Coursera-Stanford-ML-Notes

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1 Week 1: Introduction

1.1 Overview

- Machine Learning: "A computer program is said to learn from experience E with respect to some class of tasks T and performance measure P, if its performance at tasks in T, as measured by P, improves with experience E."
- Supervised Learning: know what our correct output looks like
  - Regression: want continuous output
  - Classification: want discrete output
- Unsupervised Learning: little or no idea what our results should look like
  - Clustering: find groups according to similarity in various variables
  - Nonclustering: find structure in chaos

2 Week 2: Linear Regression with Multiple Variables

2.1 Overview

- Use linear regression for continuous output
- Choose gradient descent if many features (million+) because the inverse matrix required for the normal equation can become expensive to compute
- Normal equation will directly compute theta
- Normalize features if using gradient descent

2.2 Notation

$$\begin{aligned} m &= number\ of\ samples \\ n &= number\ of\ feature \\ x &= (n \times 1) \\ X &= (m \times n) \\ X_j &= (m \times 1) \\ \theta &= (n \times 1) \\ \theta_j &= (1 \times 1) \end{aligned}$$

2.3 Gradient Descent

$$\begin{aligned} \text{Hypothesis Function} \qquad && h_\theta(x) &= \theta^\intercal \times x \\ \text{Vectorized Hypothesis Function} \qquad && h_\theta(X) &= X \cdot \theta \\ \text{Linear Regression Cost Function} \qquad && J(\theta) &= \frac{1}{2m} \sum (h_\theta(X) - y)^2 \\ \text{Derivative of Linear Regression CF wrt } \theta_j \qquad && \frac{\partial}{\partial \theta_j} J(\theta) &= \frac{1}{m} \sum (h_\theta(X) - y)\ .*\ X_j \\ \text{Change in } \theta_j \qquad && \theta_j &= \theta_j - \alpha \frac{\partial}{\partial \theta_j} \\ && &= \theta_j - \alpha \frac{1}{m} \sum (h_\theta(X) - y)\ .*\ X_j \\ \text{Vectorized Change in } \theta \qquad && \theta &= \theta - \alpha \frac{1}{m} X^\intercal (X \cdot \theta - y) \end{aligned}$$

2.4 Normal Equation

$$\theta = (X^\intercal \cdot X)^{\text{-}1} \cdot X^\intercal \cdot y$$

3 Week 3: Logistic Regression

3.1 Overview

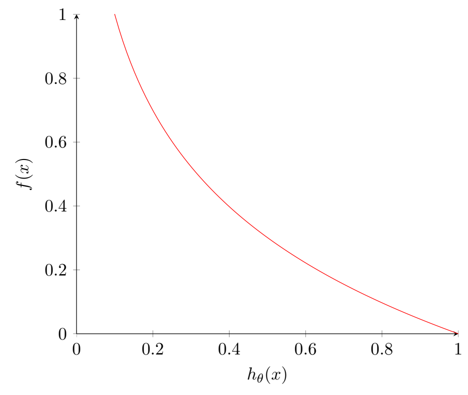
- Use logistic regression for discrete output (classification)
  - $h_\theta(x) = (y = 1|x; \theta)$; gives probability that the output is 1 given x
  - Sigmoid/Logistic function maps any real number to (0, 1)
  - Logarithm turns sum into product, allowing easier differentiation without altering search space
- For multi-class classification, use one-vs-all
  - Pick class i that maximizes $h_\theta^i(x)$
- Overfitting is when learned hypothesis fits training data well but fails to generalize; underfitting is when doesn't fit training data
- Address overfitting by reducing number of features, model selection, and regularization
  - Regularization results in simpler hypothesis and less overfitting
  - Extremely large λ will result in underfitting and gradient descent will fail to converge
  - Do not regularize λ_0
- Use other prewritten optimization algorithims (conjugate gradient, BFGS, L-BFGS) because they are faster

