Homework 1
UCLA-CS180-S18

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1 Question 1

$$f_6(n) = 2^{(logn)^{0.9}}$$
$$f_1(n) = n^3$$
$$f_5(n) = 2^{3\sqrt{n}}$$
$$f_4(n) = 2^{nlogn}$$
$$f_3(n) = n(logn)^{1000}$$
$$f_2(n) = 1000n^5/2$$

2 Question 2

Initial values $a = 10110100, b = 10111101$

Split up number $a_1 = 1011, a_0 = 0100$

Split up number $b_1 = 1011, b_0 = 1101$

Compute $a_1 + a_0$

$$\begin{array}{r} 1011 \\ +\,0100 \\ \hline 1111 \end{array}$$

Compute $b_1 + b_0$

$$\begin{array}{r} {\scriptstyle 1\,1\,1} \\ 1011 \\ +\ \ 1101 \\ \hline 11000 \end{array}$$

Recursively multiply a_1, b_1

1111001(See next page)

Recursively multiply a_0, b_0

110100(See third page from here)

Recursively multiply $a_1 + a_0, b_1 + b_0$

101101000(See four pages from here)

Compute $(a_1 * b_0 + a_0 * b_1) = (a_1 + a_0) * (b_1 + b_0) - (a_1 * b_1) - (a_0 * b_0)$

$$\begin{array}{r} 101101000 \\ -\quad 1111001 \\ -\quad\ \ 110100 \\ \hline 10111011 \end{array}$$

Shift $2^n * (a_1 * b_1)$

111100100000000

Shift $2^{n/2} * (a_1 * b_0 + a_0 * b_1)$

101110110000

Add $2^n * (a_1 * b_1) + 2^{n/2} * (a_1 * b_0 + a_0 * b_1) + (a_0 * b_0)$

1000010011100100

Initial values $a_1 = 1011, b_1 = 1011$

Split up number $a_{11} = 10, a_{10} = 11$

Split up number $b_{11} = 10, b_{10} = 11$

Compute $a_{11} + a_{10}$

$$\begin{array}{r} 10 \\ +\ 11 \\ \hline 101 \end{array}$$

Compute $b_{11} + b_{10}$

$$\begin{array}{r} 10 \\ +\ 11 \\ \hline 101 \end{array}$$

Recursively multiply a_{11}, b_{11}

$$\begin{array}{r} 10 \\ *\ 10 \\ \hline 00 \\ 100 \\ \hline 100 \end{array}$$

Recursively multiply a_{10}, b_{10}

$$\begin{array}{r} 11 \\ *\ 11 \\ \hline {\scriptstyle 1} \\ 011 \\ 110 \\ \hline 1001 \end{array}$$

Recursively multiply $a_{11} + a_{10}, b_{11} + b_{10}$

11001(See next page)

Compute $(a_{11} * b_{10} + a_{10} * b_{11}) = (a_{11} + a_{10}) * (b_{11} + b_{10}) - (a_{11} * b_{11}) - (a_{10} * b_{10})$

$$\begin{array}{r} 11001 \\ -\ \ 100 \\ -\ 1001 \\ \hline 1100 \end{array}$$

Shift $2^n * (a_{11} * b_{11})$

1000000

Shift $2^{n/2} * (a_{11} * b_{10} + a_{10} * b_{11})$

110000

Add $2^n * (a_{11} * b_{11}) + 2^{n/2} * (a_{11} * b_{10} + a_{10} * b_{11}) + (a_{10} * b_{10})$

1111001

Initial values $a_{11+10} = 101, b_{11+10} = 101$

Split up number $a_{11+10,1} = 10, a_{11+10,0} = 1$

Split up number $b_{11+10,1} = 10, b_{11+10,0} = 1$

Compute $a_{11+10,1} + a_{11+10,0}$

$$\begin{array}{r} 10 \\ +\ \ 1 \\ \hline 11 \end{array}$$

Compute $b_{11+10,1} + b_{11+10,0}$

$$\begin{array}{r} 10 \\ +\ \ 1 \\ \hline 11 \end{array}$$

Recursively multiply $a_{11+10,1}, b_{11+10,1}$

$$\begin{array}{r} 10 \\ *\ \ 10 \\ \hline 00 \\ 100 \\ \hline 100 \end{array}$$

Recursively multiply $a_{11+10,0}, b_{11+10,0}$

$$\begin{array}{r} 1 \\ *1 \\ \hline 1 \end{array}$$

Recursively multiply $a_{11+10,1} + a_{11+10,0}, b_{11+10,1} + b_{11+10,0}$

$$\begin{array}{r} 11 \\ *\ \ 11 \\ \hline {\scriptstyle 1} \\ 011 \\ 110 \\ \hline 1001 \end{array}$$

