



Search and Optimization 22/23

Assignment

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1 Context

Eat to compete

As a sports enthusiast who has been playing tennis for many years, I have always wondered how to optimize my meal to combine performance while maintaining a reasonable budget.

The scientific literature provides several quantitative recommendations in terms of nutrition for athletes.

The challenge of this assignment is to develop a sports meal while minimizing the cost of the meal so that a student can use this model.

All the listed quantitative values of the different foods in the problem are available at the end of this assignment with links to the different product pages. The same goes for the daily recommendations mentioned in this assignment.

2 Linear Programming Problem

2.1 Problem Description

To get an idea of a sports meal of a tennis player, we will base ourselves on the diet of the champion Rafael Nadal who details his diet in an article. According to the article, his diet is based on Bread, Ham, Fish, Meat, Fresh Veggies, Olive Oil, Paella and Shrimp.

For this first problem, we will restrict our problem to two decision variables. This implies to restrict the number of available foods. So we will base ourselves on an important source of proteins: fish and an important source of carbohydrates: pasta.

Here is a summary table of all the nutritional information :

	Pasta (100 g)	Fish (100g)
Kcal	176	323
Carbohydrates	35.7	4.1
Fat	0.7	24.6
Protein	5.8	21.4
Cost	£0.155	£0.238

Nutritional information and cost of our problem

The daily intake of an athlete is on average 3000 calories per day, and it is considered that lunch represents 1/3 of the athlete's caloric intake, that is 1000 calories.

“The Dietary Guidelines for Americans suggest that the optimal macronutrient ratios for adults are as follows:

- Carbohydrates: 45–65% of calories
- Protein: 10–35% of calories
- Fat: 20–35% of calories”

([Dietary Guidelines for Americans, 2020–2025](#)Trusted Source)

By following the recommendations, we obtain a minimum of 137.6 grams of carbs, 56.3 grams of proteins, 30.6 grams of fat and a minimum total of 1000 calories for this meal.

We also impose a maximum limit of 1000 grams of pasta and fish to make sense of our problem.

2.2 Decision variables

Considering a meal based on pasta and fish, the two decision variables are:

X1: The quantity in grams of pasta.

X2: the quantity in grams of fish

2.3 Objective function

Our objective is to minimize the costs of the diet so that a student can follow it :

Objective function:

$$\text{Minimize } 0,0155 \cdot X1 + 0.0238 \cdot X2$$

2.4 Problem constraints

The constraints associated with this problem are based on the minimum requirements in terms of - calories, carbohydrate, fat and protein per day.

Type	Constraints
Kcal	$1.76X_1 + 3.23 * X_2 \geq 1000$
Carbohydrate	$0.357 * X_1 + 0.041 \geq 137.6$
Fat	$0.007 * X_1 + 0.246 * X_2 \geq 30.6$
Protein	$5.8 * X_1 + 21.4 * X_2 \geq 56.3$
Non negativity constraints and limitations:	$X_1 \geq 0$ and $X_2 \geq 0$ $X_1 \leq 1000$ $X_2 \leq 1000$

Summary table of the problem constraints

2.5 Mathematical formulation

Finally, here is the final formulation of our problem.

Intuitive formulation:

Minimize

$$0.00155 * X_1 + 0.00238 * X_2$$

Subject to

$$\begin{aligned} 1.76X_1 + 3.23 * X_2 &\geq 1000 \\ 0.357 * X_1 + 0.041 * X_2 &\geq 137,6 \\ 0.007 * X_1 + 0.246 * X_2 &\geq 30,6 \\ 5.8 * X_1 + 21.4 * X_2 &\geq 56,3 \end{aligned}$$

Non negativity constraint: $X_1, X_2 \geq 0$

Formal formulation :

