

Here I changed the space and time coordinate indices from j and n to more straight forward x and t .

0.1. Stensils

Forward Euler method:

$$U_x^{t+1} = \sigma U_{x-1}^t + (1 - 2\sigma)U_x^t + \sigma U_{x+1}^t \quad (0.1)$$

Backward Euler method:

$$-\sigma U_{x-1}^{t+1} + (1 + 2\sigma)U_x^{t+1} - \sigma U_{x+1}^{t+1} = U_x^t \quad (0.2)$$

Crank-Nicolson method, $\sigma = \frac{D\Delta t}{\Delta x}$:

$$-\frac{\sigma}{2}U_{x-1}^{t+1} + (1 + \sigma)U_x^{t+1} - \frac{\sigma}{2}U_{x+1}^{t+1} = \frac{\sigma}{2}U_{x-1}^t + (1 - \sigma)U_x^t + \frac{\sigma}{2}U_{x+1}^t \quad (0.3)$$

0.2. Backward Euler

$$-\sigma U_{x-1}^{t+1} + (1 + 2\sigma)U_x^{t+1} - \sigma U_{x+1}^{t+1} = U_x^t \quad (0.4)$$

$$U_0^{t+1} = -\frac{D}{2k_u\Delta x} + \frac{1}{2}\sqrt{\left(\frac{D}{k_u\Delta x}\right)^2 + 4\frac{D}{k_u\Delta x}U_1^{t+1} + \Gamma^{t+1}} \quad (0.5)$$

$$U_L^{t+1} = -\frac{D}{2k_d\Delta x} + \frac{1}{2}\sqrt{\left(\frac{D}{k_d\Delta x}\right)^2 + 4\frac{D}{k_d\Delta x}U_{L-1}^{t+1}} \quad (0.6)$$