Coursera-Stanford-ML-Notes

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20 June 2017 - ? July 2017

Contents

1 Week 1: Introduction

1.1 Overview

– Machine Learning: "A computer program is said to learn from experience E with respect to some class of tasks T and performance measure P, if its performance at tasks in T, as measured by P, improves with experience E."
– Supervised Learning: know what our correct output looks like
  - Regression: want continuous output
  - Classification: want discrete output
– Unsupervised Learning: little or no idea what our results should look like
  - Clustering: find groups according to similarity in various variables
  - Nonclustering: find structure in chaos

2 Week 2: Linear Regression with Multiple Variables

2.1 Overview

– Use linear regression for continuous output
– Choose gradient descent if many features (million+) because the inverse matrix required for the normal equation can become expensive to compute
– Normal equation will directly compute theta
– Normalize features if using gradient descent

2.2 Symbols

$$
\begin{aligned}
m &= number\ of\ samples \\
n &= number\ of\ feature \\
x &= (n \times 1) \\
X &= (m \times n) \\
X_j &= (m \times 1) \\
\theta &= (n \times 1) \\
\theta_j &= (1 \times 1)
\end{aligned}
$$

2.3 Gradient Descent

$$
\begin{aligned}
\text{Hypothesis Function} \qquad & h_\theta(x) = \theta^\intercal \times x \\
\text{Vectorized Hypothesis Function} \qquad & h_\theta(X) = X \cdot \theta \\
\text{Linear Regression Cost Function} \qquad & J(\theta) = \frac{1}{2m} \sum (h_\theta(X) - y)^2 \\
\text{Derivative of Linear Regression CF wrt } \theta_j \qquad & \frac{\partial}{\partial \theta_j} J(\theta) = \frac{1}{m} \sum (h_\theta(X) - y) \ .* X_j \\
\text{Change in } \theta_j \qquad & \theta_j = \theta_j - \alpha \frac{\partial}{\partial \theta_j} \\
& \quad = \theta_j - \alpha \frac{1}{m} \sum (h_\theta(X) - y) \ .* X_j \\
\text{Vectorized Change in } \theta \qquad & \theta = \theta - \alpha \frac{1}{m} X^\intercal (X \cdot \theta - y)
\end{aligned}
$$

2.4 Normal Equation

$$\theta = (X^\intercal \cdot X)^{-1} \cdot X^\intercal \cdot y$$

3 Week 3: Logistic Regression

3.1 Overview

- Use logistic regression for discrete output (classification)
  - $h_\theta(x) = (y = 1|x;\theta)$; gives probability that the output is 1 given x
  - Sigmoid/Logistic function maps any real number to (0, 1)
  - For multi-class classification, use one-vs-all
  - Pick class i that maximizes $h_\theta^i(x)$
- Overfitting is when learned hypothesis fits training data well but fails to generalize
  - Underfitting is when doesn't fit training data
- Address overfitting by reducing number of features, model selection, and regularization
  - Regularization results in simpler hypothesis and less overfitting
  - Extremely large λ will result in underfitting and gradient descent will fail to converge
  - Do not regularize λ_0
- Use other prewritten optimization algorithims (conjugate gradient, BFGS, L-BFGS) because they are faster

3.2 Logistic Regression Hypothesis Function

$$
\begin{aligned}
\text{Sigmoid/Logistic Function} \qquad & g(z) = \frac{1}{1+e^{-z}} \\
\text{Hypothesis Function} \qquad & h_\theta(x) = g(\theta^\intercal x) \\
& \quad\;\;\; = \frac{1}{1+e^{-\theta^\intercal x}}
\end{aligned}
$$