3.2 Logistic Regression Hypothesis Function

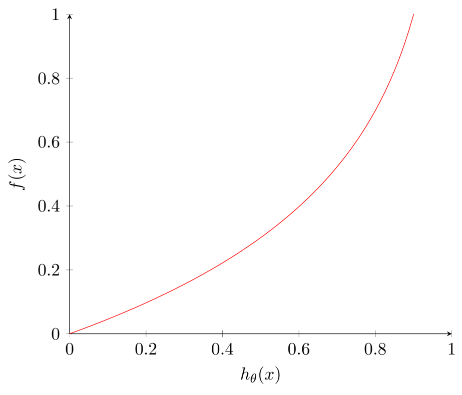
$$\begin{aligned}
\text{Sigmoid/Logistic Function} \qquad & g(z) = \frac{1}{1+e^{-z}} \\
\text{Hypothesis Function} \qquad & h_\theta(x) = g(\theta^\intercal x) \\
& \qquad = \frac{1}{1+e^{-\theta^\intercal x}}
\end{aligned}$$

3.3 Logistic Regression Cost Function

$$\begin{aligned}
Cost(h_\theta(x), y) &= \begin{cases} -\log(h_\theta(x)) \text{ if y} = 1 \\ -\log(1-h_\theta(x)) \text{ if y} = 0 \end{cases} \\
&= -y\log(h_\theta(x)) - (1-y)\log(1-h_\theta(x)) \\
J(\theta) &= \frac{1}{m}\sum_{i=1}^{m} \text{Cost}(h_\theta(x^i), y^i) \\
J(\theta) &= \frac{-1}{m}\sum_{i=1}^{m} \left[y^i \log(h_\theta(x^i)) + (1-y^i)\log(1-h_\theta(x^i))\right]
\end{aligned}$$



(a) if y=1



(b) if y=0

3.4 Proof of Logistic Regression Cost Function Derivative

$$
\begin{aligned}
J(\theta) &= \frac{-1}{m}\sum_{i=1}^{m}[y^i\log(h_\theta(x^i)) + (1-y^i)\log(1-h_\theta(x^i))] \\
\log(h_\theta(x^i)) &= \log(\frac{1}{1+e^{-\theta x^i}}) = -\log(1+e^{-\theta x^i}) \\
\log(1-h_\theta(x^i)) &= \log(1-\frac{1}{1+e^{-\theta x^i}}) = \log(e^{-\theta x^i}) - \log(1+e^{-\theta x^i}) = -\theta x^i - \log(1+e^{-\theta x^i}) \\
J(\theta) &= -\frac{1}{m}\sum_{i=1}^{m}\left[-y^i(\log(1+e^{-\theta x^i})) + (1-y^i)(-\theta x^i - \log(1+e^{-\theta x^i}))\right] \\
&= -\frac{1}{m}\sum_{i=1}^{m}\left[y^i\theta x^i - \theta x^i - \log(1+e^{-\theta x^i})\right] \\
&= -\frac{1}{m}\sum_{i=1}^{m}\left[y^i\theta x^i - \log(e^{\theta x^i}) - \log(1+e^{-\theta x^i})\right] \\
&= -\frac{1}{m}\sum_{i=1}^{m}\left[y^i\theta x^i - \log(1+e^{\theta x^i})\right] \\
\frac{\partial}{\partial\theta_j}y^i\theta x^i &= y^i x^i_j \\
\frac{\partial}{\partial\theta_j}\log(1+e^{\theta x^i}) &= \frac{x^i_j e^{\theta x^i}}{1+e^{\theta x^i}} \\
&= \frac{x^i_j}{1+e^{-\theta x^i}} \\
&= x^i_j h_\theta(x^i) \\
\frac{\partial}{\partial\theta_j}J(\theta) &= -\frac{1}{m}\sum_{i=1}^{m}\left[y^i x^i_j - x^i_j h_\theta(x^i)\right] \\
\frac{\partial}{\partial\theta_j}J(\theta) &= \frac{1}{m}\sum_{i=1}^{m}\left[h_\theta(x^i) - y^i\right]x^i_j
\end{aligned}
$$

Credit[1]

[1] https://math.stackexchange.com/questions/477207/derivative-of-cost-function-for-logistic-regression