Minimize

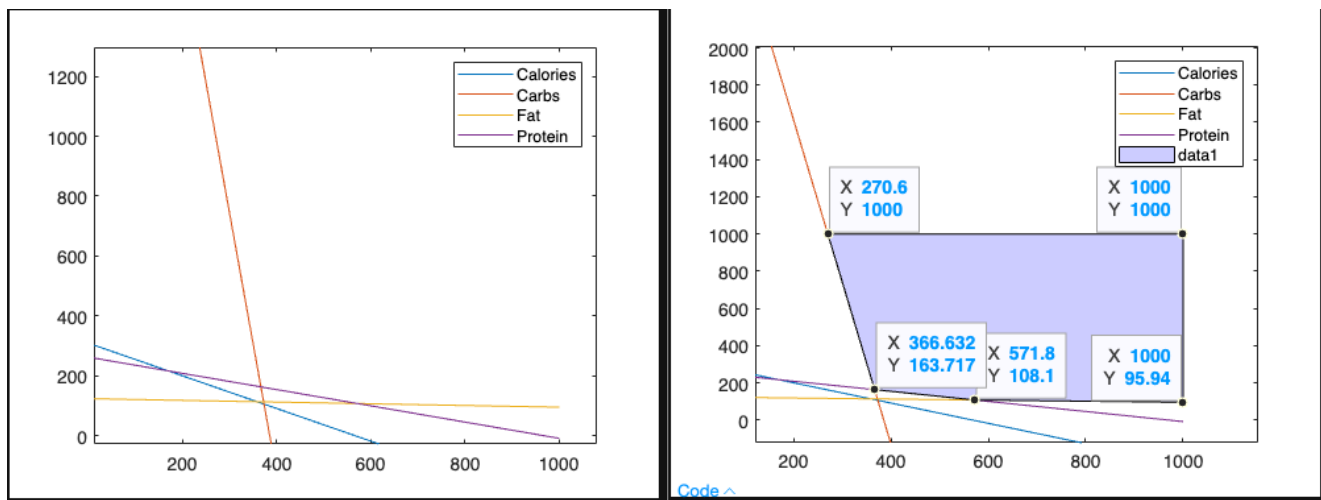
$$0.0155 * X_1 + 0.0238 * X_2$$

Subject to

$$\begin{aligned} -1.76X_1 - 3.23 * X_2 &\leq -1000 \\ -0.357X_1 - 0.041X_2 &\leq -137,6 \\ -0.007X_1 - 0.246X_2 &\leq -30,6 \\ -5.8X_1 - 21.4X_2 &\leq -56,3 \\ X_1, X_2 &\geq 0 \\ X_1, X_2 &\leq 1000 \end{aligned}$$

2.6 Graphical Method

To solve this problem graphically, we plot all the constraints of our problem and define the possible solution area.



Graphical representation of the "Eat to compete" problem

We can clearly see on the graph that the constraints on carbs, fat and protein are determining on the definition of the feasible area in addition to the limitation of X_1 and X_2 to 1000 maximum

The blue area defines the feasible area. It is known that the optimal solution is one of the Corner-point feasible solutions. We then set up the optimality test to determine which one is optimal

CPF Solution	Its Adjacent CPF Solutions	Optimality test
(270.6,1000)	(1000,1000) And (366.6,163.7)	$Z = 2.79$
(366.6,163.7)	(270.6,1000) And (571.8,108.1)	$Z = 0.958$
(571.8,108.1)	(366.6,163.7) And (1000,95.94)	$Z = 1.14$
(1000,95.94)	(571.8,108.1) And (1000,1000)	$Z = 1.78$
(1000,1000)	(270.6,1000) And (1000,95.94)	$Z = 3.93$

Optimality test table

Thus, by graphical resolution, the optimal solution is to consume 366.6 grams of pasta and 163.7 grams of fish for a cost of £0.96.

2.7 Matlab Implementation

Attention to display the graphs, the function of the course plotregion has been used and it is to include in the code. It has not been put in the code below because it is considered as plagiarism.

```
f = [0.00155 0.00238];

A = [-1.76 -3.23 ; -0.357 -0.041 ; -0.007 -0.246;
-0.058 -0.214]
b = [-1000 -137.6 -29.6 -56.3];
lb = [0,0];
ub = [1000;1000]
[val,fval] = linprog(f,A,b,[],[],lb)

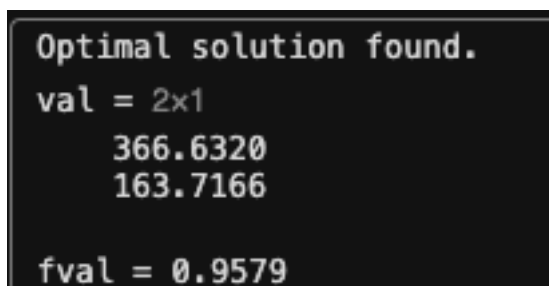
clear all
f = [0.00155 0.00238];

A = [1.76 3.23 ; 0.357 0.041 ; 0.007 0.246;
0.058 0.214]

b = [1000 137.6 30.6 56.3]

lb = [0,0];
ub = [1000;1000]
```

2.8 Solution



```
Optimal solution found.
val = 2x1
    366.6320
    163.7166

fval = 0.9579
```

Thus, by eating 366.6 grams of pasta and 163.7g of meat, we meet all the necessary calorie and macronutrient constraints while minimizing the cost of the meal £0.96. We find the same result as by graphic resolution, logical.

