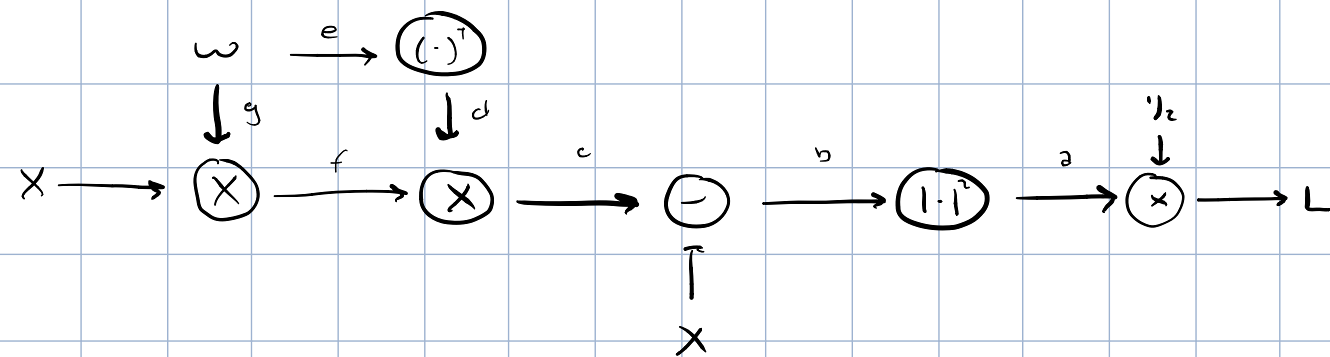
1) a) This minimization finds a W that preserves info about x because it is equivalent to PCA where $h = f(x) = W^T x$, $g(h) = Wh$. Also, we can view it in terms of SVD and the transpose - the rotations get reversed and only the scaling is left. Also $W^T W = I$ if W is orthogonal b/c W^T would be equal to W^{-1}.

b)



c) Take the derivatives with respect to both pathes and add them together. This is due to the total derivatives rule.

d) $L(a) = \frac{1}{2} a = \frac{1}{2} |W^T W x - x|^2$

$a(b) = |b|^2$

$b(c) = c - x$

$c(d, f) = d \cdot f$

$d(e) = e^T$

$f(g) = g \cdot x$

$$\frac{\partial L}{\partial w_1} = \frac{\partial d}{\partial e} \frac{\partial c}{\partial d} \frac{\partial b}{\partial c} \frac{\partial a}{\partial b} \frac{\partial L}{\partial a}$$

$$\left(\left(+1 \; 2(W^T W x - x) \tfrac{1}{2} \right) (Wx)^T \right)^T$$

$$= \left((W^T W x - x)(Wx)^T \right)^T$$

$$= (Wx)(W^TWx - x)^T$$

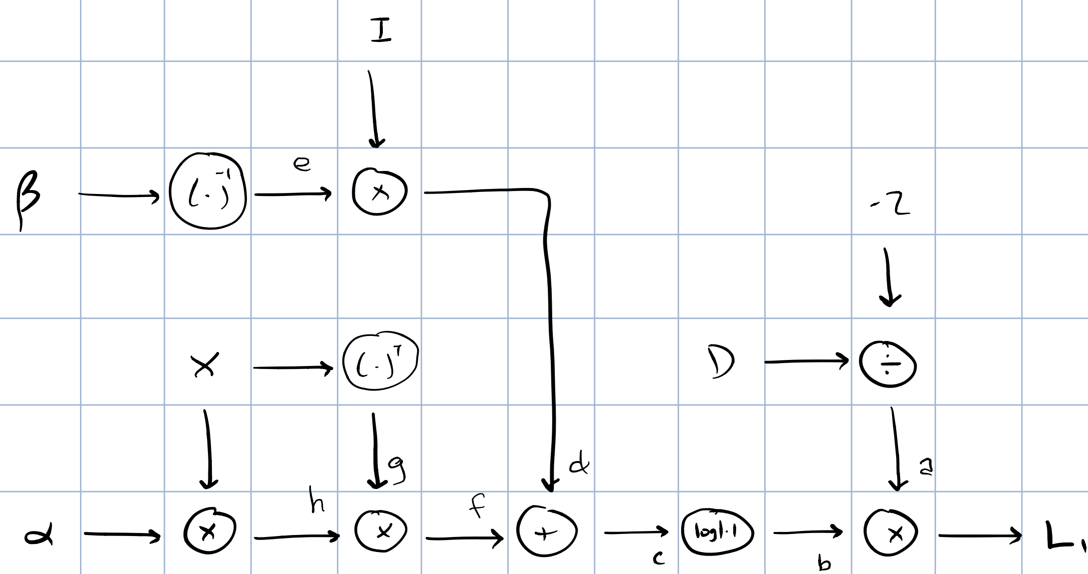
$$\frac{\partial L}{\partial w_2} = \frac{\partial f}{\partial g}\ \frac{\partial c}{\partial f}\ \frac{\partial b}{\partial c}\ \frac{\partial a}{\partial b}\ \frac{\partial L}{\partial a}$$

$$= W\ (-1)\ 2(W^TWx - x)\tfrac{1}{2}\ X^T$$

$$= W(W^TWx - x)X^T$$

$$\nabla_W L = \nabla_{W_1} L + \nabla_{W_2} L = (Wx)(W^TWx - x)^T + W(W^TWx - x)x^T$$

