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Constraining Cosmological Parameters with MCMC Analysis of Type Ia Supernova Observations

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Abstract

In cosmology, a major issue that cosmologists have been grappling with for decades is the problem of dark energy, an unseen substance whose associated values will determine the fate of the universe. By constraining the universal matter density $\boldsymbol{\Omega_M}$, dark energy density $\boldsymbol{\Omega_\Lambda}$, and the equation of state $\boldsymbol{w}$, it is possible to determine the rate of expansion of the universe, $\boldsymbol{H_0}$, measured in km/s/Mpc. This can be achieved by observing the redshift and apparent magnitude of Type Ia supernovae, which are otherwise known as 'standard candles', given that they each emit the same luminosity, and thus the same absolute magnitudes. The observed redshifts of these supernovae become stretched by the expansion of space – using these values to calculate a theoretical apparent magnitude for each supernova will show a discrepancy between the theoretical and observed apparent magnitudes. This discrepancy can then be used to constrain the aforementioned cosmological parameters. The following study uses data from the 2012 Lawrence Berkeley National Laboratory Supernova Cosmology Project, conducted by Suzuki et al.

Calculations

Astrophysics

Friedmann-Robertson-Walker Equation:

$$ds^2 = -c^2dt^2 + a(t)^2\left(\frac{1}{1-kr^2}dr^2 + r^2 d\Omega\right)$$

Necessary Relationships to find Co-Moving Distance:

$$H = \frac{\dot{a}}{a}$$

$$\frac{1}{a(t_e)} = (1+z)$$

Co-Moving Distance:

$$r(z,\{p_i\}) = cH_0{}^{-1}\int_0^z \frac{dz'}{\sqrt{\Omega_M(1+z')^3+\Omega_\Lambda(1+z')^{3(1+w)}+\Omega_R(1+z')^4-\Omega_K(1+z')^2}}$$

Luminosity Distance:

$$d_L(z,\{p_i\}) = r(z)(1+z)$$

Apparent Magnitude:

$$m(z,\{p_i\}) = 5log_{10}[H_0 d_L(z,\{p_i\})] + \mathcal{M}$$

Statistics

Chi Squared:

$$X^2 = \left(m - m^{th}\right)^T Cov^{-1}\left(m - m^{th}\right)$$

Likelihood:

$$\mathcal{L} = \frac{1}{(2\pi)^{n/2}|det(Cov)|^{1/2}} exp\left[-\frac{1}{2}X^2\right]$$

Legend

c : Speed of light (m/s)
a : Spacetime stretching factor
$a(t_e)$: Spacetime stretching factor at time of emitted light
k : Spacetime curvature factor
H : Hubble constant (km/s/Mpc)
H_0 : Time invariant Hubble constant (in frame of reference) (km/s/Mpc)
z : Redshift
Ω_M : Matter density
Ω_Λ : Dark energy density
Ω_R : Radiation density
Ω_K : Cosmological curvature
w : ratio of cosmological pressure to energy density
$\mathcal{M}$: related to absolute magnitude of Type Ia supernovae
m : observed apparent magnitude of each supernova
m^{th} : redshift-calculated apparent magnitude of each supernova
Cov : Covariance matrix of observations
n : number of supernovae in the dataset