On the other hand, it is important to note the lack of realism in the problem. Indeed, it is difficult to imagine a meal consisting only of pasta and meat without any sauce or condiment.

2.9 Sensitivity Analysis

Let's now observe what happens, by changing some parameters of our problem. What happens if we decrease the protein demand by 1 unit, i.e., a demand of 55.3.

We notice that the constraint on the protein demand is a binding constraint.

Indeed, we obtain ($X_1 = 367.2$, $X_2 = 158.9$ and $Z = 0.947$)

Shadow price: $Z - Z = 0.011$

What happens if we decrease the demand for carbs by 1 unit, i.e., a demand of 55.3.

We notice that the constraint on protein demand is a binding constraint.

Indeed, we obtain ($X_1 = 363.7$, $X_2 = 164.5$ and $Z = 0.955$)

Shadow price: $Z - Z = 0.003$

Constraint on calories and fat are not binding the optimal solution. Constraints on the price of pasta and fish are imposed by the supplier Tesco, we can't change them.

We notice that changing some constraints does not significantly change the results. Moreover, it does not make sense to lower the daily recommendations because we would no longer be following the recommendations of our article.

3 Mixed-integer Programming Problem

In the rest of this assignment, we will evolve our initial problem into a mixed integer linear programming problem.

To perform well, we consider in our lunch, the recommendations in vitamin C. We would like to consume the recommended daily dose in one meal. After research, some fruits contain a lot of vitamin C, especially apples and oranges. We want to eat at least one apple and one orange for this meal. We also add meat as a credible alternative to fish in terms of protein for variety. We even want to impose a minimum value of 80g on the meat.

3.1 Problem Description

	Pasta (100g)	Fish(100g)	Meat(100g)	Apple, 1 unit	Orange, 1 unit
Kcal	176	323	106	70.89	41
Carbohydrates	35.7	4.1	0	15.69	8.2
Fat	0.7	24.6	1.1	0.13	0.2
Protein	5.8	21.4	24	0.53	0.8
Cost	£0.155	£0.238	£0.875	0.18	0.30
Vitamine C	0	0	0	6.3	52

Table of data

3.2 Decision variables

The five decision variables of our problem are:

X1: The quantity in grams of pasta

X2: the quantity in grams of fish

X3: The quantity in grams of meat

X4: the number unit of apple

X5: the number unit of Orange

3.3 Objective function

The objective function of our problem is always to minimize the cost of our meal.

Minimize

$$0.00155*X1 + 0.00238*X2 + 0.00875*X3 + 0.18*X4 + 0.30*X5$$

3.4 Problem constraints

The constraints associated with this problem are based on the minimum requirements in terms of - calories, carbohydrate, fat, protein and vitamins per day.

Type	Constraints
Kcal	$1.76X_1 + 3.23 * X_2 + 1.06*X_3+0.7089*X_4 + 0.41*X_5 \geq 1000$
Carbohydrate	$0.357*X_1 + 0.041*X_2 +0.1569*X_4 + 0.082*X_5 \geq 137.6$
Fat	$0.007*X_1 + 0.246*X_2+ 0.011*X_3+ 0.0013*X_4 + 0.002*X_5 \geq 30.6$
Protein	$0.058*X_1 + 0.214*X_2 + 0.24*X_3 + 0.0053*X_4+ 0.008*X_5 \geq 56.3$
Vitamines	$6.3*X_4 + 52*x_5 \geq 90$
Non negativity constraints and ressource limitation:	$X_1 \geq 0$ and $X_1 \leq 1000$ $X_2 \geq 80$ and $X_2 \leq 1000$ $X_3 \geq 80$ and $X_3 \leq 1000$
Integer constraint	$X_4 \in \mathbb{N}^*$ and $X_5 \in \mathbb{N}^*$

Summary table of the problem constraints

3.5 Mathematical formulation

The formulation of our optimization problem is as follows:

Intuitive formulation :

Minimize

$$0.00155*X_1 + 0.00238*X_2 + 0.00875*X_3 + 0.18*X_4 + 0.30*X_5$$

Subject to

$$1.76*X_1 + 3.23 * X_2 + 1.06*X_3+0.7089*X_4 + 0.41*X_5 \geq 1000$$

$$0.357*X_1 + 0.041*X_2 +0.1569*X_4 + 0.082*X_5 \geq 137.6$$

$$0.007*X_1 + 0.246*X_2+ 0.011*X_3+ 0.0013*X_4 + 0.002*X_5 \geq 30.6$$

$$0.058*X_1 + 0.214*X_2 + 0.24*X_3 + 0.0053*X_4+ 0.008*X_5 \geq 56.3$$

$$6.3*X_4 + 52*x_5 \geq 90$$

Non negativity constraints:

$$X_1, X_2, X_3, X_4, X_5 \geq 0, X_4 \in \mathbb{N}^*, X_5 \in \mathbb{N}^*$$

Limitation of resources:

$X_1, X_2, X_3 < 1000$
 $X_4, X_5 < 5$
 $X_2, X_3 \geq 80$

Formal format :

Minimize

$$0.0155 \cdot X_1 + 0.0238 \cdot X_2 + 0.00875 \cdot X_3 + 0.18 \cdot X_4 + 0.30 \cdot X_5$$

Subject to -

$$\begin{aligned}
 &1.76 \cdot X_1 - 3.23 \cdot X_2 - 1.06 \cdot X_3 + 0.7089 \cdot X_4 - 0.41 \cdot X_5 \leq -1000 \\
 &-0.357 \cdot X_1 - 0.041 \cdot X_2 - 0.1569 \cdot X_4 - 0.082 \cdot X_5 \leq -137.6 \\
 &-0.007 \cdot X_1 - 0.246 \cdot X_2 - 0.011 \cdot X_3 - 0.0013 \cdot X_4 - 0.002 \cdot X_5 \leq -30.6 \\
 &-0.058 \cdot X_1 - 0.214 \cdot X_2 - 0.24 \cdot X_3 - 0.0053 \cdot X_4 - 0.008 \cdot X_5 \leq -56.3 \\
 &-6.3 \cdot X_4 - 52 \cdot X_5 \leq -90 \\
 &X_1, X_2, X_3, X_4, X_5 \geq 0 \\
 &X_4 \in \mathbb{N}^*, X_5 \in \mathbb{N}^* \\
 &X_1, X_2, X_3 < 1000 \\
 &X_3 \geq 80
 \end{aligned}$$

3.6 Matlab Implementation

```
x1 = optimvar('x1',1,'LowerBound',0,'UpperBound',1000);
x2= optimvar('x2',1,'LowerBound',80,'UpperBound',1000);
x3= optimvar('x3',1,'LowerBound',80 , 'UpperBound',1000);
x4 = optimvar('x4', 'LowerBound',1,'UpperBound',5,'Type','integer');
x5 = optimvar('x5', 'LowerBound',1,'UpperBound',5,'Type','integer');
prob = optimproblem('Objective',0.00155*x1 + 0.00238*x2 + 0.00875*x3 + 0.18*x4 +
0.30*x5);
prob.Constraints.cons1 = 1.76*x1 + 3.23 * x2 + 1.06*x3+0.7089*x4 +
0.41*x5>=1000;
prob.Constraints.cons2 = 0.357*x1 + 0.041*x2 +0.1569*x4 + 0.082*x5>=137.6;
prob.Constraints.cons3 = 0.007*x1 + 0.246*x2+ 0.011*x3+ 0.0013*x4 + 0.002*x5 >=
30.6;
prob.Constraints.cons4 = 0.058*x1 + 0.214*x2 + 0.24*x3 + 0.0053*x4+ 0.008*x5>=
56.3;
prob.Constraints.cons5 = 6.3*x4 + 52*x5 >=90;
problem = prob2struct(prob);
[sol,fval,exitflag,output] = intlinprog(problem)
x1 = optimvar('x1',1,'LowerBound',0,'UpperBound',1000);
x2= optimvar('x2',1,'LowerBound',0,'UpperBound',1000);
x3= optimvar('x3',1,'LowerBound',0 , 'UpperBound',1000);
x4 = optimvar('x4', 'LowerBound',0,'UpperBound',5,'Type','integer');
x5 = optimvar('x5', 'LowerBound',0,'UpperBound',5,'Type','integer');
prob = optimproblem('Objective',0.00155*x1 + 0.00238*x2 + 0.00875*x3 + 0.18*x4 +
0.30*x5);
prob.Constraints.cons1 = 1.76*x1 + 3.23 * x2 + 1.06*x3+0.7089*x4 +
0.41*x5>=1000;
prob.Constraints.cons2 = 0.357*x1 + 0.041*x2 +0.1569*x4 + 0.082*x5>=137.6;
prob.Constraints.cons3 = 0.007*x1 + 0.246*x2+ 0.011*x3+ 0.0013*x4 + 0.002*x5 >=
30.6;
prob.Constraints.cons4 = 0.058*x1 + 0.214*x2 + 0.24*x3 + 0.0053*x4+ 0.008*x5>=
56.3;
prob.Constraints.cons5 = 6.3*x4 + 52*x5 >=90;
problem = prob2struct(prob);
[sol,fval,exitflag,output] = intlinprog(problem)
```