2) a) $L_1 = -\frac{D}{2} \log|\alpha XX^T + \beta^{-1} I|$



b) $L_1 = ba$

$b = \log|c|$

$c = f + d$

$f = h \cdot g$

$h = \alpha X$

$g = X^T$

$\frac{\partial L_1}{\partial b} = a = -\frac{D}{2}$

$\frac{\partial b}{\partial c} = K^{-T}$

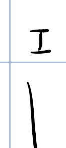
$\frac{\partial c}{\partial f} = 1$

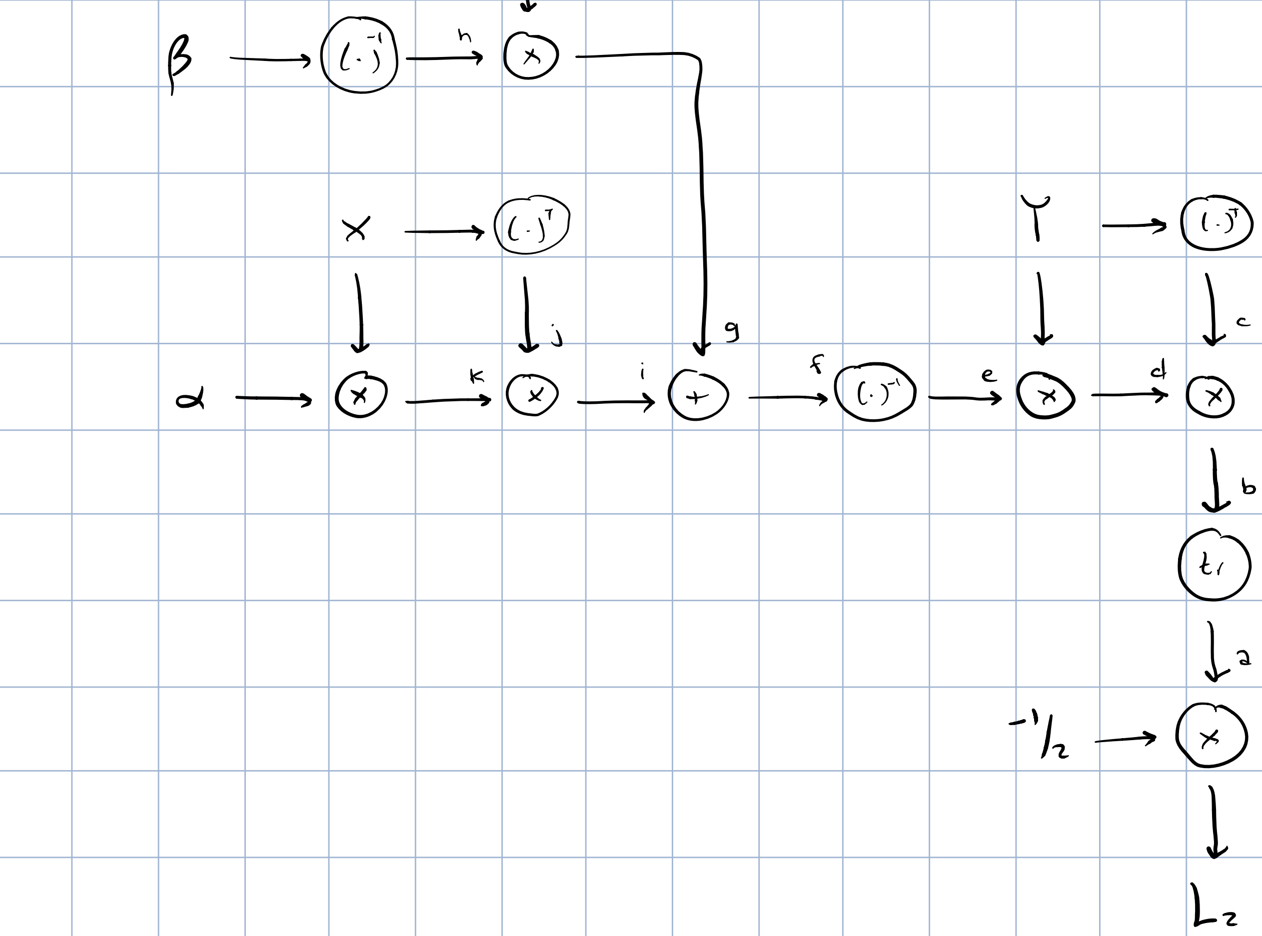
$\frac{\partial f}{\partial g} = h = \alpha X$ $\qquad \frac{\partial f}{\partial h} = g^T = (X^T)^T$

$\frac{\partial g}{\partial X} = I$ $\qquad \frac{\partial h}{\partial X} = \alpha$

$$\frac{\partial L_1}{\partial X} = -\frac{D}{2}K^{-T}(\alpha X + \alpha X) = -DK^{-T}\alpha X$$

c) $L_2 = -\frac{1}{2}\mathrm{tr}\,(\alpha XX^T + \beta^{-1}I)^{-1}YY^T$





d) $\frac{\partial L_2}{\partial X} = \frac{\partial K}{\partial X} \frac{\partial L_2}{\partial K}$

$$\frac{\partial L_2}{\partial K} = -K^{-T} \frac{\partial L_2}{\partial K^{-1}} K^{-T}$$

$$\frac{\partial L_2}{\partial K^{-1}} = \frac{\partial}{\partial K^{-1}} \left(-1/2\, tr\left(KYY^T\right)\right)$$

$$= -1/2\, YY^T$$

$$\frac{\partial K}{\partial X} = \left(\alpha X + \alpha X\right) \longleftarrow \text{from above}$$

$$\frac{\partial L_2}{\partial X} = \left(2\alpha X\right) K^{-T} \left(1/2\, YY^T\right) K^{-T}$$

$$= \alpha X K^{-T} YY^T K^{-T}$$

e) $\frac{\partial L}{\partial X} = \frac{\partial L_1}{\partial X} + \frac{\partial L_2}{\partial X}$

$$= D K^{-T} \alpha X + \alpha X K^{-T} YY^T K^{-T}$$

This is the 2-layer neural network workbook for ECE 239AS Assignment #3

Please follow the notebook linearly to implement a two layer neural network.

