Coursera-Stanford-ML-Notes

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20 June 2017 - ? July 2017

Contents

1 Week 1: Introduction

1.1 Overview

- Machine Learning: "A computer program is said to learn from experience E with respect to some class of tasks T and performance measure P, if its performance at tasks in T, as measured by P, improves with experience E."
- Supervised Learning: know what our correct output looks like
  - Regression: want continuous output
  - Classification: want discrete output
- Unsupervised Learning: little or no idea what our results should look like
  - Clustering: find groups according to similarity in various variables
  - Nonclustering: find structure in chaos

2 Week 2: Linear Regression with Multiple Variables

2.1 Overview

– Use linear regression for continuous output
– Choose gradient descent if many features (million+) because the inverse matrix required for the normal equation can become expensive to compute
– Normal equation will directly compute theta
– Normalize features if using gradient descent

2.2 Notation

$$
\begin{aligned}
m &= number\ of\ samples \\
n &= number\ of\ feature \\
x &= (n \times 1) \\
X &= (m \times n) \\
X_j &= (m \times 1) \\
\theta &= (n \times 1) \\
\theta_j &= (1 \times 1)
\end{aligned}
$$

2.3 Gradient Descent

Hypothesis Function	$h_\theta(x) = \theta^\intercal \times x$
Vectorized Hypothesis Function	$h_\theta(X) = X \cdot \theta$
Linear Regression Cost Function	$J(\theta) = \frac{1}{2m} \sum (h_\theta(X) - y)^2$
Derivative of Linear Regression CF wrt θ_j	$\frac{\partial}{\partial \theta_j} J(\theta) = \frac{1}{m} \sum (h_\theta(X) - y)\ .*\ X_j$
Change in θ_j	$\theta_j = \theta_j - \alpha \frac{\partial}{\partial \theta_j}$
	$= \theta_j - \alpha \frac{1}{m} \sum (h_\theta(X) - y)\ .*\ X_j$
Vectorized Change in θ	$\theta = \theta - \alpha \frac{1}{m} X^\intercal (X \cdot \theta - y)$

2.4 Normal Equation

$$\theta = (X^\intercal \cdot X)^{-1} \cdot X^\intercal \cdot y$$

3 Week 3: Logistic Regression

3.1 Overview

- Use logistic regression for discrete output (classification)
  - $h_\theta(x) = (y = 1|x; \theta)$; gives probability that the output is 1 given x
  - Sigmoid/Logistic function maps any real number to (0, 1)
  - Logarithm turns sum into product, allowing easier differentiation without altering search space
- For multi-class classification, use one-vs-all
  - Pick class i that maximizes $h_\theta^i(x)$
- Overfitting is when learned hypothesis fits training data well but fails to generalize; underfitting is when doesn't fit training data
- Address overfitting by reducing number of features, model selection, and regularization
  - Regularization results in simpler hypothesis and less overfitting
  - Extremely large λ will result in underfitting and gradient descent will fail to converge
  - Do not regularize λ_0
- Use other prewritten optimization algorithims (conjugate gradient, BFGS, L-BFGS) because they are faster

3.2 Logistic Regression Hypothesis Function

$$\begin{aligned} \text{Sigmoid/Logistic Function} \quad & g(z) = \frac{1}{1 + e^{-z}} \\ \text{Hypothesis Function} \quad & h_\theta(x) = g(\theta^\intercal x) \\ & = \frac{1}{1 + e^{-\theta^\intercal x}} \end{aligned}$$

3.3 Logistic Regression Cost Function

$$\begin{aligned} Cost(h_\theta(x), y) &= \begin{cases} -\log(h_\theta(x)) \text{ if y} = 1 \\ -\log(1 - h_\theta(x)) \text{ if y} = 0 \end{cases} \\ &= -y \log(h_\theta(x)) - (1 - y) \log(1 - h_\theta(x)) \\ J(\theta) &= \frac{1}{m} \sum_{i=1}^{m} \text{Cost}(h_\theta(x^i), y^i) \\ J(\theta) &= \frac{-1}{m} \sum_{i=1}^{m} \left[y^i \log(h_\theta(x^i)) + (1 - y^i) \log(1 - h_\theta(x^i)) \right] \end{aligned}$$