Data

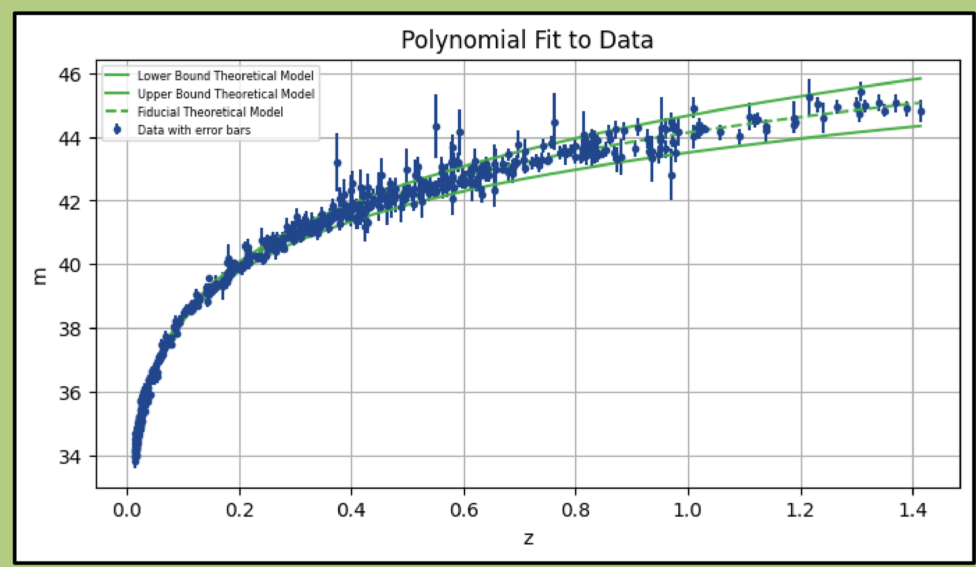

Figure 1

- The different values that $\boldsymbol{\Omega_M}$, $\boldsymbol{\Omega_\Lambda}$, and $\boldsymbol{w}$ can take on will change the theoretical apparent magnitude calculated by the observed Type Ia supernovae redshifts.
- The upper bound in this plot is calculated with $\boldsymbol{\Omega_M} = 0$ and $\boldsymbol{\Omega_\Lambda} = 1$, and vice-versa for the lower bound. The dashed line in the middle uses the estimated values for these cosmological parameters.

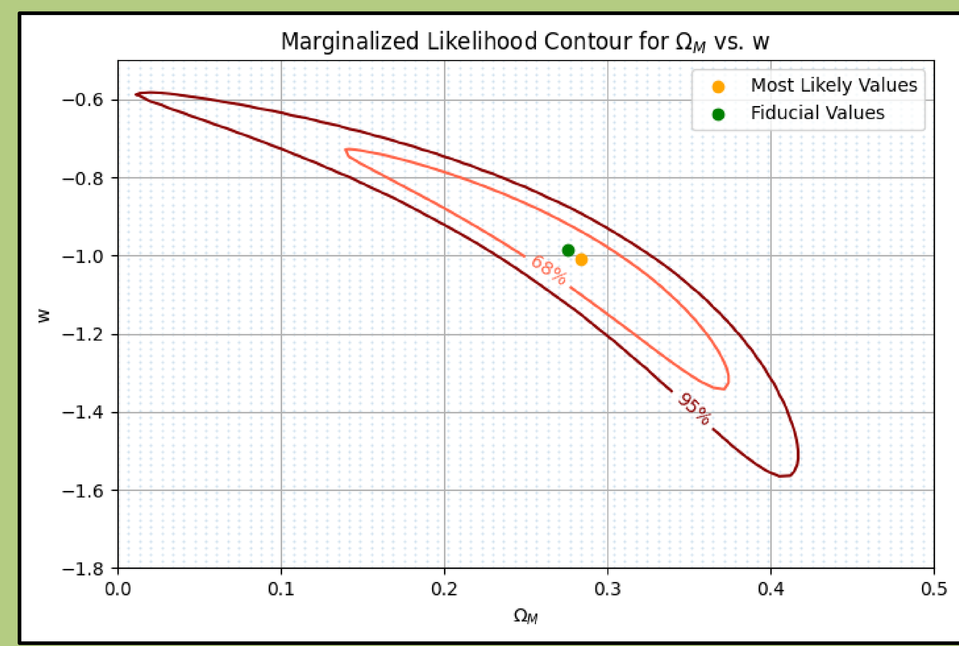

Figure 2

- Letting $\boldsymbol{\Omega_\Lambda} = 1 - \boldsymbol{\Omega_M}$ and varying $\boldsymbol{\Omega_M}$ and $\boldsymbol{w}$ simultaneously in a grid returns the 68% and 95% likelihood contours for the two parameters.