3.5 Regularization

Regularizing Term	$\lambda \sum_{j=1}^{n} \theta_j^2$
Regularized Linear Regression CF	$J(\theta) = \frac{1}{2m} \sum_{i=1}^{m} (h_\theta(x^i) - y^i)^2 + \lambda \sum_{j=1}^{n} \theta_j^2$
Regularized Logistic Regression CF	$J(\theta) = \frac{-1}{m} \sum_{i=1}^{m} \left[y^i \log(h_\theta(x^i)) + (1 - y^i) \log(1 - h_\theta(x^i)) \right] + \frac{\lambda}{2m} \sum_{j=1}^{n} \theta_j^2$
Regularized GD (Lin/Log Regression)	$\begin{cases} \theta_0 = \theta_j - \alpha \left[\frac{1}{m} \sum_{i=1}^{m} (h_\theta(x^i) - y^i) x_0^i \right] \\ \theta_j = \theta_j - \alpha \left[\frac{1}{m} \sum_{i=1}^{m} (h_\theta(x^i) - y^i) x_j^i + \frac{\lambda}{m} \theta_j \right] \text{ (j=1,2,...,n)} \end{cases}$
Regularized Normal Equation	$\theta = (X^\intercal X + \lambda \begin{bmatrix} 0 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{bmatrix}_{n+1,n+1})^{-1} X^\intercal y$

4 Week 4: Artificial Neural Networks Representation

4.1 Overview

- Neural networks allow for non-linear classification in situations with many features
  - Necessary b/c 100 features at 3rd level polynomials generates 170k features, which quickly becomes intractable
  - "One learning algorithm" hypothesis; you can see with your tongue : brain learns using one algorithm, not thousands of different programs
  - Can have multiple hidden layers
  - Can have multiple outputs (one-vs-all for multi-class classification)
  - If network has s_j units in layer j and s_{j+1} units in layer $j+1$, then Θ^j will be of dimension $s_{j+1} \times s_j + 1$
    - The +1 comes from the addition in $\Theta^{(j)}$ of the bias node, x_0 and $\Theta_0^{(j)}$
- Forward Propogation is used to predict based on learned parameters
- Bias node gives each node a trainable constant value
  - Allows bias weight to shift the activation curve left/right
  - Other weights affect steepness
- Fun fact: For image recognition, particular order of pixels does not matter for ANN (but does for humans), you just need to keep the convention the same

4.2 ANN Notation

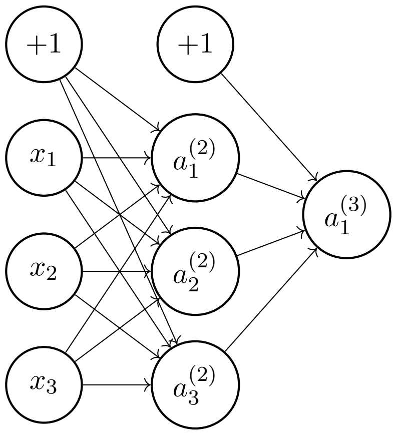
- $g(x)$: sigmoid function
- $\Theta^{(j)}$: matrix of weights controlling function mapping from layer j to layer j+1; each layer gets own Θ^j
- $\Theta_{j,0}, ..., \Theta_{j,n}$ weights corresponding to the inputs $a_0, ..., a_n$ going into z_j
- $w_{0,j}, ..., w_{n,j}$ weights corresponding to the inputs $a_0, ..., a_n$ going into z_j
- $z_k^{(j)}$: encompasses parameters inside of g function
- $a_i^{(j)}$: "activation" of unit i in layer j

4.3 ANN Equations

$$\begin{aligned} z_k^{(j)} &= \Theta_{k,0}^{(j-1)} x_0 + \Theta_{k,1}^{(j-1)} x_1 + ... + \Theta_{k,n}^{(j-1)} x_n \\ &= \Theta_{k,0}^{(j-1)} a_0^{(j-1)} + \Theta_{k,1}^{(j-1)} a_1^{(j-1)} + ... + \Theta_{k,n}^{(j-1)} a_n^{(j-1)} \\ z^{(j)} &= \Theta^{(j-1)} a^{(j-1)} \\ a^{(j)} &= g(z^{(j)}) \end{aligned}$$