3.7 Solution

```
sol = 5x1
    371.8782
    110.2096
    80.0000
    1.0000
    2.0000

fval = 2.3187
```

Matlab solution

We have obtained an optimal solution. By consuming 371.9 g of pasta, 110.2g of fish, 80g of meat, an apple and two oranges, we obtain a meal that respects all our constraints in terms of macronutrients, calories and vitamins for a meal cost of £2.3 per meal.

3.8 Sensitivity Analysis

By imposing 80g of meat, we notice that the solution found is 80g of meat also, if we modify this constraint, it is very likely to affect our solution. This is understandable because meat, despite an interesting protein contribution, has a high cost and brings little fat, it appears intuitively less interesting than the solution with fish.

If we were to remove the condition on the quantity of meat, we would obtain a solution without meat:

```
sol = 5x1
    365.7160
    163.8653
         0
    1.0000
    2.0000

fval = 1.7369
```

Solution without minimum constraint on the meat

Another interesting point is that if we remove the minimal constraint of an apple, we notice that we reduce the price of the meal by 6% and that we do not need to consume the apple to reach the quota of vitamin C, the two oranges are sufficient.

However, if we remove the minimal constraint of one orange and leave the one on the apple, the solution still contains two oranges.

In other words, the minimal constraint of an apple is binding the optimal solution while the one on the orange is not.

Thus, it is interesting to remove the constraint of an apple and it will avoid having to consume 3 fruits.

sol = 5x1	
	1
1	372.3185
2	110.2023
3	80.0000
4	0
5	2.0000
fval = 2.1394	

Solution without minimum constraint on the apple

As for the previous problem, changing the constraint on fish or pasta by one unit does not bring any significant change on the solution of the problem

After sensitivity analysis, we can ask ourselves what the solution would be if we impose no minimum quantity on any food and see if it seems plausible.

Matlab implementation:

```
x1 = optimvar('x1',1,'LowerBound',0,'UpperBound',1000);
x2= optimvar('x2',1,'LowerBound',0,'UpperBound',1000);
x3= optimvar('x3',1,'LowerBound',0,'UpperBound',1000);
x4 = optimvar('x4','LowerBound',0,'UpperBound',5,'Type','integer');
x5 = optimvar('x5','LowerBound',0,'UpperBound',5,'Type','integer');
prob = optimproblem('Objective',0.00155*x1 + 0.00238*x2 + 0.00875*x3 + 0.18*x4 + 0.30*x5);
prob.Constraints.cons1 = 1.76*x1 + 3.23 * x2 + 1.06*x3+0.7089*x4 + 0.41*x5>=1000;
prob.Constraints.cons2 = 0.357*x1 + 0.041*x2 +0.1569*x4 + 0.082*x5>=137.6;
prob.Constraints.cons3 = 0.007*x1 + 0.246*x2+ 0.011*x3+ 0.0013*x4 + 0.002*x5 >= 30.6;
prob.Constraints.cons4 = 0.058*x1 + 0.214*x2 + 0.24*x3 + 0.0053*x4+ 0.008*x5>= 56.3;
prob.Constraints.cons5 = 6.3*x4 + 52*x5 >=90;
problem = prob2struct(prob);
[sol,fval,exitflag,output] = intlinprog(problem)
```

```

sol = 5x1
      366.1667
      163.7679
           0
           0
           2.0000

fval = 1.5573