3.4 Proof of Logistic Regression Cost Function Derivative

$$
\begin{aligned}
J(\theta) &= \frac{-1}{m}\sum_{i=1}^{m}[y^i \log(h_\theta(x^i)) + (1-y^i)\log(1-h_\theta(x^i))] \\
\log(h_\theta(x^i)) &= \log(\frac{1}{1+e^{-\theta x^i}}) = -\log(1+e^{-\theta x^i}) \\
\log(1-h_\theta(x^i)) &= \log(1-\frac{1}{1+e^{-\theta x^i}}) = \log(e^{-\theta x^i}) - \log(1+e^{-\theta x^i}) = -\theta x^i - \log(1+e^{-\theta x^i}) \\
J(\theta) &= -\frac{1}{m}\sum_{i=1}^{m}\left[-y^i(\log(1+e^{-\theta x^i})) + (1-y^i)(-\theta x^i - \log(1+e^{-\theta x^i}))\right] \\
&= -\frac{1}{m}\sum_{i=1}^{m}\left[y^i\theta x^i - \theta x^i - \log(1+e^{-\theta x^i})\right] \\
&= -\frac{1}{m}\sum_{i=1}^{m}\left[y^i\theta x^i - \log(e^{\theta x^i}) - \log(1+e^{-\theta x^i})\right] \\
&= -\frac{1}{m}\sum_{i=1}^{m}\left[y^i\theta x^i - \log(1+e^{\theta x^i})\right] \\
\frac{\partial}{\partial\theta_j} y^i\theta x^i &= y^i x_j^i \\
\frac{\partial}{\partial\theta_j}\log(1+e^{\theta x^i}) &= \frac{x_j^i e^{\theta x^i}}{1+e^{\theta x^i}} \\
&= \frac{x_j^i}{1+e^{-\theta x^i}} \\
&= x_j^i h_\theta(x^i) \\
\frac{\partial}{\partial\theta_j} J(\theta) &= -\frac{1}{m}\sum_{i=1}^{m}\left[y^i x_j^i - x_j^i h_\theta(x^i)\right] \\
\frac{\partial}{\partial\theta_j} J(\theta) &= \frac{1}{m}\sum_{i=1}^{m}\left[h_\theta(x^i) - y^i\right] x_j^i
\end{aligned}
$$

3.5 Regularization

Regularizing Term $\quad \lambda\sum_{j=1}^{n}\theta_j^2$

Regularized Linear Regression CF $\quad J(\theta) = \frac{1}{2m}\sum_{i=1}^{m}(h_\theta(x^i)-y^i)^2 + \lambda\sum_{j=1}^{n}\theta_j^2$

Regularized Logistic Regression CF $\quad J(\theta) = \frac{-1}{m}\sum_{i=1}^{m}\left[y^i\log(h_\theta(x^i)) + (1-y^i)\log(1-h_\theta(x^i))\right] + \frac{\lambda}{2m}\sum_{j=1}^{n}\theta_j^2$

Regularized GD (Lin/Log Regression)

$$
\begin{cases}
\theta_0 = \theta_j - \alpha\left[\frac{1}{m}\sum_{i=1}^{m}(h_\theta(x^i)-y^i)x_0^i\right] \\
\theta_j = \theta_j - \alpha\left[\frac{1}{m}\sum_{i=1}^{m}(h_\theta(x^i)-y^i)x_j^i + \frac{\lambda}{m}\theta_j\right] \text{ (j=1,2,...,n)}
\end{cases}
$$

Regularized Normal Equation

$$
\theta = (X^\intercal X + \lambda \begin{bmatrix} 0 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{bmatrix}_{n+1,n+1})^{-1} X^\intercal y
$$

4 Week 4: Artificial Neural Networks Representation

4.1 Overview

– Neural networks allow for non-linear classification in situations with many features
  - Necessary b/c 100 features at 3rd level polynomials generates 170k features, which quickly becomes intractable
  - "One learning algorithm" hypothesis; you can see with your tongue : brain learns using one algorithm, not thousands of different programs
  - Can have multiple hidden layers
  - Can have multiple outputs (one-vs-all for multi-class classification)
  - If network has s_j units in layer j and s_{j+1} units in layer $j+1$, then Θ^j will be of dimension $s_{j+1} \times s_j + 1$
    – The +1 comes from the addition in $\Theta^{(j)}$ of the bias node, x_0 and $\Theta_0^{(j)}$
– Forward Propogation is used to predict based on learned parameters
– Bias node gives each node a trainable constant value
  - Allows bias weight to shift the activation curve left/right
  - Other weights affect steepness

4.2 Notation

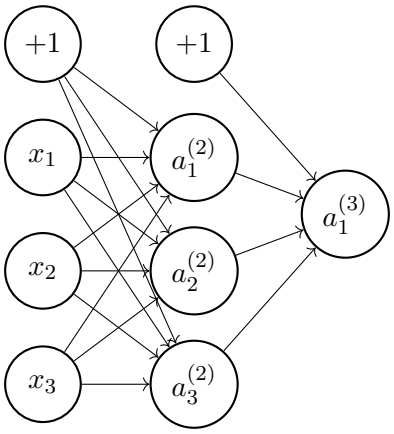
– $g(x)$: sigmoid function
– $\Theta^{(j)}$: matrix of weights controlling function mapping from layer j to layer j+1; each layer gets own Θ^j
– $\Theta_{j,0}, ..., \Theta_{j,n}$ weights corresponding to the inputs $a_0, ..., a_n$ going into z_j
– $w_{0,j}, ..., w_{n,j}$ weights corresponding to the inputs $a_0, ..., a_n$ going into z_j
– $z_k^{(j)}$: encompasses parameters inside of g function
– $a_i^{(j)}$: "activation" of unit i in layer j