4.4 Sample Three Layer System

$$\begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{bmatrix} \rightarrow \begin{bmatrix} a_1^{(2)} \\ a_2^{(2)} \\ a_3^{(2)} \end{bmatrix} \rightarrow h_\Theta(x)$$



$$\begin{aligned}
a_1^{(2)} &= g(\Theta_{1,0}^{(1)} x_0 + \Theta_{1,1}^{(1)} x_1 + \Theta_{1,2}^{(1)} x_2 + \Theta_{1,3}^{(1)} x_3) \\
a_2^{(2)} &= g(\Theta_{2,0}^{(1)} x_0 + \Theta_{2,1}^{(1)} x_1 + \Theta_{2,2}^{(1)} x_2 + \Theta_{2,3}^{(1)} x_3) \\
a_3^{(2)} &= g(\Theta_{3,0}^{(1)} x_0 + \Theta_{3,1}^{(1)} x_1 + \Theta_{3,2}^{(1)} x_2 + \Theta_{3,3}^{(1)} x_3) \\
h_\Theta(x) &= g(\Theta_{1,0}^{(2)} a_0^{(2)} + \Theta_{1,1}^{(2)} a_1^{(2)} + \Theta_{1,2}^{(2)} a_2^{(2)} + \Theta_{1,3}^{(2)} a_3^{(2)}) \\
&= g(z^3) \\
&= a_1^{(3)}
\end{aligned}$$

5 Week 5: Artificial Neural Network Learning

5.1 Notation

- L : total number of layers in the network
- s_l : number of units (not counting bias unit) in layer l
- K : number of output units/classes
- $h_\Theta(x)_k$: hypothesis that results in the kth output
- δ_k : Error signal

5.2 Cost Function

$$J(\Theta) = -\frac{1}{m}\sum_{i=1}^{m}\sum_{k=1}^{K}\left[y_k^{(i)}\log((h_\Theta(x^{(i)}))_k) + (1-y_k^{(i)})\log(1-(h_\Theta(x^{(i)}))_k)\right] + \frac{\lambda}{2m}\sum_{l=1}^{L-1}\sum_{i=1}^{s_l}\sum_{j=1}^{s_{l+1}}(\Theta_{j,i}^{(l)})^2$$

Picking training example	$\sum_{i=1}^{m}$
Picking output node	$\sum_{k=1}^{K}$
Picking layer	$\sum_{l=1}^{L-1}$
Picking node	$\sum_{i=1}^{s_l}$
Picking Θ	$\sum_{j=1}^{s_{l+1}}$

5.3 Backpropagation Algorithm

1. Set $a(1) := x(t)$
2. Perform forward propagation to compute $a(l)$ for $l = 2, 3, , L$
3. Using $y^{(t)}$, compute $\delta^L = a^{(L)} - y^{(t)}$
4. Compute $\delta^{(L-1)}, \delta^{(L-2)}, \ldots, \delta^{(2)}$ using $\delta^{(l)} = ((\Theta^{(l)})^T\delta^{(l+1)})\ .*\ a^{(l)}\ .*\ (1-a^{(l)})$
5. $\Delta_{i,j}^{(l)} := \Delta_{i,j}^{(l)} + a_j^{(l)}\delta_i^{(l+1)}$ or with vectorization $\Delta^{(l)} := \Delta^{(l)} + \delta^{(l+1)}(a^{(l)})^\intercal$
6. $\frac{\partial}{\partial\Theta_{ij}^{(l)}}J(\Theta) = D_{ij}^{(l)}$ where $\begin{cases} D_{i,j}^{(l)} := \frac{1}{m}\Delta_{i,j}^{(l)} \text{ (j = 0)} \\ D_{i,j}^{(l)} := \frac{1}{m}\left(\Delta_{i,j}^{(l)} + \lambda\Theta_{i,j}^{(l)}\right) \text{ (j} \neq \text{0)} \end{cases}$

5.4 Backpropagation Derivation - Base Case

$$\frac{\partial C}{\partial w_{ij}} = \frac{\partial C}{\partial z_j}\frac{\partial z_j}{\partial w_{ij}}$$

Cost C varies wrt input accumulator z_j, z_j varies wrt w_{ij} (Chain Rule)

$$\frac{\partial C}{\partial z_j} = \frac{\partial}{\partial z_j}(y_j - a_j)^2$$

Cost defined as $(y_j - a_j)^2$

$$= \frac{\partial}{\partial z_j}(y_j - g(z_j))^2$$

$$= -2(y_j - g(z_j))g'(z_j)$$

$$g'(z_j) = \frac{d}{dz_j}\frac{1}{1+e^{-z_j}}$$

Derivative of logistic function

$$= \frac{1}{1+e^{-z_j}}\frac{e^{-z_j}}{1+e^{-z_j}}$$

$$= g(z_j)(1 - g(z_j))$$

$$\frac{\partial C}{\partial z_j} = -2(y_j - g(z_j))g(z_j)(1 - g(z_j))$$

$$= -2(y_j - a_j)a_j(1 - a_j)$$

$$\frac{\partial z_j}{\partial w_{ij}} = \frac{\partial}{\partial w_{ij}}\sum_q a_q w_{q,j}$$