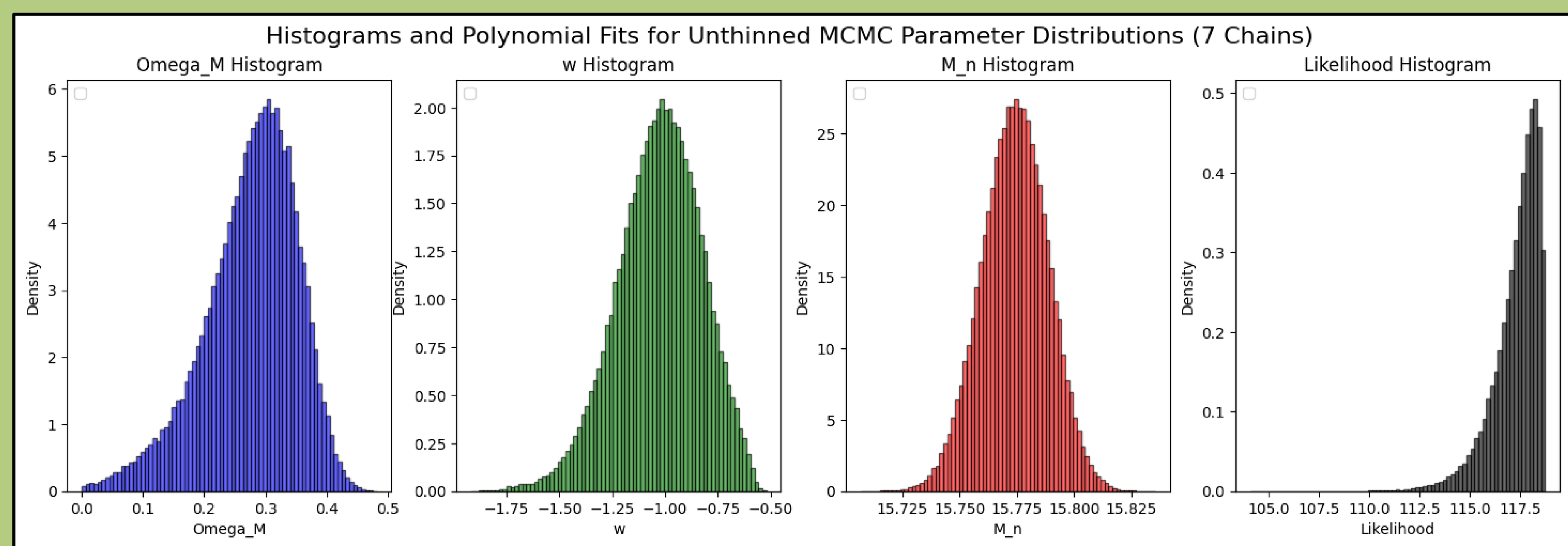

Figure 3

- Running a Markov Chain Monte Carlo (MCMC) algorithm with 1,000,000 likelihoods returns the above distributions for $\boldsymbol{\Omega_M}$, $\boldsymbol{w}$, $\boldsymbol{\mathcal{M}}$, and the associated likelihoods for a given combination of their values.
- The MCMC algorithm optimizes the runtime on the calculation of these likelihoods, allowing for a higher number of computations, and thus a more precise constraining of the parameters.