```

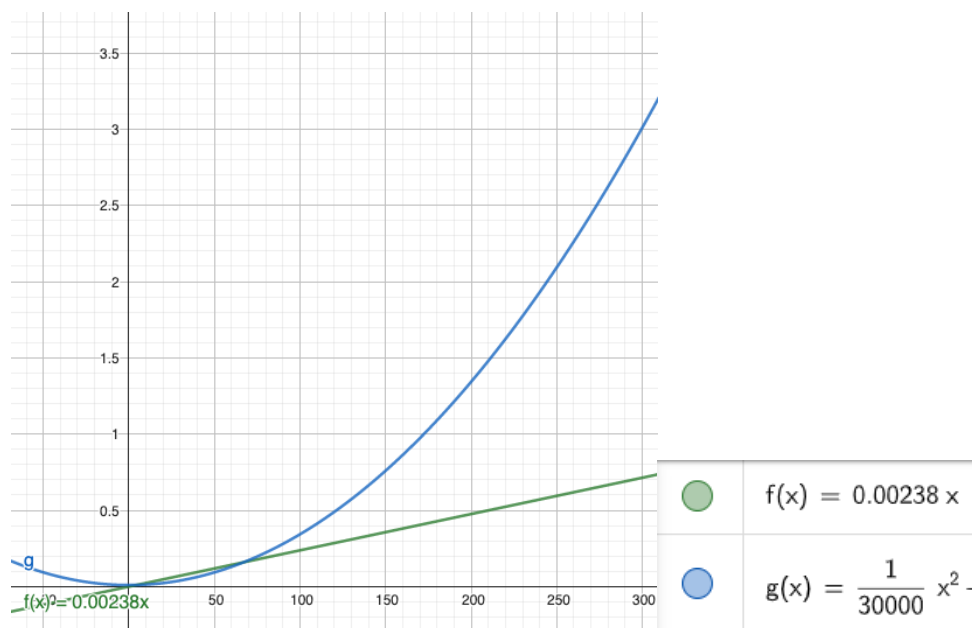
Best Solution of the problem in terms of cost

After studying the sensitivity analysis, we obtain the best possible result of this problem in terms of cost by consuming 366.1 grams of pasta, 163.8 grams of pasta, and two oranges for a cost of £1.6. This solution seems more plausible to implement for a meal.

4 Non-Linear Programming Problem

4.1 Problem Description

Our problem is evolving, the fish is a victim of the crisis and inflation in the UK, the price no longer follows a linear law but evolves as follows and becomes very quickly more expensive than before



Non-linear evolution of the fish price (function g) compared to the linear evolution (function f)

The nutritional information and foods remain the same.

	Pasta (100 grams)	Fish (100 grams)	Meat(100 grams)	Apple, 1 unit	Orange, 1 unit
Kcal	176	323	106	70.89	41
Carbohydrates	35.7	4.1	0	15.69	8.2
Fat	0.7	24.6	1.1	0.13	0.2
Protein	5.8	21.4	24	0.53	0.8
Cost	£0.155	£0.238	£0.875	0.18	0.30
Vitamine C	0	0	0	6.3	52

Table of data

4.2 Decision variables

The five decision variables of our problem are:

X1: The quantity in grams of pasta

X2: the quantity in grams of fish

X3: The quantity in grams of meat

X4: the number unit of apple

X5: the number unit of Orange

4.3 Objective function

Due to inflation, the objective function becomes:

Minimize

$$0.00155*x_1 + (1/30000) * x_2.^2 + 0.00875*x_3 + 0.18*x_4 + 0.30*x_5$$

4.4 Problem constraints

The constraints associated with this problem are based on the minimum requirements in terms of - calories, carbohydrate, fat, protein and vitamins per day.

Type	Constraints
Kcal	$1.76X_1 + 3.23 * X_2 + 1.06*X_3 + 0.7089*X_4 + 0.41*X_5 \geq 1000$
Carbohydrate	$0.357*X_1 + 0.041*X_2 + 0.1569*X_4 + 0.082*X_5 \geq 137.6$
Fat	$0.007*X_1 + 0.246*X_2 + 0.011*X_3 + 0.0013*X_4 + 0.002*X_5 \geq 30.6$
Protein	$0.058*X_1 + 0.214*X_2 + 0.24*X_3 + 0.0053*X_4 + 0.008*X_5 \geq 56.3$
Vitamines	$6.3*X_4 + 52*x_5 > 90$
Non negativity constraints:	$X_1 \geq 0$ and $X_2 \geq 0$ and $X_3 > 0$
Integer constraint	$X_4 \in \mathbb{N}^*$ and $X_5 \in \mathbb{N}^*$

Summary table of the problem constraints

4.5 Mathematical formulation

Formal format :

Minimize

$$0.00155*X1 + (1/30000)*x2.^2 + 0.00875*X3 + 0.18*X4 + 0.30*X5$$

Subject to -

$$\begin{aligned} 1.76*X1 - 3.23 * X2 - 1.06*X3+0.7089*X4 - 0.41*X5 &\leq -1000 \\ -0.357*X1 - 0.041*X2 - 0.1569*X4 - 0.082*X5 &\leq -137.6 \\ -0.007*X1 - 0.246*X2 - 0.011*X3 - 0.0013*X4 - 0.002*X5 &\leq -30.6 \\ -0.058*X1 - 0.214*X2 - 0.24*X3 - 0.0053*X4- 0.008*X5 &\leq -56.3 \\ -6.3*X4 - 52*x5 &\leq -90 \\ X1,X2,X3,X4,X5 &\geq 0 \\ X4 \in \mathbb{N}, X5 \in \mathbb{N} \end{aligned}$$

