by the averaged model. In the averaged model, all the circuit variables are the averaged value of the actual circuit variables. The state-space averaging method [6] has been applied to model the dynamic characteristics of the switching converter under duty ratio control and the small-signal characteristics of current-programmed converters. In [6], this averaged model is expressed by a set of state-space differential equations. Canonical circuit and small-signal transfer functions can be derived from the state-space averaged model.

*Yan-Fei Liu is now with Bell Northern Research Ltd., Ottawa, Canada
0-7803-1993-1/94 $4.00 © 1994 IEEE

Actually, for every set of state-space differential equations, a circuit topology can be found that has the same state-space differential equations. This argument provides another possibility to express the averaged model, i.e., using the averaged-circuit model, in which every circuit variable is the averaged value of the corresponding instantaneous variable. This alternative approach for dynamic modelling is used in this paper.

In Boost converter, as shown in Fig.1, the non-linear components are the active switch (transistor, MOSFET, etc.) and the diode. The current flowing through the active switch is a square wave. The switch current equals to the inductor current when the switch is on and the switch current equals to zero when it is off. The voltage across the diode is also a square wave. It equals to the output voltage when the active switch is on and equals to zero when the active switch is off.

Based on the above observation and the fact that it is the averaged inductor current and output voltage that are most interesting, it is proposed in the paper that the large-signal averaged-circuit model of the current-programmed converters can be obtained by:

(a) replacing the active switch by a current source with its value equal to the averaged current flowing through it
(b) replacing the diode by a voltage source with its value equal to the averaged voltage across it
(c) keeping all the other parts of the power stage unchanged.

In the current-programmed control [1], the switch is turned on by clock and is turned off when the inductor current reaches a threshold value determined by the control signal. Therefore, this threshold value now becomes the control variable and the duty ratio is only controlled indirectly.

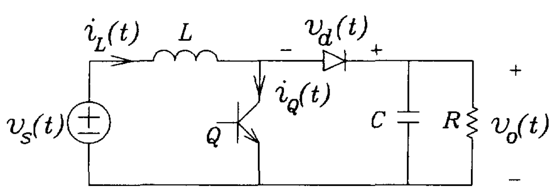



Fig.1 Boost converter

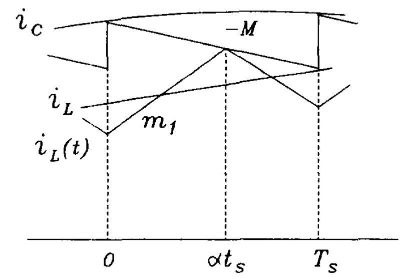



Fig.2 Detailed waveforms of current-programmed control

Fig.2 gives the detailed relationship between the control signal, $i_c$, and the inductor current, $i_L(t)$ [3]. The state-space averaged inductor current $i_L$ can be expressed as [3]:

$$i_L = i_c - \frac{1}{2} m_1 \alpha T_s - \alpha M T_s \tag{1}$$

where $m_1$ is the slope of the rising segment of the inductor current, $-M$ is the slope of the artificial ramp and $T_s$ is the switching period. Equation (1) gives the relation between the control signal $i_c$, averaged inductor current $i_L$, the duty ratio $\alpha$ and some other circuit variables.

In the current-programmed control, the duty ratio $\alpha$ is no longer the control variable but can be expressed from (1) as:

$$\alpha = \frac{i_c - i_L}{\frac{1}{2} m_1 T_s + M T_s} \tag{2}$$

Equation (2) holds true for all switching converters. For Boost converter, as shown in Fig.2, $m_1$ is:

$$m_1 = v_s / L \tag{3}$$

where L is the value of the inductor.

In a switching converter, the nonlinear components are the active switch Q and the diode D. Switch Q can be modelled by a current source $i_Q$ with its value equal to the average current flowing through it and diode D can be modelled by a voltage source $v_d$ with its value equal to the average voltage across it. For the Boost converter, $i_Q$ and $v_d$ are:

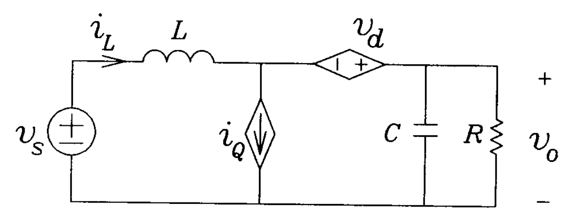
$$i_Q = \alpha i_L , \quad v_d = \alpha v_o \tag{4}$$