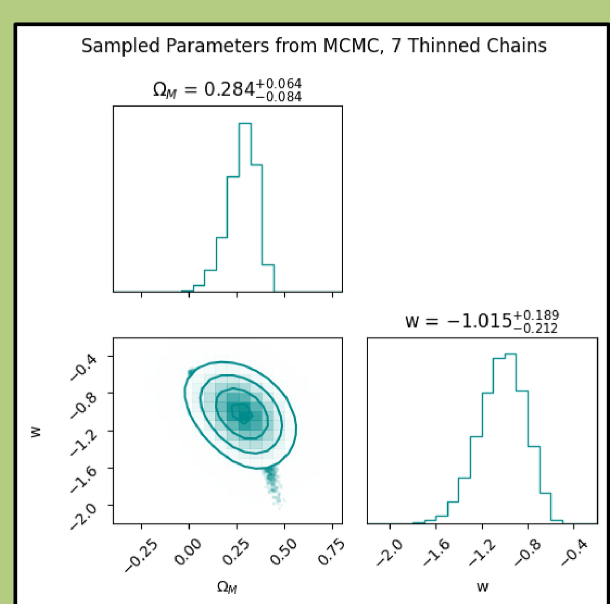

Figure 4

- Using the py.corner package returns a contour plot of the distribution of $\boldsymbol{\Omega_M}$ versus $\boldsymbol{w}$, as shown on the left.
- The likelihoods associated with each combination of $\boldsymbol{\Omega_M}$ and $\boldsymbol{w}$ values are converted into posterior values by py.corner, which gives the estimated value of each parameter and their uncertainties – shown at the top of each individual parameter distribution plot.

Results

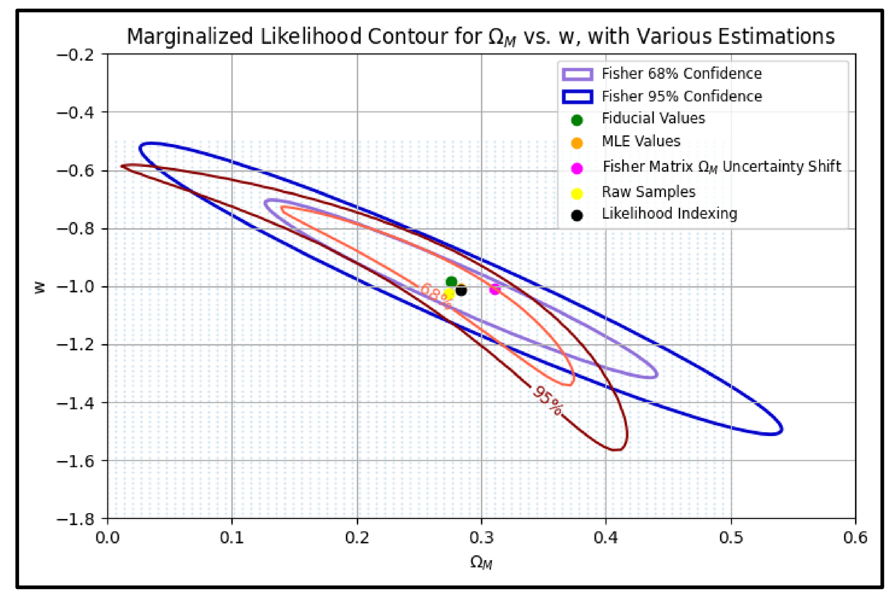

Figure 5

- The Fisher Information Matrix ellipses generated using the constrained values for the cosmological parameters appears to follow the likelihood contours closely, but deviates from them at their tails, showing the non-gaussianity in the parameter distributions.

Ω_M

Estimation Method	Estimation
MAP (Chains)	$0.284 \pm^{0.06}_{0.08}$
Chain Overall Average	$0.275 \pm^{0.08}_{0.05}$
MLE (Grid)	$0.284 \pm^{0.03}_{0.03}$
Fiducial	$0.276\pm^{0.02}_{0.02}$

w

Estimation Method	Estimation
MAP (Chains)	$-1.015\pm^{0.19}_{0.21}$
Chain Overall Average	$-1.028\pm^{0.07}_{0.12}$
MLE (Grid)	$-1.009\pm^{0.07}_{0.09}$
Fiducial	$-0.985\pm^{0.07}_{0.08}$

Conclusions

The final estimated values of $\boldsymbol{\Omega_M}$ and $\boldsymbol{w}$ by MAP (Maximum a Posteriori) estimation, of 0.284 and -1.100 respectively closely align with the values found by Suzuki et al. in the LBNL Supernova Cosmology Project, of 0.276 and -0.985. These estimations indicate a dark energy dominated universe. which explains the observed acceleration in the expansion of space. If this rate of expansion continues forever, it is theorized that the universe will end in a 'Big Rip', where eventually atoms will no longer be able to hold together due to the overpowering force of dark energy, expanding the space in between protons and neutrons.

Acknowledgements

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References

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