Please print out the workbook entirely when completed.

We thank Serena Yeung & Justin Johnson for permission to use code written for the CS 231n class (cs231n.stanford.edu). These are the functions in the cs231n folders and code in the jupyer notebook to preprocess and show the images. The classifiers used are based off of code prepared for CS 231n as well.

The goal of this workbook is to give you experience with training a two layer neural network.

In [1]:

```
import random
import numpy as np
from cs231n.data_utils import load_CIFAR10
import matplotlib.pyplot as plt

%matplotlib inline
%load_ext autoreload
%autoreload 2

def rel_error(x, y):
    """ returns relative error """
    return np.max(np.abs(x - y) / (np.maximum(1e-8, np.abs(x) + np.abs(y))))
```

Toy example

Before loading CIFAR-10, there will be a toy example to test your implementation of the forward and backward pass

In [2]:

```
from nndl.neural_net import TwoLayerNet
```

In [3]:

```
# Create a small net and some toy data to check your implementations.
# Note that we set the random seed for repeatable experiments.

input_size = 4
hidden_size = 10
num_classes = 3
num_inputs = 5

def init_toy_model():
    np.random.seed(0)
    return TwoLayerNet(input_size, hidden_size, num_classes, std=1e-1)

def init_toy_data():
    np.random.seed(1)
    X = 10 * np.random.randn(num_inputs, input_size)
    y = np.array([0, 1, 2, 2, 1])
    return X, y

net = init_toy_model()
X, y = init_toy_data()
```

Compute forward pass scores

In [4]:
```
## Implement the forward pass of the neural network.

# Note, there is a statement if y is None: return scores, which is why
# the following call will calculate the scores.
scores = net.loss(X)
print('Your scores:')
print(scores)
print()
print('correct scores:')
correct_scores = np.asarray([
    [-1.07260209,  0.05083871, -0.87253915],
    [-2.02778743, -0.10832494, -1.52641362],
    [-0.74225908,  0.15259725, -0.39578548],
    [-0.38172726,  0.10835902, -0.17328274],
    [-0.64417314, -0.18886813, -0.41106892]])
print(correct_scores)
print()

# The difference should be very small. We get < 1e-7
print('Difference between your scores and correct scores:')
print(np.sum(np.abs(scores - correct_scores)))
```

```
Your scores:
[[-1.07260209  0.05083871 -0.87253915]
 [-2.02778743 -0.10832494 -1.52641362]
 [-0.74225908  0.15259725 -0.39578548]
 [-0.38172726  0.10835902 -0.17328274]
 [-0.64417314 -0.18886813 -0.41106892]]

correct scores:
[[-1.07260209  0.05083871 -0.87253915]
 [-2.02778743 -0.10832494 -1.52641362]
 [-0.74225908  0.15259725 -0.39578548]
 [-0.38172726  0.10835902 -0.17328274]
 [-0.64417314 -0.18886813 -0.41106892]]

Difference between your scores and correct scores:
3.381231204052648e-08
```

Forward pass loss

In [5]:
```
loss, _ = net.loss(X, y, reg=0.05)
correct_loss = 1.071696123862817

# should be very small, we get < 1e-12
print('Difference between your loss and correct loss:')
print(np.sum(np.abs(loss - correct_loss)))
```

```
Difference between your loss and correct loss:
0.0
```

In [6]:
```
print(loss)
```

```
1.071696123862817
```

Backward pass

Implements the backwards pass of the neural network. Check your gradients with the gradient check utilities provided.

In [7]:
```
from cs231n.gradient_check import eval_numerical_gradient

# Use numeric gradient checking to check your implementation of the backward pas
s.
# If your implementation is correct, the difference between the numeric and
# analytic gradients should be less than 1e-8 for each of W1, W2, b1, and b2.

loss, grads = net.loss(X, y, reg=0.05)

# these should all be less than 1e-8 or so
for param_name in grads:
    f = lambda W: net.loss(X, y, reg=0.05)[0]
    param_grad_num = eval_numerical_gradient(f, net.params[param_name], verbose=F
alse)
    print('{} max relative error: {}'.format(param_name, rel_error(param_grad_num
, grads[param_name])))
```

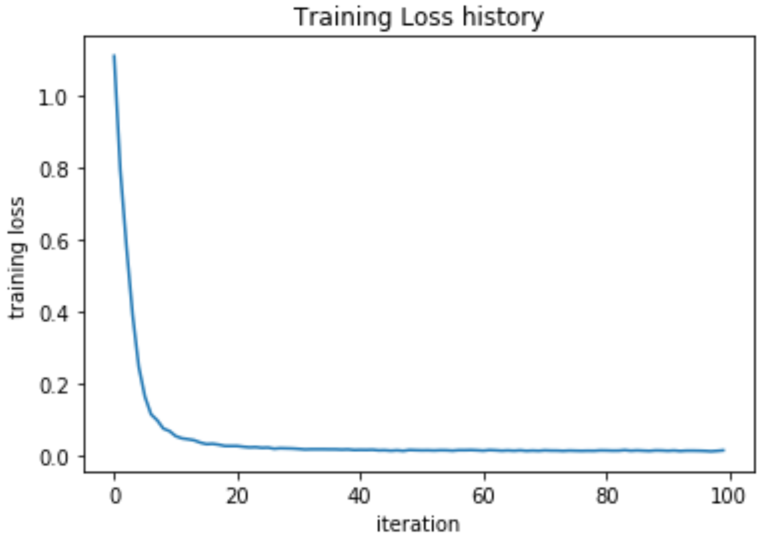
```
W2 max relative error: 2.9632233460136427e-10
b2 max relative error: 1.8392106647421603e-10
W1 max relative error: 1.283286893046317e-09
b1 max relative error: 3.1726799962069797e-09
```