Since the duty ratio $\alpha$ is no longer the control variable in the current-programmed control, instead, it can be expressed by (2). Substituting (2) into (4) and also considering (3), the averaged switch current $i_Q$ and averaged diode voltage $v_d$ under current-programmed control can be found as:

$$i_Q = \alpha i_L = i_L \frac{i_c - i_L}{\frac{T_s}{2L} v_s + M T_s} \tag{5a}$$

$$v_d = \alpha v_o = v_o \frac{i_c - i_L}{\frac{T_s}{2L} v_s + M T_s} \tag{5b}$$

In Boost converter, Fig.1, replacing the switch Q by the current $i_Q$, and diode by the voltage $v_d$, (5), the averaged-circuit model for current-programmed Boost converter is obtained, as shown in Fig.3. This model is valid for large-signal analysis because no small-signal assumption is made during the above derivation.




$$i_Q = i_L \frac{i_c - i_L}{\frac{T_s}{2L} v_s + M T_s} \qquad v_d = v_o \frac{i_c - i_L}{\frac{T_s}{2L} v_s + M T_s}$$

Fig.3 Large-signal averaged-circuit model for current-programmed Boost converter

The large-signal characteristics of the current-programmed Boost converter can be analyzed from the large-signal averaged-circuit model, Fig.3. One way to do so is to use the state-space differential equations derived from the averaged-circuit model, i.e.,

$$L \frac{d i_L}{d t} = v_s - v_o \left( 1 - \frac{i_c - i_L}{\frac{T_s}{2L} v_s + M T_s} \right) \tag{6a}$$

$$C \frac{d v_o}{d t} = i_L \left( 1 - \frac{i_c - i_L}{\frac{T_s}{2L} v_s + M T_s} \right) - \frac{v_o}{R} \tag{6b}$$

By integrating (6) for different conditions, the response of the averaged inductor current and output voltage can be calculated and consequently the large-signal dynamic characteristics can be investigated.

Another method to study the large-signal characteristics is to use a circuit analysis software package, such as PSPICE, because the averaged-circuit model can be entered into such a software directly. The designer needs only to specify the configuration of the circuit and the parameters, the simulation software can formulate the circuit equations and give the corresponding results.

One advantage of the proposed technique is that the mathematical derivation and the effort to obtain the large-signal dynamic characteristics of the current-programmed converters are minimized so that the designer can concentrate his effort on how to improve the performance to meet the specifications. Another advantage is that the effect of the parasitic parameters, such as winding resistor of the inductor, equivalent series resistor (ESR) of the filter capacitor, etc., can be evaluated easily when either methods discussed above is employed to analyze the large-signal characteristics.

## III. CHARACTERISTICS OF CURRENT-PROGRAMMED BOOST CONVERTER

Based on the averaged-circuit model proposed in the previous section, the steady-state, small-signal and large-signal dynamic characteristics of the current-programmed Boost converter are investigated in this section.

### 3.1 *Steady-State Relation*

In the steady state, the voltage across the inductor and the current through capacitor are zero and all the circuit variables are time-invariant. The steady-state averaged-circuit model is shown in Fig.4, where $I_Q$ and $V_d$ are:

$$I_Q = \frac{I_L ( I_c - I_L )}{\frac{T_s}{2L} V_s + M T_s} \tag{7a}$$

$$V_d = \frac{V_o ( I_c - I_L )}{\frac{T_s}{2L} V_s + M T_s} \tag{7b}$$

From (7) and noticing that $V_s = V_o - V_d$, the steady-state output voltage can be solved from the following equation:

$$V_o^3 + \left( \frac{T_s}{2L} R V_s^2 + M T_s R V_s - I_c R V_s \right) V_o - \left( \frac{T_s}{2L} R V_s + M T_s R \right) V_s^2 = 0 \tag{8}$$

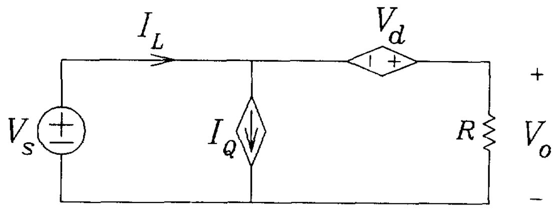



Fig.4 Steady-state model

Equation (8) shows that in the current-programmed Boost converter, the steady-state output voltage is no longer proportional to the supply voltage. The load resistor R, filter inductor L, switching period $T_s$, etc., will also affect the output voltage. The relation between the output voltage $V_o$ and the control signal $I_c$ is given in Fig.5 when $V_s$ =10V, L=278$\mu$H, R=10$\Omega$, $T_s$=40$\mu$H and M=0.045A/$\mu$S. It shows that the output voltage is almost linearly proportional to the control signal in this case.