Definition of z_j as sum of previous node's inputs and their weights

$$= a_i$$

$$\frac{\partial C}{\partial w_{ij}} = -2(y_j - a_j)a_j(1 - a_j)(a_i)$$

$$= -\Delta_j a_i$$

5.5 Backpropagation Derivation - Recursive Case

$$\frac{\partial C}{\partial w_{i,j}} = \sum_k\left(\frac{\partial C}{\partial z_k}\frac{\partial z_k}{\partial a_j}\frac{\partial a_j}{\partial z_j}\frac{\partial z_j}{\partial w_{i,j}}\right)$$

C depends on z_k, z_k depends on a_j, a_j depends on z_j, z_j depends on $w_{i.j}$

$$\frac{\partial C}{\partial z_k} = -\Delta_k$$

$$\frac{\partial z_k}{\partial a_j} = \frac{\partial}{\partial a_j}\sum_s a_s w_{s,k}$$

$$= w_{j,k}$$

$$\frac{\partial a_j}{\partial z_j} = \frac{\partial}{\partial z_j}g(z_j)$$

$$= g(z_j)(1 - g(z_j))$$

$$= a_j(1 - a_j)$$

$$\frac{\partial z_j}{\partial w_{i,j}} = \frac{\partial}{\partial w_{i,j}}\sum_q a_q w_{q,j}$$

$$= a_i$$

$$\frac{\partial C}{\partial w_{i,j}} = \sum_k(-\Delta_k w_{j,k} a_j(1 - a_j)a_i)$$

$$= \sum_k(-\Delta_k w_{j,k})a_j(1 - a_j)a_i$$

$$= -\Delta_j a_i$$

Credit [2] [3] [4]

5.6 Backpropagation Intuition

δ_k is the error signal from the output

$$\delta_k = (a_k - t_k)g_k'(z_k)$$

so, error wrt output weights is

$$\frac{\partial E}{\partial w_{j,k}} = \delta_k a_j$$

similarly, error wrt internal weights is

$$\frac{\partial E}{\partial w_{i,j}} = \delta_j a_i$$

the error is determined by following layer's error, so it may also be understood as

$$\frac{\partial E}{\partial w_{i,j}} = g_j'(z_j) \sum_k (\delta_k w_{j,k}) a_i$$

5.7 Gradient Checking

- Gradient checking ensures that backpropagation is actually working
  - Turn off gradient checking once backpropagation is verified to work
- Approximate the derivative of cost function using slope
  - Pick $\epsilon = 10^{-4}$
  - Check all Θ_j

$$\frac{\partial}{\partial \Theta_j} J(\Theta) \approx \frac{J(\Theta_1, \ldots, \Theta_j + \epsilon, \ldots, \Theta_n) - J(\Theta_1, \ldots, \Theta_j - \epsilon, \ldots, \Theta_n)}{2\epsilon}$$

5.8 Random Initialization

- Need Symmetry Breaking
  - All hidden units would receive the same signal and the same updates
  - Results in finding only one feature, redundantly copied many times over
- Fixed by randomly initializing each $\Theta_{ij}^{(l)}$ to a value in $[-\epsilon, \epsilon]$
  - A good choice of ϵ init is $\epsilon = \frac{\sqrt{6}}{\sqrt{L_{in} + L_{out}}}$, where $L_{in} = s_l$ and $L_{out} = s_{l+1}$, the number of units in the layers adjacent to $\Theta^{(l)}$

[2] https://pdfs.semanticscholar.org/69c2/814c7f8ca16aa836fd32e3c4975f8208e63f.pdf
[3] https://stats.stackexchange.com/questions/94387/how-to-derive-errors-in-neural-network-with-the-backpropagation-algorithm
[4] https://theclevermachine.wordpress.com/2014/09/06/derivation-error-backpropagation-gradient-descent-for-neural-networks/