3.3 Logistic Regression Cost Function

$$
\begin{aligned}
Cost(h_\theta(x), y) &= \begin{cases} -\log(h_\theta(x)) \text{ if y = 1} \\ -\log(1-h_\theta(x)) \text{ if y = 0} \end{cases} \\
&= -y\log(h_\theta(x)) - (1-y)\log(1-h_\theta(x)) \\
J(\theta) &= \frac{1}{m}\sum_{i=1}^{m} \text{Cost}(h_\theta(x^i), y^i) \\
J(\theta) &= \frac{-1}{m}\sum_{i=1}^{m} \left[y^i\log(h_\theta(x^i)) + (1-y^i)\log(1-h_\theta(x^i))\right]
\end{aligned}
$$

3.4 Proof of Logistic Regression Cost Function Derivative

$$
\begin{aligned}
J(\theta) &= \frac{-1}{m}\sum_{i=1}^{m}[y^i \log(h_\theta(x^i)) + (1-y^i)\log(1-h_\theta(x^i))] \\
\log(h_\theta(x^i)) &= \log(\frac{1}{1+e^{-\theta x^i}}) = -\log(1+e^{-\theta x^i}) \\
\log(1-h_\theta(x^i)) &= \log(1-\frac{1}{1+e^{-\theta x^i}}) = \log(e^{-\theta x^i}) - \log(1+e^{-\theta x^i}) = -\theta x^i - \log(1+e^{-\theta x^i}) \\
J(\theta) &= -\frac{1}{m}\sum_{i=1}^{m}\left[-y^i(\log(1+e^{-\theta x^i})) + (1-y^i)(-\theta x^i - \log(1+e^{-\theta x^i}))\right] \\
&= -\frac{1}{m}\sum_{i=1}^{m}\left[y^i\theta x^i - \theta x^i - \log(1+e^{-\theta x^i})\right] \\
&= -\frac{1}{m}\sum_{i=1}^{m}\left[y^i\theta x^i - \log(e^{\theta x^i}) - \log(1+e^{-\theta x^i})\right] \\
&= -\frac{1}{m}\sum_{i=1}^{m}\left[y^i\theta x^i - \log(1+e^{\theta x^i})\right] \\
\frac{\partial}{\partial\theta_j} y^i\theta x^i &= y^i x^i_j \\
\frac{\partial}{\partial\theta_j}\log(1+e^{\theta x^i}) &= \frac{x^i_j e^{\theta x^i}}{1+e^{\theta x^i}} \\
&= \frac{x^i_j}{1+e^{-\theta x^i}} \\
&= x^i_j h_\theta(x^i) \\
\frac{\partial}{\partial\theta_j} J(\theta) &= -\frac{1}{m}\sum_{i=1}^{m}\left[y^i x^i_j - x^i_j h_\theta(x^i)\right] \\
\frac{\partial}{\partial\theta_j} J(\theta) &= \frac{1}{m}\sum_{i=1}^{m}\left[h_\theta(x^i) - y^i\right] x^i_j
\end{aligned}
$$

3.5 Regularization

Regularizing Term $\lambda\sum_{j=1}^{n}\theta_j^2$

Regularized Linear Regression CF $J(\theta) = \frac{1}{2m}\sum_{i=1}^{m}(h_\theta(x^i)-y^i)^2 + \lambda\sum_{j=1}^{n}\theta_j^2$

Regularized Logistic Regression CF $J(\theta) = \frac{-1}{m}\sum_{i=1}^{m}\left[y^i\log(h_\theta(x^i)) + (1-y^i)\log(1-h_\theta(x^i))\right] + \frac{\lambda}{2m}\sum_{j=1}^{n}\theta_j^2$

Regularized GD (Lin/Log Regression) $\begin{cases} \theta_0 = \theta_j - \alpha\left[\frac{1}{m}\sum_{i=1}^{m}(h_\theta(x^i)-y^i)x^i_0\right] \\ \theta_j = \theta_j - \alpha\left[\frac{1}{m}\sum_{i=1}^{m}(h_\theta(x^i)-y^i)x^i_j + \frac{\lambda}{m}\theta_j\right] \text{ (j=1,2,...,n)} \end{cases}$

Regularized Normal Equation $\theta = (X^\intercal X + \lambda \begin{bmatrix} 0 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{bmatrix}_{n+1,n+1})^{-1}X^\intercal y$