Training the network

Implement neural_net.train() to train the network via stochastic gradient descent, much like the softmax and SVM.

In [8]:
```
net = init_toy_model()
stats = net.train(X, y, X, y,
            learning_rate=1e-1, reg=5e-6,
            num_iters=100, verbose=False)

print('Final training loss: ', stats['loss_history'][-1])

# plot the loss history
plt.plot(stats['loss_history'])
plt.xlabel('iteration')
plt.ylabel('training loss')
plt.title('Training Loss history')
plt.show()
```

```
Final training loss:  0.01449786458776595
```



Classify CIFAR-10

Do classification on the CIFAR-10 dataset.

In [9]:
```python
from cs231n.data_utils import load_CIFAR10

def get_CIFAR10_data(num_training=49000, num_validation=1000, num_test=1000):
    """
    Load the CIFAR-10 dataset from disk and perform preprocessing to prepare
    it for the two-layer neural net classifier. These are the same steps as
    we used for the SVM, but condensed to a single function.
    """
    # Load the raw CIFAR-10 data
    cifar10_dir = 'cifar-10-batches-py'
    X_train, y_train, X_test, y_test = load_CIFAR10(cifar10_dir)

    # Subsample the data
    mask = list(range(num_training, num_training + num_validation))
    X_val = X_train[mask]
    y_val = y_train[mask]
    mask = list(range(num_training))
    X_train = X_train[mask]
    y_train = y_train[mask]
    mask = list(range(num_test))
    X_test = X_test[mask]
    y_test = y_test[mask]

    # Normalize the data: subtract the mean image
    mean_image = np.mean(X_train, axis=0)
    X_train -= mean_image
    X_val -= mean_image
    X_test -= mean_image

    # Reshape data to rows
    X_train = X_train.reshape(num_training, -1)
    X_val = X_val.reshape(num_validation, -1)
    X_test = X_test.reshape(num_test, -1)

    return X_train, y_train, X_val, y_val, X_test, y_test


# Invoke the above function to get our data.
X_train, y_train, X_val, y_val, X_test, y_test = get_CIFAR10_data()
print('Train data shape: ', X_train.shape)
print('Train labels shape: ', y_train.shape)
print('Validation data shape: ', X_val.shape)
print('Validation labels shape: ', y_val.shape)
print('Test data shape: ', X_test.shape)
print('Test labels shape: ', y_test.shape)
```

```
Train data shape:  (49000, 3072)
Train labels shape:  (49000,)
Validation data shape:  (1000, 3072)
Validation labels shape:  (1000,)
Test data shape:  (1000, 3072)
Test labels shape:  (1000,)
```

Running SGD

If your implementation is correct, you should see a validation accuracy of around 28-29%.

In [10]:
```
input_size = 32 * 32 * 3
hidden_size = 50
num_classes = 10
net = TwoLayerNet(input_size, hidden_size, num_classes)

# Train the network
stats = net.train(X_train, y_train, X_val, y_val,
            num_iters=1000, batch_size=200,
            learning_rate=1e-4, learning_rate_decay=0.95,
            reg=0.25, verbose=True)

# Predict on the validation set
val_acc = (net.predict(X_val) == y_val).mean()
print('Validation accuracy: ', val_acc)

# Save this net as the variable subopt_net for later comparison.
subopt_net = net
```

```
iteration 0 / 1000: loss 2.302757518613176
iteration 100 / 1000: loss 2.302120159207236
iteration 200 / 1000: loss 2.2956136007408703
iteration 300 / 1000: loss 2.2518259043164135
iteration 400 / 1000: loss 2.188995235046776
iteration 500 / 1000: loss 2.1162527791897747
iteration 600 / 1000: loss 2.064670827698217
iteration 700 / 1000: loss 1.9901688623083942
iteration 800 / 1000: loss 2.002827640124685
iteration 900 / 1000: loss 1.9465176817856495
Validation accuracy:  0.283
```

Questions:

The training accuracy isn't great.

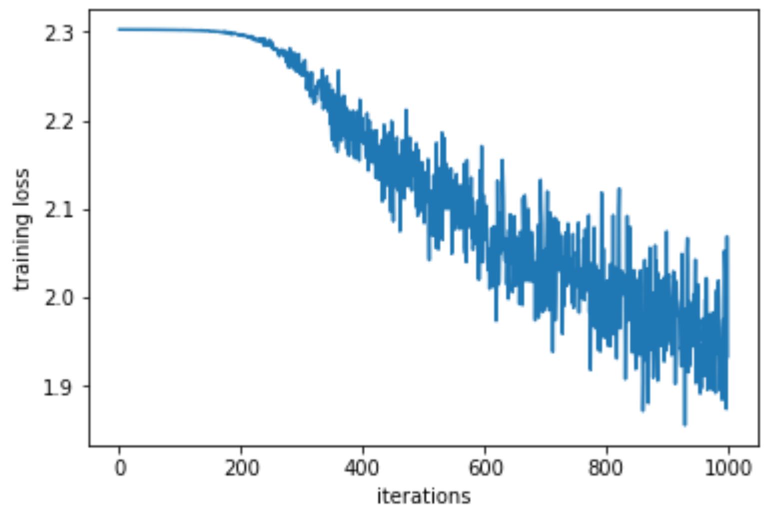
(1) What are some of the reasons why this is the case? Take the following cell to do some analyses and then report your answers in the cell following the one below.