Compute $(a_{11+10,1} * b_{11+10,0} + a_{11+10,0} * b_{11+10,1})$

$= (a_{11+10,1} + a_{11+10,0}) * (b_{11+10,1} + b_{11+10,0})$

$- (a_{11+10,1} * b_{11+10,1})$

$- (a_{11+10,0} * b_{11+10,0})$

$$\begin{array}{r} 1001 \\ -\ \ 100 \\ -\ \ \ \ 1 \\ \hline 100 \end{array}$$

Shift $2^n * (a_{11+10,1} * b_{11+10,1})$

10000

Shift $2^{n/2} * (a_{11+10,1} * b_{11+10,0} + a_{11+10,0} * b_{11+10,1})$

1000

Add $2^n * (a_{11+10,1} * b_{11+10,1}) + 2^{n/2} * (a_{11+10,1} * b_{11+10,0} + a_{11+10,0} * b_{11+10,1}) + (a_{11+10,0} * b_{11+10,0})$

11001

Initial values $a_0 = 0100, b_0 = 1101$

Split up number $a_{01} = 01, a_{00} = 00$

Split up number $b_{01} = 11, b_{00} = 01$

Compute $a_{01} + a_{00}$

$$\begin{array}{r} 01 \\ +\,00 \\ \hline 01 \end{array}$$

Compute $b_{01} + b_{00}$

$$\begin{array}{r} {\scriptstyle 1} \\ 11 \\ +\;\;01 \\ \hline 100 \end{array}$$

Recursively multiply a_{01}, b_{01}

$$\begin{array}{r} 01 \\ *\;\;11 \\ \hline 01 \\ 010 \\ \hline 011 \end{array}$$

Recursively multiply a_{00}, b_{00}

$$\begin{array}{r} 00 \\ *\,01 \\ \hline 00 \\ 00 \\ \hline 00 \end{array}$$

Recursively multiply $a_{01} + a_{00}, b_{01} + b_{00}$

100(1 multiplied by x is x)

Compute $(a_{01} * b_{00} + a_{00} * b_{01}) = (a_{01} + a_{00}) * (b_{01} + b_{00}) - (a_{01} * b_{01}) - (a_{00} * b_{00})$

$$\begin{array}{r} 100 \\ -\;\;11 \\ -\;\;00 \\ \hline 01 \end{array}$$

Shift $2^n * (a_{01} * b_{01})$

110000

Shift $2^{n/2} * (a_{01} * b_{00} + a_{00} * b_{01})$

100

Add $2^n * (a_{01} * b_{01}) + 2^{n/2} * (a_{01} * b_{00} + a_{00} * b_{01}) + (a_{00} * b_{00})$

110100

Initial values $a_{1+0} = 1111, b_{1+0} = 11000$

Split up number $a_{1+0,1} = 11, a_{1+0,0} = 11$

Split up number $b_{1+0,1} = 110, b_{1+0,0} = 00$

Compute $a_{1+0,1} + a_{1+0,0}$

$$\begin{array}{r} {\scriptstyle 1} \\ 11 \\ +\ 11 \\ \hline 110 \end{array}$$

Compute $b_{1+0,1} + b_{1+0,0}$

$$\begin{array}{r} 110 \\ +\ 00 \\ \hline 110 \end{array}$$

Recursively multiply $a_{1+0,1}, b_{1+0,1}$

10010(See next page)

Recursively multiply $a_{1+0,0}, b_{1+0,0}$

$$\begin{array}{r} 11 \\ *\ 00 \\ \hline 00 \\ 000 \\ \hline 000 \end{array}$$

Recursively multiply $a_{1+0,1} + a_{1+0,0}, b_{1+0,1} + b_{1+0,0}$

100100(See second page from here)