6 Week 6: Advice for Machine Learning

6.1 Overview

– Training set: 60%
– Cross validation set: 20%
– Test set: 20%
– Randomize your data and plot learning curves to determine high/low variance

6.2 Evauating Hypothesis

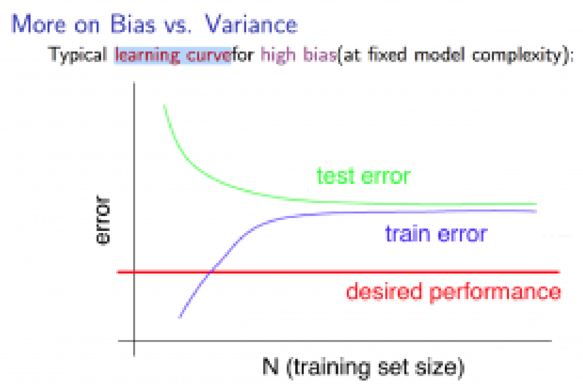
Linear regression $$J_{test}(\Theta) = \frac{1}{2m_{test}} \sum_{i=1}^{m_{test}} (h_\Theta(x_{test}^{(i)}) - y_{test}^{(i)})^2$$

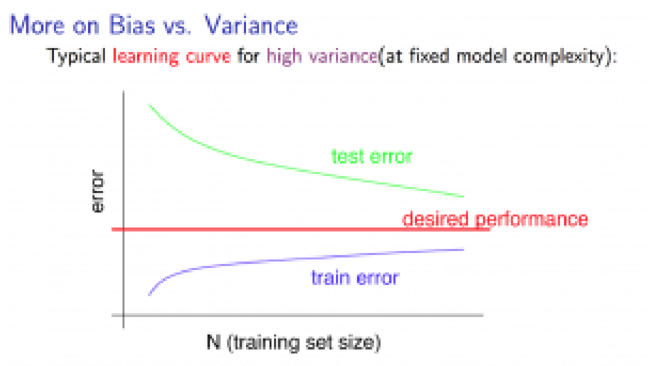
Classification $$err(h_\Theta(x), y) = \begin{cases} 1 \text{ if } h_\Theta(x) \geq 0.5 \text{ and } y = 0 \\ 1 \text{ if } h_\Theta(x) < 0.5 \text{ and } y = 1 \\ 0 \text{ otherwise} \end{cases}$$

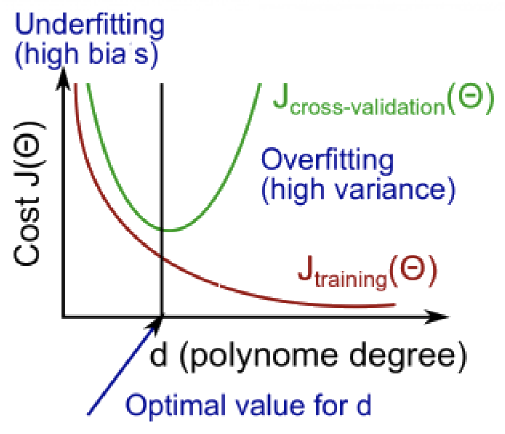
Test Error $$= \frac{1}{m_{test}} \sum_{i=1}^{m_{test}} err(h_\Theta(x_{test}^{(i)}), y_{test}^{(i)})$$

6.3 Bias, Variance, Learning Curves

– High variance is when overfitting data
  - Get more training examples, try a smaller set of features, increase lambda
– High bias is when underfitting data
  - Add features, add polynomial features, decrease lambda
– Small neural networks are more prone to underfitting and are cheaper
– Large neural networks are more prone to overfitting and are more expensive







6.4 Error Analysis

– Difficult to know which feature is actually most helpful
– Start with simple algorithm and test it on cross validation data
– Look at learning curves to decide where to focus
– Manually look at errors in cross validation data to spot trends
– Iterate towards improvement according to a single numerical value of error

6.5 Precision and Recall

– Better metric for skewed classes, where there is only a very small chance of $y = 1$
– Precision $= \frac{\text{true positives}}{\text{predicted positives}}$
– Recall $= \frac{\text{true positives}}{\text{actual positives}}$
– Tradeoff between precision and recall by changing threshold
  - Predict 1 if $h_\theta(x) \geq$ threshold