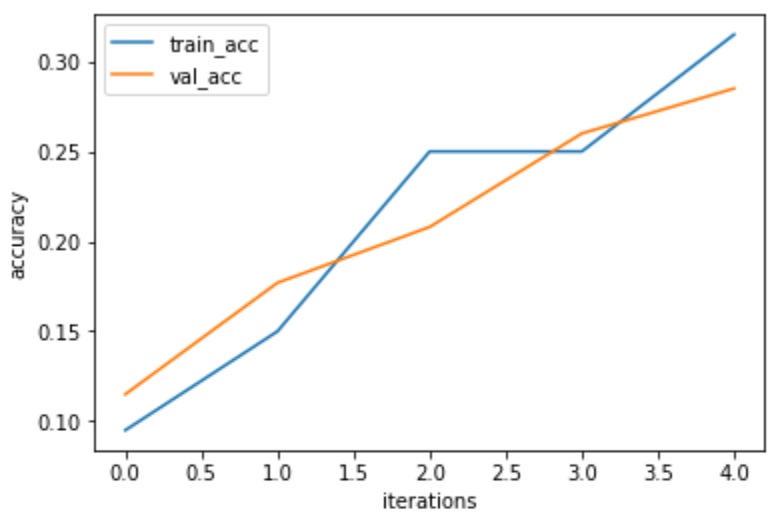
(2) How should you fix the problems you identified in (1)?

In [11]:
```
stats['train_acc_history']
```

Out[11]:
```
[0.095, 0.15, 0.25, 0.25, 0.315]
```

In [12]:
```
# ================================================================ #
# YOUR CODE HERE:
#   Do some debugging to gain some insight into why the optimization
#   isn't great.
# ================================================================ #

# Plot the loss function and train / validation accuracies
plt.plot(stats['loss_history'])
plt.xlabel('iterations')
plt.ylabel('training loss')
plt.show()

train_plt = plt.plot(stats['train_acc_history'], label='train_acc')
val_plt = plt.plot(stats['val_acc_history'], label='val_acc')
plt.xlabel('iterations')
plt.ylabel('accuracy')
plt.legend(handles=[train_plt[0], val_plt[0]])
plt.show()
# ================================================================ #
# END YOUR CODE HERE
# ================================================================ #
```





Answers:

(1) Network needs more iterations; this is evidenced by the fact that the training loss hasn't flattened out yet.

(2) Train the network on more iterations.

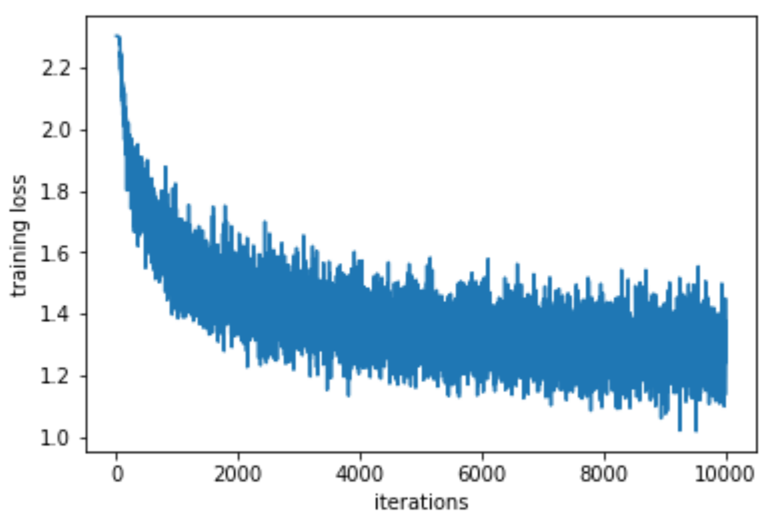
Optimize the neural network

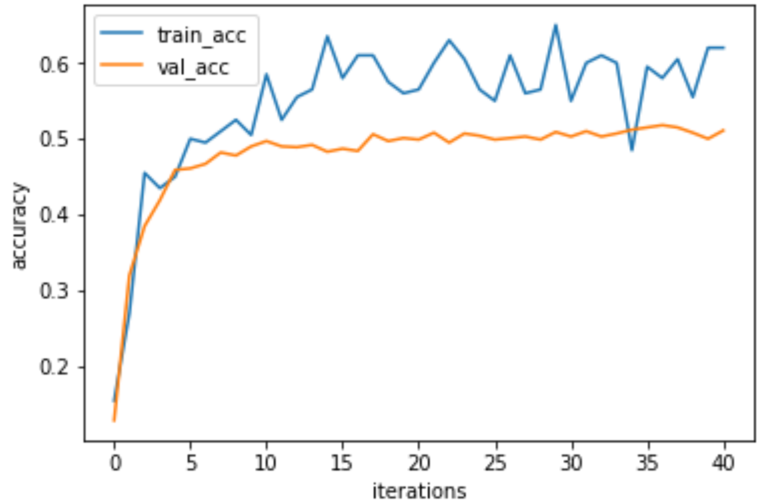
Use the following part of the Jupyter notebook to optimize your hyperparameters on the validation set. Store your nets as best_net.