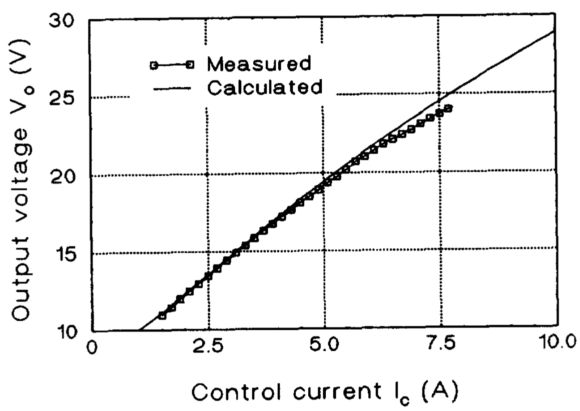



Fig.5 Calculated and measured steady-state output voltage

### 3.2 *Small-Signal Transfer Function*

In the averaged-circuit model, Fig.3, the switch current $i_Q$ and diode $v_d$ are the only non-linear components and they can be linearized at small-signal assumption. The small-signal models for $i_Q$ and $v_d$ can be obtained by substituting all the circuit variables in (5) with its steady-state value (expressed in capital letter) and small-signal perturbation (expressed in small letter with a hat above it), i.e., $x=X+\hat{x}$, and neglecting the products of the small-signal perturbation. The results can be expressed as:

$$\hat{i}_Q = \bar{\alpha}\hat{i}_L + a_c\hat{i}_c - a_L\hat{i}_L - a_s\hat{v}_s \tag{9}$$

$$\hat{v}_d = \bar{\alpha}\hat{v}_o + b_c\hat{i}_c - b_L\hat{i}_L - b_s\hat{v}_s \tag{10}$$

where $\bar{\alpha} = (V_o - V_s)/V_o$ is the steady-state duty ratio and

$$a_L = a_c = I_L/a, \quad a_s = (T_s/2L)*(\alpha I_L/a) \tag{11a}$$

$$b_L = b_c = (1-\alpha)Ra_c, \quad b_s = (1-\alpha)Ra_s \tag{11b}$$

$$a = V_s*T_s/(2L) + MT_s \tag{11c}$$

From the above expressions, the small-signal averaged-circuit model can be obtained and is shown in Fig.6, where the expressions for $\hat{i}_Q$ and $\hat{v}_d$ are given in (9) and (10), respectively. The small-signal transfer function can be derived easily from this small-signal model as:

$$\hat{v}_o = \frac{H_V(s)\hat{v}_s + H_I(s)\hat{i}_c}{s^2LC + (\frac{L}{R} + b_LC)s + (1-\bar{\alpha})(2a_L + 1 - \bar{\alpha})} \tag{12}$$

where $H_V(s)$ and $H_I(s)$ are expressed as:

$$H_V(s) = [a_L + (1-\bar{\alpha})b_s][1 + \frac{sa_sL}{a_L + (1-\bar{\alpha})b_s}] \tag{12a}$$

$$H_I(s) = a_cR(1-\bar{\alpha})^2[1 - \frac{sL}{(1-\bar{\alpha})^2R}] \tag{12b}$$

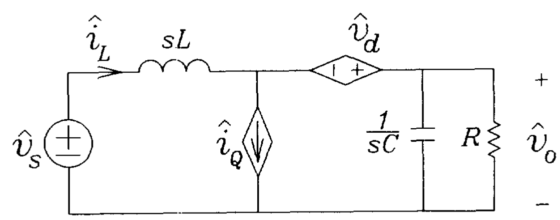



Fig.6 Small-signal model

Although the expression of the small-signal transfer function (12) appears to be different from what has been derived from the y-parameter model [4], they are actually the same because it can be proved that these two expressions can be simplified into a same one. The validity of the small-signal model is thus verified.

It is well known that under the current programmed control, more damping is introduced. This can also be observed from the small-signal transfer function (12). In (12), another term, $b_cCs$ is added to the term sL/R, so that the poles of the system becomes real under the current control.

Other interesting small-signal characteristics, such as the current loop gain, input impedance, output impedance, can all be derived from the small-signal model given in Fig.6.

### *3.3 Large-Signal Characteristics*

The large-signal characteristics of the current-programmed Boost converter can be analyzed using the averaged-circuit model shown in Fig.3. The phase-plane trajectory and time domain response are used here. The circuit parameters for the analysis are the same as before and the filter capacitor is C=163μF and the sum of winding resistor of the inductor and the sampling resistor is 0.065Ω. An evaluation version of PSPICE is used to obtain the large-signal response.