4.6 Matlab Implementation

```
clear all;
x1 = optimvar('x1','LowerBound',0,'UpperBound',1000);
x2= optimvar('x2','LowerBound',0,'UpperBound',1000);
x3= optimvar('x3','LowerBound',0,'UpperBound',1000);
x4 = optimvar('x4','LowerBound',0,'UpperBound',5,'Type','integer');
x5 = optimvar('x5','LowerBound',0,'UpperBound',5,'Type','integer');

prob = optimproblem('Objective',0.00155*x1 + (1/30000)*x2.^2 + 0.00875*x3 +
0.18*x4 + 0.30*x5, 'ObjectiveSense', 'min');

prob.Constraints.cons1 = 1.76*x1 + 3.23 * x2 + 1.06*x3+0.7089*x4 +
0.41*x5>=1000;
prob.Constraints.cons2 = 0.357*x1 + 0.041*x2 +0.1569*x4 + 0.082*x5>=137.6;
prob.Constraints.cons3 = 0.007*x1 + 0.246*x2+ 0.011*x3+ 0.0013*x4 + 0.002*x5 >=
30.6;
prob.Constraints.cons4 = 0.058*x1 + 0.214*x2 + 0.24*x3 + 0.0053*x4+ 0.008*x5>=
56.3;
prob.Constraints.cons5 = 6.3*x4 + 52*x5 >=90;

problem = prob2struct(prob);
problem.options = optimoptions('ga',
'MaxStallGenerations',200,'MaxGenerations',400);
[val, fval] = ga(problem)
```

4.7 Solution

```
val = 1x5
      570.9721  108.1168    0.1055    1.0000    2.0000
fval = 2.0556
```

Optimal solution approximated by the genetic algorithm

The optimal solution approximated is to consume 570 grams of pasta, 108 grams of fish, 0.10 grams of meat, one apple and two oranges per meal for a total cost of £2.1.

4.8 Sensitivity Analysis

Of course, it makes no sense to consume 0.1 gram of meat in a meal. Hence, we can consider the consumption of meat equal to 0 in our meal without unbalancing our meal.

We notice by this problem that the increase of the price of fish has direct repercussions on our problem, the solution found by the genetic algorithm implies to consume less fish because of its price, and consequently, this decrease is compensated by an increase of the quantity of pasta to consume. On the other hand, meat remains a non-preferred choice even with the increase of fish, it is preferable to increase the portion of pasta rather than to opt for meat.

The sensitivity analysis on fruits leads to the same conclusions as the previous problem. We also notice that the increase of the fish price has a direct consequence on the cost of the optimal solution. The cost of an ideal meal increases by 0.30£ compared to the previous problem.

The solution found requires 571 grams of pasta, which is starting to be substantial. It can be interesting to look at the evolution of the optimal solution if we add a constraint on the maximum amount of pasta in a logic of realism.

Let's see what happens if we limit the amount of pasta to 400 grams and we remove the minimal constraint of an apple.

```
clear all;
x1 = optimvar('x1','LowerBound',0,'UpperBound',400);
x2 = optimvar('x2','LowerBound',0,'UpperBound',1000);
x3 = optimvar('x3','LowerBound',0,'UpperBound',1000);
x4 = optimvar('x4','LowerBound',0,'UpperBound',5,'Type','integer');
x5 = optimvar('x5','LowerBound',0,'UpperBound',5,'Type','integer');

prob = optimproblem('Objective',0.00155*x1 + (1/30000)*x2.^2 + 0.00875*x3 +
0.18*x4 + 0.30*x5, 'ObjectiveSense', 'min');

prob.Constraints.cons1 = 1.76*x1 + 3.23 * x2 + 1.06*x3+0.7089*x4 +
0.41*x5>=1000;
prob.Constraints.cons2 = 0.357*x1 + 0.041*x2 +0.1569*x4 + 0.082*x5>=137.6;
prob.Constraints.cons3 = 0.007*x1 + 0.246*x2+ 0.011*x3+ 0.0013*x4 + 0.002*x5 >=
30.6;
```

```

prob.Constraints.cons4 = 0.058*x1 + 0.214*x2 + 0.24*x3 + 0.0053*x4+ 0.008*x5>=
56.3;
prob.Constraints.cons5 = 6.3*x4 + 52*x5 >=90;

problem = prob2struct(prob);
problem.options = optimoptions('ga',
'MaxStallGenerations',200,'MaxGenerations',400);
[val, fval] = ga(problem)