Compute $(a_{1+0,1} * b_{1+0,0} + a_{1+0,0} * b_{1+0,1})$

$= (a_{1+0,1} + a_{1+0,0}) * (b_{1+0,1} + b_{1+0,0})$

$- (a_{1+0,1} * b_{1+0,1})$

$- (a_{1+0,0} * b_{1+0,0})$

$$\begin{array}{r} 100100 \\ -\ 10010 \\ -\ \ 000 \\ \hline 10010 \end{array}$$

Shift $2^n * (a_{1+0,1} * b_{1+0,1})$

100100000

Shift $2^{n/2} * (a_{1+0,1} * b_{1+0,0} + a_{1+0,0} * b_{1+0,1})$

1001000

Add $2^n * (a_{1+0,1} * b_{1+0,1})$

$+ 2^{n/2} * (a_{1+0,1} * b_{1+0,0} + a_{1+0,0} * b_{1+0,1})$

$+ (a_{1+0,0} * b_{1+0,0})$

101101000

Initial values $a_{1+0,1} = 11, b_{1+0,1} = 110$

Split up number $a_{1+0,1,1} = 1, a_{1+0,1,0} = 1$

Split up number $b_{1+0,1,1} = 11, b_{1+0,1,0} = 0$

Compute $a_{1+0,1,1} + a_{1+0,1,0}$

$$\begin{array}{r} {\scriptstyle 1} \\ 01 \\ +\ \ 1 \\ \hline 10 \end{array}$$

Compute $b_{1+0,1,1} + b_{1+0,1,0}$

$$\begin{array}{r} 11 \\ +\ \ 0 \\ \hline 11 \end{array}$$

Recursively multiply $a_{1+0,1,1}, b_{1+0,1,1}$

$$\begin{array}{r} 1 \\ *11 \\ \hline 1 \\ 10 \\ \hline 11 \end{array}$$

Recursively multiply $a_{1+0,1,0}, b_{1+0,1,0}$

$$\begin{array}{r} 1 \\ *0 \\ \hline 0 \end{array}$$

Recursively multiply $a_{1+0,1,1} + a_{1+0,1,0}, b_{1+0,1,1} + b_{1+0,1,0}$

$$\begin{array}{r} 10 \\ *\ \ 11 \\ \hline 10 \\ 100 \\ \hline 110 \end{array}$$

Compute $(a_{1+0,1,1} * b_{1+0,1,0} + a_{1+0,1,0} * b_{1+0,1,1}) = (a_{1+0,1,1}$
$+ a_{1+0,1,0}) * (b_{1+0,1,1} + b_{1+0,1,0})$
$- (a_{1+0,1,1} * b_{1+0,1,1}) - (a_{1+0,1,0} * b_{1+0,1,0})$

$$\begin{array}{r} 110 \\ -\ \ 11 \\ -\ \ \ 0 \\ \hline 11 \end{array}$$

Shift $2^n * (a_{1+0,1,1} * b_{1+0,1,1})$

1100

Shift $2^{n/2} * (a_{1+0,1,1} * b_{1+0,1,0} + a_{1+0,1,0} * b_{1+0,1,1})$

110

Add $2^n * (a_{1+0,1,1} * b_{1+0,1,1}) + 2^{n/2} * (a_{1+0,1,1} * b_{1+0,1,0} + a_{1+0,1,0} * b_{1+0,1,1}) + (a_{1+0,1,0} * b_{1+0,1,0})$

10010

Initial values $a_{1+0,1} + a_{1+0,0} = 110, b_{1+0,1} + b_{1+0,0} = 110$

Split up number $a_{1+0,1*1+0,0;1} = 11, a_{1+0,1*1+0,0;0} = 0$

Split up number $b_{1+0,1*1+0,0;1} = 11, b_{1+0,1*1+0,0;0} = 0$

Compute $a_{1+0,1*1+0,0;1} + a_{1+0,1*1+0,0;0}$

$$\begin{array}{r} 11 \\ +\ \ 0 \\ \hline 11 \end{array}$$

Compute $b_{1+0,1*1+0,0;1} + b_{1+0,1*1+0,0;0}$

$$\begin{array}{r} 11 \\ +\ \ 0 \\ \hline 11 \end{array}$$

Recursively multiply $a_{1+0,1*1+0,0;1}, b_{1+0,1*1+0,0;1}$

$$\begin{array}{r} 11 \\ *\ \ 11 \\ \hline 11 \\ 110 \\ \hline 1001 \end{array}$$