Fig.7 gives the phase-plane trajectory for different initial conditions when the control signal $I_c$ is fixed at 5A. The value of x-axis and y-axis is the dynamic value of the output voltage and inductor current, respectively, i.e., the actual value minus the steady-state one. It can be observed from the phase-plane trajectory that the current-programmed Boost converter has two separate real eigenvalues at the vicinity of the operating point. The change of the inductor current is much faster than that of the output voltage. This phenomenon is understandable because in the current-programmed control, it is the inductor current that we try to control directly.

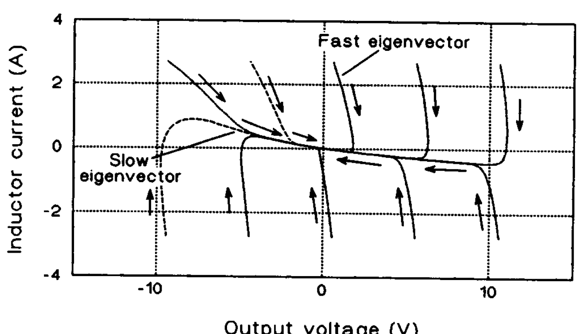



Fig.7 Phase-plane trajectory for different initial conditions

When the control signal steps between $i_c$=3A and $i_c$=6A, the simulated phase-plane trajectory of the current-programmed Boost converter is shown in Fig.8 (solid line). It can be observed from Fig.8 that when the control signal steps from 3A to 6A, inductor current increases at first (from left bottom corner to left top corner) and then decreases to its new steady state (right top corner). This phenomenon implies an overshoot in the time domain. On the other hand, the output voltage actually decreases at first and then

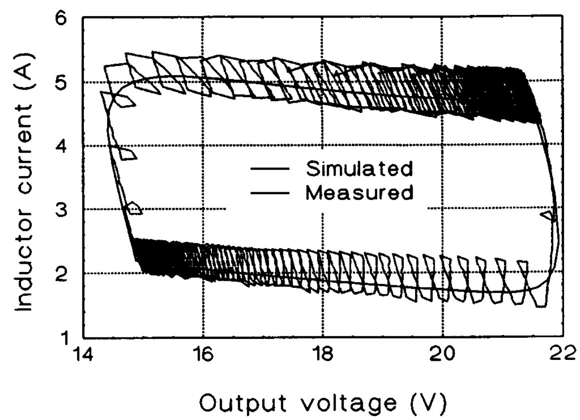



Fig.8 Simulated and measured phase-plane trajectory ($I_{con}$ steps between 3A and 6A)

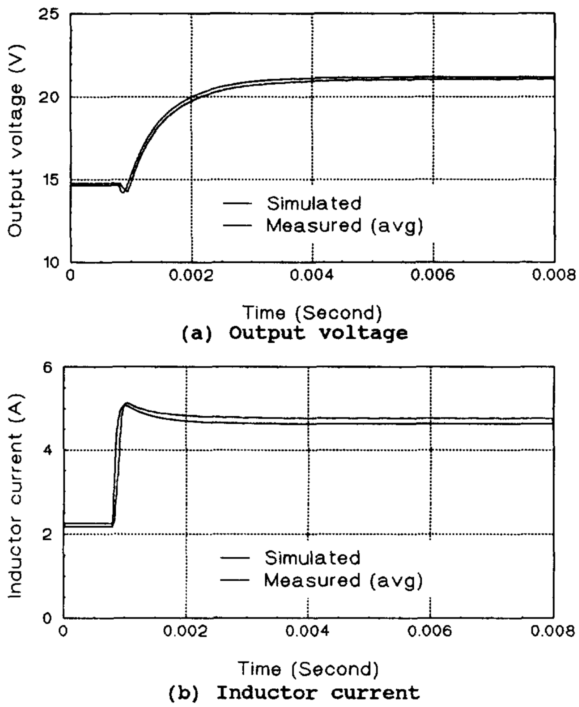
increases. This phenomenon implies a right half plane zero in the control-to-output transfer function, as been shown in the previous section. Similarly, when the control signal steps from 6A to 3A, the inductor decreases at first and then settle down at the new steady state and the output voltage increases at first and then decreases to the new steady state value.

For the time domain, when the control signal $i_c$ steps from 3A to 6A, the response of the output voltage and the inductor current is given in Fig.9. The dip of the output voltage immediately after the control signal steps is observed clearly. The overshoot of the inductor current is also observed clearly.

In this section, the characteristics of the current-programmed Boost converter is analyzed thoroughly using the large-signal averaged circuit model. The steady state and small-signal transfer functions can all be derived from the large-signal model conveniently. The large-signal behaviour is analyzed by both the phase plane trajectory and the time domain response when the control signal has a large-signal step change.