In [13]:
```python
best_net = None # store the best model into this

# ================================================================ #
# YOUR CODE HERE:
#   Optimize over your hyperparameters to arrive at the best neural
#   network.  You should be able to get over 50% validation accuracy.
#   For this part of the notebook, we will give credit based on the
#   accuracy you get.  Your score on this question will be multiplied by:
#      min(floor((X - 28%)) / %22, 1)
#   where if you get 50% or higher validation accuracy, you get full
#   points.
#
#   Note, you need to use the same network structure (keep hidden_size = 50)!
# ================================================================ #
input_size = 32 * 32 * 3
hidden_size = 50
num_classes = 10
net = TwoLayerNet(input_size, hidden_size, num_classes)

# Train the network
# Changed num_iters from 1000 to 10000
# Changed learning_rate from 1e-4 to 5e-4
stats = net.train(X_train, y_train, X_val, y_val,
            num_iters=10000, batch_size=200,
            learning_rate=5e-4, learning_rate_decay=0.95,
            reg=0.25, verbose=True)

# Predict on the validation set
val_acc = (net.predict(X_val) == y_val).mean()
print('Validation accuracy: ', val_acc)

plt.plot(stats['loss_history'])
plt.xlabel('iterations')
plt.ylabel('training loss')
plt.show()

train_plt = plt.plot(stats['train_acc_history'], label='train_acc')
val_plt = plt.plot(stats['val_acc_history'], label='val_acc')
plt.xlabel('iterations')
plt.ylabel('accuracy')
plt.legend(handles=[train_plt[0], val_plt[0]])
plt.show()

# ================================================================ #
# END YOUR CODE HERE
# ================================================================ #
best_net = net
```

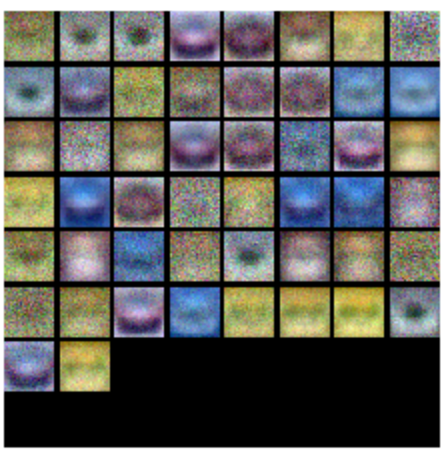
```
iteration 0 / 10000: loss 2.3027667167979295
iteration 100 / 10000: loss 2.150700404323184
iteration 200 / 10000: loss 1.879372419292919
iteration 300 / 10000: loss 1.9023182130827285
iteration 400 / 10000: loss 1.7807142164056704
iteration 500 / 10000: loss 1.8181766769644458
iteration 600 / 10000: loss 1.6414543683069336
iteration 700 / 10000: loss 1.6940316212826791
iteration 800 / 10000: loss 1.6196747665938407
iteration 900 / 10000: loss 1.6406450319379673
iteration 1000 / 10000: loss 1.5120425119723269
iteration 1100 / 10000: loss 1.494769579559985
iteration 1200 / 10000: loss 1.531886187331089
iteration 1300 / 10000: loss 1.547155005442258
iteration 1400 / 10000: loss 1.5132851154937077
iteration 1500 / 10000: loss 1.453939234243126
iteration 1600 / 10000: loss 1.461623947034721
iteration 1700 / 10000: loss 1.547346288758514
iteration 1800 / 10000: loss 1.3682624643370256
iteration 1900 / 10000: loss 1.5815310032238992
iteration 2000 / 10000: loss 1.4679435396646277
iteration 2100 / 10000: loss 1.4049537929212006
iteration 2200 / 10000: loss 1.3817505548517754
iteration 2300 / 10000: loss 1.4495089637987175
iteration 2400 / 10000: loss 1.328465196790276
iteration 2500 / 10000: loss 1.4310819970683502
iteration 2600 / 10000: loss 1.5498635334664423
iteration 2700 / 10000: loss 1.5402525253364188
iteration 2800 / 10000: loss 1.4329429930813415
iteration 2900 / 10000: loss 1.4112411804077145
iteration 3000 / 10000: loss 1.3973912564253008
iteration 3100 / 10000: loss 1.572805577184825