– F_1 score $= 2\frac{PR}{P+R}$
  - Maximize F_1 score for picking threshold

6.6 Large Data Sets

– Use large data (many training examples) if a human can confidently predict the results from the feature set
– Learning algorithms with many features/hidden units may need more data

7 Week 7: Support Vector Machines

7.1 Overview

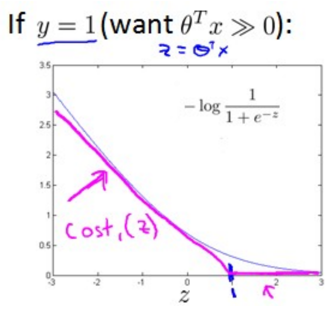
– Change cost function and lambda, but otherwise is basically logistic regression
– Creates larger margins
– Use prewritten SVM's because they have computational tricks to make them efficient enough to work
– Use feature scaling
– Large C: lower bias, higher variance
– Large σ: higher bias, lower variance, varies more smoothly
– If n is small and m is intermediate, use SVM w/ gaussian kernel
– If n is small and m is large, add features and use logistic regression or SVM w/o kernel

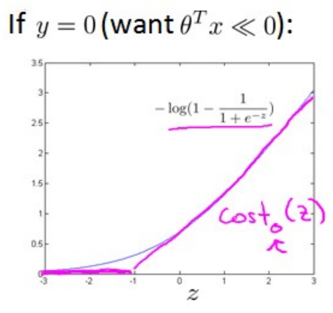
7.2 Large Margin Classification

$$\min_\theta \, C \sum_{i=1}^{m} \left[y^{(i)} cost_1(\theta^\intercal x^{(i)}) + (1 - y^{(i)}) cost_0(\theta^\intercal x^{(i)}) \right] + \frac{1}{2} \sum_{i=1}^{n} \theta_j^2$$

7.3 Math behind SVM

– $\theta^\intercal x^{(i)} = p^{(i)} |\theta|$
– Try to maximize $p^{(i)}$ so that $|\theta|$ can be minimized





7.4 Kernels

– Compute features according to proximity to landmarks $l^{(i)}$
  - Pick landmarks based on training data
  - Gaussian Kernel: $f_1 = \text{similarity}(x, l^{(1)}) = \exp(-\frac{|x - l^{(1)}|^2}{2\sigma^2})$
– Other kernels exist, like linear or polynomial; all must satisfy Mercer's Theorem

8 Week 8: Unsupervised Learning

8.1 Overview

- Unsupervised learning will find structure in data
- K-Means is a way to cluster
  - Pick number of clusters K by hand or application
  - K-Means can get stuck in local optima, especially for small numbers of clusters
    - Run algorithm many times, pick cluster with lowest cost

8.2 K-Means Notation

- K: number of clusters
- $x^{(1)}$: training examples $\in \mathbb{R}$
- $\{x^{(1)}, x^{(2)}, ..., x^{(m)}\}$: training set
- μ_k: average of points assigned to cluster k
- $c^{(i)}$: index of cluster centroid closest to $x^{(i)}$

8.3 K-Means Distortion Function/Optimization Objective

$$\min\ J(c^{(1)}, ..., c^{(m)}, \mu_1, ..., \mu_K) = \frac{1}{m}\sum_{i=1}^{m} |x^{(i)} - \mu_{c^{(i)}}|^2$$

8.4 K-Means Algorithm

1. Randomly initialized K cluster centroids according to K training examples
2. $c^{(i)}$:= index of cluster centroid closest to $x^{(i)}$
3. μ_k := average of points assigned to cluster centroid k

8.5 Dimensionality Reduction: Principal Component Analysis

- Useful for data compression, visualizations, and speeding up computation
- PCA projects data onto lower dimensional surface
  - Preprocess data with feature scaling and mean normalization
  - Try to maintain at least 85% variance, preferably 95%+
- Don't use PCA to address overfitting

8.6 PCA Notation

- Covariance matrix: Σ
- Number of principal components: k
- $[U, S, V] = \text{svd}(\Sigma)$
  - Matrix of eigenvectors: U
  - $U_{reduce} = U(:, 1 : k)$
- New inputs: $z = U_{reduce}^{\intercal} * x$

