Here I changed the space and time coordinate indices from j and n to more straight forward x and t.

0.1. Different σ

The σ is defined in the finite difference approximation of the diffusion equation ??. For the Crank-Nicolson method, the second order derivative is approximated as half sum of both forward and backward Euler approximations.

$$\frac{u_x^{t+1} - u_x^t}{\Delta t} = \frac{D}{2\Delta x^2} \left((u_{x+1}^{t+1} - 2u_x^{t+1} + u_{x-1}^{t+1}) + (u_{x+1}^t - 2u_x^t + u_{x-1}^t) \right) \tag{0.1}$$

So the $\sigma = \frac{D\Delta t}{2\Delta x^2}$ is convenient.

For explicit stencils we have diffent finite difference equation:

$$\frac{u_x^{t+1} - u_x^t}{\Delta t} = \frac{D}{\Delta x^2} \left(u_{x+1}^t - 2u_x^t + u_{x-1}^t \right) \tag{0.2}$$

In this case $\sigma = \frac{D\Delta t}{\Delta x^2}$ is convenient.

0.2. Stensils

Here I use $\sigma = \frac{D\Delta t}{\Delta x^2}$, one for all.

Forward Euler method (explicit):

$$\boxed{U_x^{t+1} = \sigma U_{x-1}^t + (1 - 2\sigma) U_x^t + \sigma U_{x+1}^t} \tag{0.3}$$

Backward Euler method (implicit):

$$\boxed{-\sigma U_{x-1}^{t+1} + (1 + 2\sigma) U_x^{t+1} - \sigma U_{x+1}^{t+1} = U_x^t} \tag{0.4}$$

Crank-Nicolson method (implicit, second order in time), $\sigma = \frac{D\Delta t}{\Delta x^2}$:

$$\boxed{-\frac{\sigma}{2} U_{x-1}^{t+1} + (1 + \sigma) U_x^{t+1} - \frac{\sigma}{2} U_{x+1}^{t+1} = \frac{\sigma}{2} U_{x-1}^t + (1 - \sigma) U_x^t + \frac{\sigma}{2} U_{x+1}^t} \tag{0.5}$$

0.3. Backward Euler

$$-\sigma U_{x-1}^{t+1} + (1 + 2\sigma) U_x^{t+1} - \sigma U_{x+1}^{t+1} = U_x^t \tag{0.6}$$

$$U_0^{t+1} = -\frac{D}{2k_u \Delta x} + \frac{1}{2}\sqrt{\left(\frac{D}{k_u \Delta x}\right)^2 + 4\frac{D}{k_u \Delta x} U_1^{t+1} + \Gamma^{t+1}} \tag{0.7}$$

$$U_L^{t+1} = -\frac{D}{2k_d \Delta x} + \frac{1}{2}\sqrt{\left(\frac{D}{k_d \Delta x}\right)^2 + 4\frac{D}{k_d \Delta x} U_{L-1}^{t+1}} \tag{0.8}$$