iteration 3200 / 10000: loss 1.359371239528938
iteration 3300 / 10000: loss 1.3811747563387333
iteration 3400 / 10000: loss 1.4160773841356475
iteration 3500 / 10000: loss 1.4195009141239936
iteration 3600 / 10000: loss 1.350825460475455
iteration 3700 / 10000: loss 1.2911371561539673
iteration 3800 / 10000: loss 1.3731407597629282
iteration 3900 / 10000: loss 1.4124538383982286
iteration 4000 / 10000: loss 1.4441994055060885
iteration 4100 / 10000: loss 1.3246583909471457
iteration 4200 / 10000: loss 1.3624120124779688
iteration 4300 / 10000: loss 1.275014816492637
iteration 4400 / 10000: loss 1.3528064446794132
iteration 4500 / 10000: loss 1.3354325618133192
iteration 4600 / 10000: loss 1.3278215295604259
iteration 4700 / 10000: loss 1.3217036343822535
iteration 4800 / 10000: loss 1.3801300810517763
iteration 4900 / 10000: loss 1.3491733661265153
iteration 5000 / 10000: loss 1.3233163127059862
iteration 5100 / 10000: loss 1.4413107772170564
iteration 5200 / 10000: loss 1.2665377397971052
iteration 5300 / 10000: loss 1.3803130359901452
iteration 5400 / 10000: loss 1.309787842974287
iteration 5500 / 10000: loss 1.2774451597439764
iteration 5600 / 10000: loss 1.3270047840876893
iteration 5700 / 10000: loss 1.3499470249137107
iteration 5800 / 10000: loss 1.5044287004932146
iteration 5900 / 10000: loss 1.5049673726137842
iteration 6000 / 10000: loss 1.255532126006217
```

```
iteration 6100 / 10000: loss 1.5777025416034964
iteration 6200 / 10000: loss 1.2319064826917787
iteration 6300 / 10000: loss 1.3651920885472617
iteration 6400 / 10000: loss 1.2136066551065452
iteration 6500 / 10000: loss 1.4709769773807257
iteration 6600 / 10000: loss 1.2491033618943153
iteration 6700 / 10000: loss 1.2282269821290899
iteration 6800 / 10000: loss 1.2192034678544237
iteration 6900 / 10000: loss 1.2626648985642135
iteration 7000 / 10000: loss 1.3302038264636649
iteration 7100 / 10000: loss 1.2818110960312208
iteration 7200 / 10000: loss 1.3066055244665762
iteration 7300 / 10000: loss 1.3552036062279216
iteration 7400 / 10000: loss 1.3200377322348247
iteration 7500 / 10000: loss 1.3203374770555931
iteration 7600 / 10000: loss 1.2074896307210108
iteration 7700 / 10000: loss 1.4101472056848172
iteration 7800 / 10000: loss 1.2377970622028498
iteration 7900 / 10000: loss 1.2509436584468023
iteration 8000 / 10000: loss 1.2519056490843126
iteration 8100 / 10000: loss 1.356874812752282
iteration 8200 / 10000: loss 1.096048928736568
iteration 8300 / 10000: loss 1.1368053760281858
iteration 8400 / 10000: loss 1.2959323614794833
iteration 8500 / 10000: loss 1.206817959804406
iteration 8600 / 10000: loss 1.3717791188775452
iteration 8700 / 10000: loss 1.3697091377560717
iteration 8800 / 10000: loss 1.3641535991107125
iteration 8900 / 10000: loss 1.2475129846109128
iteration 9000 / 10000: loss 1.1826295062685948
iteration 9100 / 10000: loss 1.2716632836494195
iteration 9200 / 10000: loss 1.234386593980497
iteration 9300 / 10000: loss 1.2175029237525152
iteration 9400 / 10000: loss 1.3239823811498188
iteration 9500 / 10000: loss 1.2078180564723775
iteration 9600 / 10000: loss 1.2710874817084576
iteration 9700 / 10000: loss 1.3382242965166542
iteration 9800 / 10000: loss 1.204562418055099
iteration 9900 / 10000: loss 1.3023757043096909
Validation accuracy:  0.508
```