```

```

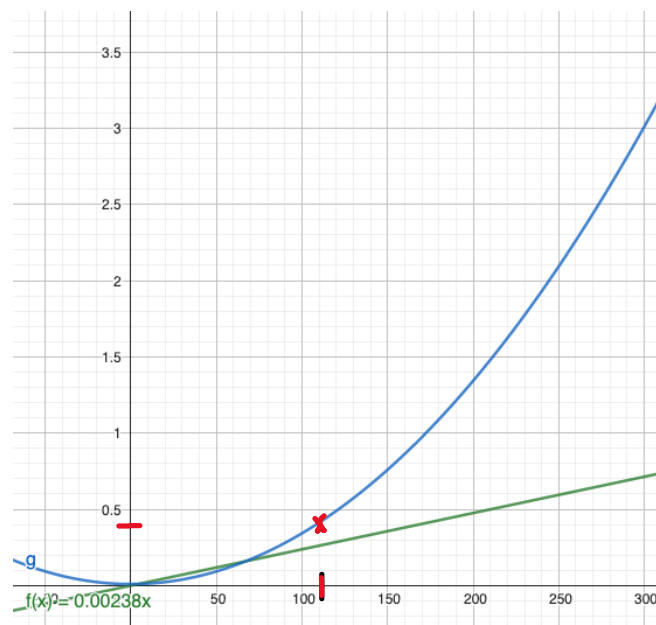
val = 1x5
    400.0000    111.2641    38.6395         0     2.0000

fval = 1.9708

```

Matlab solution

The solution is to eat 400 grams of pasta, 11 grams of fish, 38.6 grams of meat and two oranges for a cost of £2. To compensate for the limitation in pasta, it is preferable to increase the consumption of meat than fish, which seems logical because the price of fish soars beyond 110 grams purchased.



The soaring price of fish over 110 grams

5 Conclusion

The first problem allows us to give indications in terms of quantities to consume to satisfy the nutritional needs for an affordable cost for a student concerned about his food for his training.

The second problem makes the first one more complex and realistic. It allowed us to highlight the advantage of eating meat over fish on specific criteria. For according to aminogram, meat has a more complete aminogram profile than fish. Hence, it is all relative to the constraints that one puts on.

Thus, we could improve the problem by considering more parameters such as the aminogram, the glycemic index, as many interesting metrics in the constitution of a meal.

Finally, the last problem introduces an external constraint that can influence a dietary problem such as inflation in our case.

If we were to keep only one solution of this problem out of times of crisis and with the most sense for a meal. It would be to keep the one from the Mixed Integer programming problem by consuming 366.1 grams of pasta, 163.8 grams of pasta, and two oranges for a cost of £1.6.

It allows to follow the recommendations, without multiplying the food to consume for a reasonable cost.

These problems give an idea of a diet problem on a meal subject to nutritional constraints. With hindsight, this study gives an order of magnitude more than exact values to follow, it seems inconceivable to take out the scale to weigh to the tenth of grams. To improve this problem, it would be necessary to consider many more foods, to integrate condiments, to add other vitamin constraints so that this one is more realistic of a real sports meal. Furthermore, the recommended quantities can also vary according to the discipline practiced and the profile of the individual, our problem is based on an average individual.

Furthermore, offering the most cost-effective meal is interesting, but what about the variety of the food? It seems hardly conceivable to always eat the same meal at lunchtime every day.

6 Bibliography

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[2] Anon (n.d.) *Vitamin C for athletes: what you need to know.*, HealthspanElite Available at: <https://www.healthspanelite.co.uk/knowledge-hub/nutrition/vitamin-c-for-athletes-what-you-need-to-know> (Accessed: 9 November 2022).

[3] DIETARY GUIDELINES FOR AMERICANS, 2020-2025 (2021) S.I.: HEALTH AND HUMAN SERVICES.

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[5] Singh, C. (2022) *Rafael Nadal's Diet & Workout Is The Secret To Being A Grand Slam Champion.*, Boss Hunting Available at: <https://www.bosshunting.com.au/lifestyle/fitness/rafael-nadal-tennis-workout-diet-plan/> (Accessed: 9 November 2022).

All product references used in this book:

Pasta : <https://www.tesco.com/groceries/en-GB/products/254878499>

Fish : <https://www.tesco.com/groceries/en-GB/products/303088668>

Meat : <https://www.tesco.com/groceries/en-GB/products/304368923>