4 Week 4: Artificial Neural Networks Representation

4.1 Overview

- Neural networks allow for non-linear classification in situations with many features
  - Necessary b/c 100 features at 3rd level polynomials generates 170k features, which quickly becomes intractable
  - "One learning algorithm" hypothesis; you can see with your tongue : brain learns using one algorithm, not thousands of different programs
- General
  - If network has s_j units in layer j and s_{j+1} units in layer $j+1$, then Θ^j will be of dimension $s_{j+1} \times s_j + 1$
    - The +1 comes from the addition in $\Theta^{(j)}$ of the bias node, x_0 and $\Theta_0^{(j)}$
  - Can have multiple hidden layers
  - Can have multiple outputs (one-vs-all for multi-class classification)
  - If three layers:
    - Layer 1: Input nodes
    - Layer 2: Hidden/intermediate layer
    - Layer 3: Output layer
- Forward Propogation is used to predict based on learned parameters
- Bias node gives each node a trainable constant value
  - Allows bias weight to shift the activation curve left/right
  - Other weights affect steepness

4.2 Symbols

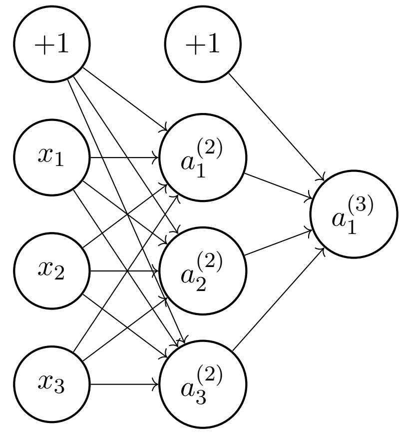
- $g(x)$: sigmoid function
- $a_i^{(j)}$: "activation" of unit i in layer j
- $\Theta^{(j)}$: matrix of weights controlling function mapping from layer j to layer j+1; each layer gets own Θ^j
- $z_k^{(j)}$: encompasses parameters inside of g function (from above)

4.3 Equations

$$\begin{aligned}
z_k^{(j)} &= \Theta_{k,0}^{(j-1)} x_0 + \Theta_{k,1}^{(j-1)} x_1 + ... + \Theta_{k,n}^{(j-1)} x_n \\
&= \Theta_{k,0}^{(j-1)} a_0^{(j-1)} + \Theta_{k,1}^{(j-1)} a_1^{(j-1)} + ... + \Theta_{k,n}^{(j-1)} a_n^{(j-1)} \\
z^{(j)} &= \Theta^{(j-1)} a^{(j-1)} \\
a^{(j)} &= g(z^{(j)})
\end{aligned}$$

4.4 Sample Three Layer System

$$\begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{bmatrix} \rightarrow \begin{bmatrix} a_1^{(2)} \\ a_2^{(2)} \\ a_3^{(2)} \end{bmatrix} \rightarrow h_\Theta(x)$$



$$\begin{aligned}
a_1^{(2)} &= g(\Theta_{1,0}^{(1)}x_0 + \Theta_{1,1}^{(1)}x_1 + \Theta_{1,2}^{(1)}x_2 + \Theta_{1,3}^{(1)}x_3) \\
a_2^{(2)} &= g(\Theta_{2,0}^{(1)}x_0 + \Theta_{2,1}^{(1)}x_1 + \Theta_{2,2}^{(1)}x_2 + \Theta_{2,3}^{(1)}x_3) \\
a_3^{(2)} &= g(\Theta_{3,0}^{(1)}x_0 + \Theta_{3,1}^{(1)}x_1 + \Theta_{3,2}^{(1)}x_2 + \Theta_{3,3}^{(1)}x_3) \\
h_\Theta(x) &= g(\Theta_{1,0}^{(2)}a_0^{(2)} + \Theta_{1,1}^{(2)}a_1^{(2)} + \Theta_{1,2}^{(2)}a_2^{(2)} + \Theta_{1,3}^{(2)}a_3^{(2)}) \\
&= g(z^3) \\
&= a_1^{(3)}
\end{aligned}$$

