Here I changed the space and time coordinate indices from j and n to more straight forward x and t.

0.1. Stensils

Forward Euler method:

$$\boxed{U_x^{t+1} = \sigma U_{x-1}^t + (1-2\sigma)U_x^t + \sigma U_{x+1}^t} \tag{0.1}$$

Backward Euler method:

$$\boxed{-\sigma U_{x-1}^{t+1} + (1+2\sigma)U_x^{t+1} - \sigma U_{x+1}^{t+1} = U_x^t} \tag{0.2}$$

Crank-Nicolson method, $\sigma = \frac{D\Delta t}{\Delta x^2}$, as an F in my oldcode:

$$\boxed{-\frac{\sigma}{2}U_{x-1}^{t+1} + (1+\sigma)U_x^{t+1} - \frac{\sigma}{2}U_{x+1}^{t+1} = \frac{\sigma}{2}U_{x-1}^t + (1-\sigma)U_x^t + \frac{\sigma}{2}U_{x+1}^t} \tag{0.3}$$

0.2. Backward Euler

$$-\sigma U_{x-1}^{t+1} + (1+2\sigma)U_x^{t+1} - \sigma U_{x+1}^{t+1} = U_x^t \tag{0.4}$$

$$U_0^{t+1} = -\frac{D}{2k_u\Delta x} + \frac{1}{2}\sqrt{\left(\frac{D}{k_u\Delta x}\right)^2 + 4\frac{D}{k_u\Delta x}U_1^{t+1} + \Gamma^{t+1}} \tag{0.5}$$

$$U_L^{t+1} = -\frac{D}{2k_d\Delta x} + \frac{1}{2}\sqrt{\left(\frac{D}{k_d\Delta x}\right)^2 + 4\frac{D}{k_d\Delta x}U_{L-1}^{t+1}} \tag{0.6}$$