4.3 Equations

$$
\begin{aligned}
z_k^{(j)} &= \Theta_{k,0}^{(j-1)} x_0 + \Theta_{k,1}^{(j-1)} x_1 + ... + \Theta_{k,n}^{(j-1)} x_n \\
&= \Theta_{k,0}^{(j-1)} a_0^{(j-1)} + \Theta_{k,1}^{(j-1)} a_1^{(j-1)} + ... + \Theta_{k,n}^{(j-1)} a_n^{(j-1)} \\
z^{(j)} &= \Theta^{(j-1)} a^{(j-1)} \\
a^{(j)} &= g(z^{(j)})
\end{aligned}
$$

4.4 Sample Three Layer System

$$
\begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{bmatrix} \rightarrow \begin{bmatrix} a_1^{(2)} \\ a_2^{(2)} \\ a_3^{(2)} \end{bmatrix} \rightarrow h_\Theta(x)
$$



$$
\begin{aligned}
a_1^{(2)} &= g(\Theta_{1,0}^{(1)} x_0 + \Theta_{1,1}^{(1)} x_1 + \Theta_{1,2}^{(1)} x_2 + \Theta_{1,3}^{(1)} x_3) \\
a_2^{(2)} &= g(\Theta_{2,0}^{(1)} x_0 + \Theta_{2,1}^{(1)} x_1 + \Theta_{2,2}^{(1)} x_2 + \Theta_{2,3}^{(1)} x_3) \\
a_3^{(2)} &= g(\Theta_{3,0}^{(1)} x_0 + \Theta_{3,1}^{(1)} x_1 + \Theta_{3,2}^{(1)} x_2 + \Theta_{3,3}^{(1)} x_3) \\
h_\Theta(x) &= g(\Theta_{1,0}^{(2)} a_0^{(2)} + \Theta_{1,1}^{(2)} a_1^{(2)} + \Theta_{1,2}^{(2)} a_2^{(2)} + \Theta_{1,3}^{(2)} a_3^{(2)}) \\
&= g(z^3) \\
&= a_1^{(3)}
\end{aligned}
$$

5 Week 5: Artificial Neural Network Learning

5.1 Notation

- L : total number of layers in the network
- s_l : number of units (not counting bias unit) in layer l
- K : number of output units/classes
- $h_\Theta(x)_k$: hypothesis that results in the kth output
- δ_k : Error signal

5.2 Cost Function

$$J(\Theta) = -\frac{1}{m}\sum_{i=1}^{m}\sum_{k=1}^{K}\left[y_k^{(i)}\log((h_\Theta(x^{(i)}))_k) + (1-y_k^{(i)})\log(1-(h_\Theta(x^{(i)}))_k)\right] + \frac{\lambda}{2m}\sum_{l=1}^{L-1}\sum_{i=1}^{s_l}\sum_{j=1}^{s_{l+1}}(\Theta_{j,i}^{(l)})^2$$

Picking training example	$\sum_{i=1}^{m}$
Picking output node	$\sum_{k=1}^{K}$
Picking layer	$\sum_{l=1}^{L-1}$
Picking node	$\sum_{i=1}^{s_l}$
Picking Θ	$\sum_{j=1}^{s_{l+1}}$

5.3 Backpropagation Algorithm

1. Set $a(1) := x(t)$
2. Perform forward propagation to compute $a(l)$ for $l = 2, 3, , L$
3. Using $y^{(t)}$, compute $\delta^L = a^{(L)} - y^{(t)}$
4. Compute $\delta^{(L-1)}, \delta^{(L-2)}, \ldots, \delta^{(2)}$ using $\delta^{(l)} = ((\Theta^{(l)})^T\delta^{(l+1)}) \ .*\ a^{(l)} \ .*\ (1-a^{(l)})$
5. $\Delta_{i,j}^{(l)} := \Delta_{i,j}^{(l)} + a_j^{(l)}\delta_i^{(l+1)}$ or with vectorization $\Delta^{(l)} := \Delta^{(l)} + \delta^{(l+1)}(a^{(l)})^\intercal$
6. $\frac{\partial}{\partial\Theta_{ij}^{(l)}}J(\Theta) = D_{ij}^{(l)}$ where $\begin{cases} D_{i,j}^{(l)} := \frac{1}{m}\Delta_{i,j}^{(l)} \text{ (j = 0)} \\ D_{i,j}^{(l)} := \frac{1}{m}\left(\Delta_{i,j}^{(l)} + \lambda\Theta_{i,j}^{(l)}\right) \text{ (j} \neq 0) \end{cases}$