5 Week 5: Artificial Neural Network Learning

5.1 Symbols

– L : total number of layers in the network
– s_l : number of units (not counting bias unit) in layer l
– K : number of output units/classes
– $h_\Theta(x)_k$: hypothesis that results in the kth output

5.2 Cost Function

$$J(\Theta) = -\frac{1}{m}\sum_{i=1}^{m}\sum_{k=1}^{K}\left[y_k^{(i)}\log((h_\Theta(x^{(i)}))_k) + (1 - y_k^{(i)})\log(1 - (h_\Theta(x^{(i)}))_k)\right] + \frac{\lambda}{2m}\sum_{l=1}^{L-1}\sum_{i=1}^{s_l}\sum_{j=1}^{s_{l+1}}(\Theta_{j,i}^{(l)})^2$$

Picks training example	$\sum_{i=1}^{m}$
Picks output node	$\sum_{k=1}^{K}$
Picks layer	$\sum_{l=1}^{L-1}$
Picks node	$\sum_{i=1}^{s_l}$
Picks Θ	$\sum_{j=1}^{s_{l+1}}$

5.3 Backpropagation Algorithm

1. Set $a(1) := x(t)$
2. Perform forward propagation to compute $a(l)$ for $l = 2, 3, , L$
3. Using $y^{(t)}$, compute $\delta^L = a^{(L)} - y^{(t)}$

4. Compute $\delta^{(L-1)}, \delta^{(L-2)}, \ldots, \delta^{(2)}$ using $\delta^{(l)} = ((\Theta^{(l)})^T \delta^{(l+1)})\ .*\ a^{(l)}\ .*\ (1 - a^{(l)})$

5. $\Delta^{(l)}_{i,j} := \Delta^{(l)}_{i,j} + a^{(l)}_j \delta^{(l+1)}_i$ or with vectorization $\Delta^{(l)} := \Delta^{(l)} + \delta^{(l+1)}(a^{(l)})^\intercal$

6. $\frac{\partial}{\partial \Theta^{(l)}_{ij}} J(\Theta) = D^{(l)}_{ij}$ where $\begin{cases} D^{(l)}_{i,j} := \dfrac{1}{m}\Delta^{(l)}_{i,j} \text{ (j = 0)} \\ D^{(l)}_{i,j} := \dfrac{1}{m}\left(\Delta^{(l)}_{i,j} + \lambda\Theta^{(l)}_{i,j}\right) \text{ (j} \neq \text{0)} \end{cases}$

5.4 Backpropagation Error Derivation

$$\delta^{(l)}_j = \frac{\partial}{\partial z^{(l)}_j} cost(t)$$
$$\delta^{(l)} = ((\Theta^{(l)})^T \delta^{(l+1)})\ .* g'(z^{(l)})$$
$$g'(z^{(l)}) = a^{(l)}\ .*\ (1 - a^{(l)})$$

5.5 Gradient Checking

- Gradient checking ensures that backpropagation is actually working
  - Turn off gradient checking once backpropagation is verified to work
- Approximate the derivative of cost function using slope
  - Pick $\epsilon = 10^{-4}$
  - Check all Θ_j

$$\frac{\partial}{\partial \Theta_j} J(\Theta) \approx \frac{J(\Theta_1, \ldots, \Theta_j + \epsilon, \ldots, \Theta_n) - J(\Theta_1, \ldots, \Theta_j - \epsilon, \ldots, \Theta_n)}{2\epsilon}$$

5.6 Random Initialization

- Need Symmetry Breaking
  - All hidden units would receive the same signal and the same updates
  - Results in finding only one feature, redundantly copied many times over
- Fixed by randomly initializing each $\Theta^{(l)}_{ij}$ to a value in $[-\epsilon, \epsilon]$