8.7 PCA: Equations

$$\Sigma = \frac{1}{m}\sum_{i=1}^{n}(x^{(i)})(x^{(i)})^{\intercal}$$

$$\text{Average squared projection error} = \frac{1}{m}\sum_{i=1}^{m}|x^{(i)} - x_{approx}^{(i)}|^2$$

$$\text{Total variance} = \frac{1}{m}\sum_{i=1}^{m}|x^{(i)}|^2$$

8.8 PCA Algorithm

1. Mean normalize and feature scaling
2. Compute covariance matrix Σ
3. Compute eigenvectors of Σ using singular value decomposition
4. Compute z

8.9 PCA K

$$\min_k \frac{\frac{1}{m}\sum_{i=1}^{m}|x^{(i)} - x_{approx}^{(i)}|^2}{\frac{1}{m}\sum_{i=1}^{m}|x^{(i)}|^2} \leq 0.01$$

alternatively, use S from svd(Σ)

$$\min_k \frac{\sum_{i=1}^{k} S_{ii}}{\sum_{i=1}^{m} S_{ii}} \geq 0.99$$

9 Week 9: Anomaly Detection

9.1 Overview

- Use Anomaly detection over Supervised learning when there a very few positive examples, there are many different types of anomalies, or when future anomalies may not look like current examples
- Gaussian Distribution
  - Positive μ shifts bellcurve right
  - Small σ increases height of bellcurve
  - For non-gaussian features, do something like $x_1 \leftarrow log(x_1)$
  - Can also create your own features; $x_3 = \frac{x_1}{x_2}$
- Multivariate Gaussian will automatically capture correlations between features
  - Is more computationally expensive bc Σ^{-1}
  - Should have $m > 10n$ to verify invertibility

9.2 Gaussion (Normal) Distribution

$$p(x) = \prod_{j=1}^{n} p(x_j; \mu_j; \sigma_j^2) = \prod_{j=1}^{n} \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$$

$$\mu_j = \frac{1}{m}\sum_{i=1}^{m} x_j^{(i)}$$

$$\sigma_j^2 = \frac{1}{m}\sum_{i=1}^{m}(x_j^{(i)} - \mu_j)^2$$

9.3 Anomaly Detection Algorithm

1. Choose features x_j indicative of anomalous examples
2. Fit parameters μ and σ
3. Compute $p(x)$; anomaly if $p(x) < \epsilon$

9.4 Multivariate Gaussion Distribution

$$p(x; \mu; \Sigma) = \frac{1}{(2\pi)^{\frac{n}{2}}|\Sigma|^{\frac{1}{2}}} \exp\left(-\frac{1}{2}(x-\mu)^{\intercal}\Sigma^{-1}(x-\mu)\right)$$

$$\mu = \frac{1}{m}\sum_{i=1}^{m} x^{(i)}$$

$$\Sigma = \frac{1}{m}\sum_{i=1}^{m}(x^{(i)} - \mu)(x^{(i)} - \mu)^{\intercal}$$

9.5 Recommender Systems Notation

- n_u: number of users
- n_m: number of movies
- $r(i,j) = 1$ if user j has rated movie i
- $y^{(i,j)}$: rating given by user j to movie i
- $\theta^{(j)}$: parameter vector for user j
- $x^{(i)}$: feature vector for movie i
- $m^{(j)}$: number of movies rated by user j

9.6 Recommender Systems Algorithm

$$\min_{\theta^{(1)},\ldots,\theta^{(n_u)}} \frac{1}{2}\sum_{j=1}^{n_u}\sum_{i:r(i,j)==1}\left((\theta^{(j)})^\intercal x^{(i)} - y^{(i,j)}\right)^2 + \frac{\lambda}{2}\sum_{j=1}^{n_u}\sum_{k=1}^{n}(\theta_k^{(j)})^2$$

$$\theta_k^{(j)} = \theta_k^{(j)} - \alpha \sum_{i:r(i,j)==1}\left((\theta^{(j)})^\intercal x^{(i)} - y^{(i,j)}\right)x_k^{(i)} \text{ k=0}$$

$$\theta_k^{(j)} = \theta_k^{(j)} - \alpha\left(\sum_{i:r(i,j)==1}\left((\theta^{(j)})^\intercal x^{(i)} - y^{(i,j)}\right)x_k^{(i)} + \lambda\theta_k^{(j)}\right)$$