5.4 Backpropagation Derivation - Base Case

$$\frac{\partial C}{\partial w_{ij}} = \frac{\partial C}{\partial z_j}\frac{\partial z_j}{\partial w_{ij}}$$ Cost C varies wrt input accumulator z_j, z_j varies wrt w_{ij} (Chain Rule)

$$\begin{aligned}\frac{\partial C}{\partial z_j} &= \frac{\partial}{\partial z_j}(y_j - a_j)^2 && \text{Cost defined as } (y_j - a_j)^2\\ &= \frac{\partial}{\partial z_j}(y_j - g(z_j))^2\\ &= -2(y_j - g(z_j))g'(z_j)\end{aligned}$$

$$\begin{aligned}g'(z_j) &= \frac{d}{dz_j}\frac{1}{1+e^{-z_j}} && \text{Derivative of logistic function}\\ &= \frac{1}{1+e^{-z_j}}\frac{e^{-z_j}}{1+e^{-z_j}}\\ &= g(z_j)(1-g(z_j))\end{aligned}$$

$$\begin{aligned}\frac{\partial C}{\partial z_j} &= -2(y_j - g(z_j))g(z_j)(1-g(z_j))\\ &= -2(y_j - a_j)a_j(1-a_j)\end{aligned}$$

$$\begin{aligned}\frac{\partial z_j}{\partial w_{ij}} &= \frac{\partial}{\partial w_{ij}}\sum_q a_q w_{q,j} && \text{Definition of } z_j \text{ as sum of previous node's inputs and their weights}\\ &= a_i\end{aligned}$$

$$\begin{aligned}\frac{\partial C}{\partial w_{ij}} &= -2(y_j - a_j)a_j(1-a_j)(a_i)\\ &= -\Delta_j a_i\end{aligned}$$

5.5 Backpropagation Derivation - Recursive Case

$$\frac{\partial C}{\partial w_{i,j}} = \sum_k\left(\frac{\partial C}{\partial z_k}\frac{\partial z_k}{\partial a_j}\frac{\partial a_j}{\partial z_j}\frac{\partial z_j}{\partial w_{i,j}}\right)$$ C depends on z_k, z_k depends on a_j, a_j depends on z_j, z_j depends on $w_{i.j}$

$$\begin{aligned}\frac{\partial C}{\partial z_k} &= -\Delta_k\\ \frac{\partial z_k}{\partial a_j} &= \frac{\partial}{\partial a_j}\sum_s a_s w_{s,k}\\ &= w_{j,k}\\ \frac{\partial a_j}{\partial z_j} &= \frac{\partial}{\partial z_j}g(z_j)\\ &= g(z_j)(1-g(z_j))\\ &= a_j(1-a_j)\\ \frac{\partial z_j}{\partial w_{i,j}} &= \frac{\partial}{\partial w_{i,j}}\sum_q a_q w_{q,j}\\ &= a_i\\ \frac{\partial C}{\partial w_{i,j}} &= \sum_k(-\Delta_k w_{j,k}a_j(1-a_j)a_i)\\ &= \sum_k(-\Delta_k w_{j,k})a_j(1-a_j)a_i\\ &= -\Delta_j a_i\end{aligned}$$

5.6 Backpropagation Intuition

δ_k is the error signal from the output

$$\delta_k = (a_k - t_k) g'_k(z_k)$$

so, error wrt output weights is

$$\frac{\partial E}{\partial w_{j,k}} = \delta_k a_j$$

similarly, error wrt internal weights is

$$\frac{\partial E}{\partial w_{i,j}} = \delta_j a_i$$

the error is determined by following layer's error, so it may also be understood as

$$\frac{\partial E}{\partial w_{i,j}} = g'_j(z_j) \sum_k (\delta_k w_{j,k}) a_i$$

5.7 Gradient Checking

– Gradient checking ensures that backpropagation is actually working
  • Turn off gradient checking once backpropagation is verified to work
– Approximate the derivative of cost function using slope
  • Pick $\epsilon = 10^{-4}$
  • Check all Θ_j

$$\frac{\partial}{\partial \Theta_j} J(\Theta) \approx \frac{J(\Theta_1, \ldots, \Theta_j + \epsilon, \ldots, \Theta_n) - J(\Theta_1, \ldots, \Theta_j - \epsilon, \ldots, \Theta_n)}{2\epsilon}$$

5.8 Random Initialization

– Need Symmetry Breaking
  • All hidden units would receive the same signal and the same updates
  • Results in finding only one feature, redundantly copied many times over
– Fixed by randomly initializing each $\Theta_{ij}^{(l)}$ to a value in $[-\epsilon, \epsilon]$

6 Sources

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