## IV. EXPERIMENTAL RESULTS

An experimental prototype of the current programmed Boost converter is breadboarded to verify the proposed large-signal model and the analytical results presented in the previous section. The circuit parameters are



(a) Output voltage

(b) Inductor current

Fig.9 Simulated and measured response of step change of $I_{con}$ (from 3A to 6A) for Boost converter

the same as before, i.e., $V_s$ = 10V, L = 278$\mu$H, R=10$\Omega$, $T_s$=40$\mu$H and M=0.045A/$\mu$S, C=163$\mu$F and the sum of the winding resistor of the inductor and the sampling resistor is 0.065$\Omega$.

Fig.5 gives the steady state characteristic of the current-programmed Boost converter. It plotted the measured and calculated (from equation (8)) relation between the output voltage $V_o$ and the control signal $I_c$ when the supply voltage is fixed at 10V. It shows that the measured value and the calculated value are very close. The small difference is introduced by the sampling resistor and winding resistor of the inductor which are not considered in (8). It is demonstrated that the steady state model derived from the averaged circuit model is valid to predict the steady state behaviour of the current programmed Boost converter.

Because the small-signal transfer function is exactly the same as derived in [4], which has been verified experimentally, the validity of the small-signal model derived from the averaged circuit model is, therefore, verified.

Phase plane trajectory and time domain response are used to verify the large-signal model. In the large-signal dynamic test, a digital scope is used to record the response of the inductor current and the output voltage. The recorded data is then plotted at the same graph with the simulated results.

When the control signal steps between 3A and 6A, the measured phase plane trajectory is plotted in Fig.8. The simulated phase plane trajectory is also plotted.

The time domain response of the measured output voltage and the inductor current are shown in Fig.9. The averaged value of the measured data is used. The simulated curves are also plotted. It is observed that the measured curve is very close to the simulated one. The output voltage dip is also observed from the measured data. The overshoot of the inductor current is also observed from the experimental data.

The experimental results presented in this section demonstrated that the proposed averaged circuit model for current-programmed Boost converter can predict the steady state and dynamic characteristics accurately.

## V. CONCLUSION

A technique to derive the large-signal model for the current-programmed Boost converter is proposed in this paper. The large-signal characteristics of the current-programmed converter can be analyzed systematically for the first time. The steady-state and small-signal dynamic characteristics can all be derived from this model. Some salient advantages of the proposed technique are:

(a) ***Simple***: the averaged-circuit model can be derived with very little mathematical manipulation.

(b) ***General***: this technique can be applied to all current-programmed DC-to-DC converters.

(c) ***Unified***: the small-signal transfer functions and the steady-state input-to-output relations can all be derived conveniently from the large-signal averaged-circuit model.

(d) ***Powerful***: the effect of the input filter, output filter, and the parasitic parameters can all be considered with little difficulty.

The same technique can also be used to derive the large-signal averaged circuit model for other PWM switching converters under current-programmed control. The key point is to find the expression for the averaged switch current $i_Q$ and averaged diode voltage $v_d$ and then replace the active switch and diode with $i_Q$ and $v_d$, respectively.

The proposed large-signal model is verified by an experimental current-programmed Boost converter using both phase-plane trajectory and time domain response. The steady-state relation is also verified.

## REFERENCES


[1] C.W. Deisch, "Simple switching control method changes power converter into a current source", IEEE Power Electronics Specialists Conference Record, 1978, pp.300-306

[2] A. Capel, G. Ferrante, D. O'Sullivan and A. Weinberg, "Application of the injected current model for the dynamic analysis of switching regulators with the new concept of $LC^3$ modulator", IEEE Power Electronics Specialists Conference Record, 1978, pp.135-147

[3] S.P. Hsu, A. Brown, L. Rensink and R.D. Middlebrook, "Modelling and analysis of switching DC-to-DC converters in constant-frequency current-programmed mode", IEEE Power Electronics Specialists Conference Record, 1979, pp.284-301

[4] R.D. Middlebrook, "Modelling current-programmed Buck and Boost regulators", IEEE Trans. Power Electronics, Vol.4, No.1, Jan. 1989, pp.36-52

[5] R. Tymerski and D. Li, "State space models for current programmed pulse width modulated converters", IEEE Power Electronics Specialists Conference Record, 1992, pp.337-344

[6] R.D. Middlebrook and S. Ćuk, "A General Unified Approach to Modelling Switching Power Converter Stages", IEEE Power Electronics Specialists Conference 1976 Record, pp.18-34