Recursively multiply $a_{1+0,1*1+0,0;0}, b_{1+0,1*1+0,0;0}$

$$\begin{array}{r} 0 \\ *0 \\ \hline 0 \end{array}$$

Recursively multiply $a_{1+0,1*1+0,0;1} + a_{1+0,1*1+0,0;0}, b_{1+0,1*1+0,0;1} + b_{1+0,1*1+0,0;0}$

$$\begin{array}{r} 11 \\ *\ \ 11 \\ \hline 11 \\ 110 \\ \hline 1001 \end{array}$$

Compute $(a_{1+0,1*1+0,0;1} * b_{1+0,1*1+0,0;0} + a_{1+0,1*1+0,0;0} * b_{1+0,1*1+0,0;1})$

$= (a_{1+0,1*1+0,0;1} + a_{1+0,1*1+0,0;0}) * (b_{1+0,1*1+0,0;1} + b_{1+0,1*1+0,0;0})$

$- (a_{1+0,1*1+0,0;1} * b_{1+0,1*1+0,0;1})$

$- (a_{1+0,1*1+0,0;0} * b_{1+0,1*1+0,0;0})$

$$\begin{array}{r} 1001 \\ -1001 \\ \hline 0 \end{array}$$

Shift $2^n * (a_{1+0,1*1+0,0;1} * b_{1+0,1*1+0,0;1})$

100100

Shift $2^{n/2} * (a_{1+0,1*1+0,0;1} * b_{1+0,1*1+0,0;0} + a_{1+0,1*1+0,0;0} * b_{1+0,1*1+0,0;1})$

0

Add $2^n * (a_{1+0,1*1+0,0;1} * b_{1+0,1*1+0,0;1}) + 2^{n/2} * (a_{1+0,1*1+0,0;1} * b_{1+0,1*1+0,0;0}$

$+ a_{1+0,1*1+0,0;0} * b_{1+0,1*1+0,0;1})$

$+ (a_{1+0,1*1+0,0;0} * b_{1+0,1*1+0,0;0})$

100100

3 Question 3

(a)

$$\begin{aligned} T(n) &= 4T(n/5) + n \\ &= n^{log_5 4} < n \\ &= \Theta(n) \end{aligned}$$

(b)

$$\begin{aligned} T(n) &= 6T(n/3) + n \\ &= n^{log_3 6} > n \\ &= \Theta(n^{log_3 6}) \end{aligned}$$

(c)

$$\begin{aligned} T(n) &= 16T(n/4) + n^2 \\ &= n^{log_4 16} == n^2 \\ &= \Theta(n^2 log n) \end{aligned}$$

4 Question 4

We iterate through the list of integers and XOR each of their kth bits together (a single column). The resulting bit's value is the same value as the missing integer's kth bit. For the next column, we only iterate through the list of integers which had the same kth bit as the previous value we found, cutting the search space in half. On the last recursive step (n==1), the resulting bit is inverted to obtain the missing integer's bit. We concatenate these bits together to obtain the integer. Each recursive step does $f(n) = n$ work due to iteration and divides the problem into one subproblem of half the original size.

```
integer find_missing_integer(list_A, n)
    bitpattern_to_integer(find_missing_integer_helper(list_A, n + 1, log(n)))

bitpattern find_missing_integer_helper(list_A, n, pos)
    if n == 1
        return invert(list_A[0][pos])

    leading_bit = 0
    list_B = []

    for i in range(0, n)
        leading_bit = leading_bit XOR list_A[i][pos]


    for i in range(0, n)
        if leading_bit == list_A[i][pos]
            append list_A[i] to list_B

    return concatenate_bits(leading_bit, find_missing_integer(list_B, n/2, pos - 1))
```

$$\begin{aligned} T(n) &= 1T(n/2) + n \\ &= n^{log_2 1} < n \\ &= \Theta(n) \end{aligned}$$