In [14]:
```
from cs231n.vis_utils import visualize_grid

# Visualize the weights of the network

def show_net_weights(net):
    W1 = net.params['W1']
    W1 = W1.T.reshape(32, 32, 3, -1).transpose(3, 0, 1, 2)
    plt.imshow(visualize_grid(W1, padding=3).astype('uint8'))
    plt.gca().axis('off')
    plt.show()

show_net_weights(subopt_net)
show_net_weights(best_net)
```



Question:

(1) What differences do you see in the weights between the suboptimal net and the best net you arrived at?

Answer:

(1) The suboptimal net looks a lot more just like averages of certain image classes, especially the car, whereas the best net is pretty abstract and grainy looking, more like certain shapes are being learned.

Evaluate on test set

In [15]:

```
test_acc = (best_net.predict(X_test) == y_test).mean()
print('Test accuracy: ', test_acc)
```

```
Test accuracy:  0.525
```

In [ ]:

```
import numpy as np
import matplotlib.pyplot as plt

"""
This code was originally written for CS 231n at Stanford University
(cs231n.stanford.edu).  It has been modified in various areas for use in the
ECE 239AS class at UCLA.  This includes the descriptions of what code to
implement as well as some slight potential changes in variable names to be
consistent with class nomenclature.  We thank Justin Johnson & Serena Yeung for
permission to use this code.  To see the original version, please visit
cs231n.stanford.edu.
"""

class TwoLayerNet(object):
  """
  A two-layer fully-connected neural network. The net has an input dimension of
  N, a hidden layer dimension of H, and performs classification over C classes.
  We train the network with a softmax loss function and L2 regularization on the
  weight matrices. The network uses a ReLU nonlinearity after the first fully
  connected layer.

  In other words, the network has the following architecture:

  input - fully connected layer - ReLU - fully connected layer - softmax

  The outputs of the second fully-connected layer are the scores for each class.
  """

  def __init__(self, input_size, hidden_size, output_size, std=1e-4):
    """
    Initialize the model. Weights are initialized to small random values and
    biases are initialized to zero. Weights and biases are stored in the
    variable self.params, which is a dictionary with the following keys:

    W1: First layer weights; has shape (H, D)
    b1: First layer biases; has shape (H,)
    W2: Second layer weights; has shape (C, H)
    b2: Second layer biases; has shape (C,)

    Inputs:
    - input_size: The dimension D of the input data.
    - hidden_size: The number of neurons H in the hidden layer.
    - output_size: The number of classes C.
    """
    self.params = {}
    self.params['W1'] = std * np.random.randn(hidden_size, input_size)
    self.params['b1'] = np.zeros(hidden_size)
    self.params['W2'] = std * np.random.randn(output_size, hidden_size)
    self.params['b2'] = np.zeros(output_size)


  def loss(self, X, y=None, reg=0.0):
    """
    Compute the loss and gradients for a two layer fully connected neural
    network.

    Inputs:
    - X: Input data of shape (N, D). Each X[i] is a training sample.
    - y: Vector of training labels. y[i] is the label for X[i], and each y[i] is
      an integer in the range 0 <= y[i] < C. This parameter is optional; if it
      is not passed then we only return scores, and if it is passed then we
```

```
      instead return the loss and gradients.
    - reg: Regularization strength.

    Returns:
    If y is None, return a matrix scores of shape (N, C) where scores[i, c] is
    the score for class c on input X[i].

    If y is not None, instead return a tuple of:
    - loss: Loss (data loss and regularization loss) for this batch of training
      samples.
    - grads: Dictionary mapping parameter names to gradients of those parameters
      with respect to the loss function; has the same keys as self.params.
    """
    # Unpack variables from the params dictionary
    W1, b1 = self.params['W1'], self.params['b1']
    W2, b2 = self.params['W2'], self.params['b2']
    N, D = X.shape

    # Compute the forward pass
    scores = None

    # ================================================================ #
    # YOUR CODE HERE:
    #   Calculate the output scores of the neural network.  The result
    #   should be (N, C). As stated in the description for this class,
    #   there should not be a ReLU layer after the second FC layer.
    #   The output of the second FC layer is the output scores. Do not
    #   use a for loop in your implementation.
    # ================================================================ #
    h1 = np.maximum([0], np.matmul(X, W1.T) + b1)
    scores = (np.matmul(h1, W2.T) + b2)
    # ================================================================ #
    # END YOUR CODE HERE
    # ================================================================ #


    # If the targets are not given then jump out, we're done
    if y is None:
      return scores

    # Compute the loss
    loss = None

    # ================================================================ #
    # YOUR CODE HERE:
    #   Calculate the loss of the neural network.  This includes the
    #   softmax loss and the L2 regularization for W1 and W2. Store the
    #   total loss in the variable loss.  Multiply the regularization
    #   loss by 0.5 (in addition to the factor reg).
    # ================================================================ #

    # scores is num_examples by num_classes
    regularization = 0.5 * reg * (np.sum(np.square(W1)) + np.sum(np.square(W2)))
    softmax = np.sum(np.log(np.sum(np.exp(scores), axis=1)) - scores[np.arange(sc
ores.shape[0]), y]) / X.shape[0]
    loss = softmax + regularization
    # ================================================================ #
    # END YOUR CODE HERE
    # ================================================================ #
    pass

    # ================================================================ #
```

```
    # YOUR CODE HERE:
    #   Implement the backward pass.  Compute the derivatives of the
    #   weights and the biases.  Store the results in the grads
    #   dictionary.  e.g., grads['W1'] should store the gradient for
    #   W1, and be of the same size as W1.
    # ================================================================ #
    grads = {}

    probabilities = np.exp(scores) / np.sum(np.exp(scores), axis=1).reshape(X.sha
pe[0], 1)
    probabilities[np.arange(N), y] -= 1
    dz = np.matmul(probabilities, W2).T * (np.matmul(X, W1.T) + b1 > 0).T / N

    grads['W2'] = (np.matmul(probabilities.T, h1) / N) + (reg * W2)
    grads['b2'] = np.sum(probabilities, axis=0) / N

    grads['W1'] = np.matmul(dz, X) + (reg * W1)
    grads['b1'] = np.sum(dz, axis=1)
    # ================================================================ #
    # END YOUR CODE HERE
    # ================================================================ #

    return loss, grads

  def train(self, X, y, X_val, y_val,
            learning_rate=1e-3, learning_rate_decay=0.95,
            reg=1e-5, num_iters=100,
            batch_size=200, verbose=False):
    """
    Train this neural network using stochastic gradient descent.

    Inputs:
    - X: A numpy array of shape (N, D) giving training data.
    - y: A numpy array f shape (N,) giving training labels; y[i] = c means that
      X[i] has label c, where 0 <= c < C.
    - X_val: A numpy array of shape (N_val, D) giving validation data.
    - y_val: A numpy array of shape (N_val,) giving validation labels.
    - learning_rate: Scalar giving learning rate for optimization.
    - learning_rate_decay: Scalar giving factor used to decay the learning rate
      after each epoch.
    - reg: Scalar giving regularization strength.
    - num_iters: Number of steps to take when optimizing.
    - batch_size: Number of training examples to use per step.
    - verbose: boolean; if true print progress during optimization.
    """
    num_train = X.shape[0]
    iterations_per_epoch = max(num_train / batch_size, 1)

    # Use SGD to optimize the parameters in self.model
    loss_history = []
    train_acc_history = []
    val_acc_history = []

    for it in np.arange(num_iters):
      X_batch = None
      y_batch = None

      # ================================================================ #
      # YOUR CODE HERE:
      #   Create a minibatch by sampling batch_size samples randomly.
      # ================================================================ #
      indices = np.random.choice(X.shape[0], batch_size)
```

```
      X_batch = X[indices,:]
      y_batch = y[indices]
      # ================================================================ #
      # END YOUR CODE HERE
      # ================================================================ #

       # Compute loss and gradients using the current minibatch
      loss, grads = self.loss(X_batch, y=y_batch, reg=reg)
      loss_history.append(loss)

      # ================================================================ #
      # YOUR CODE HERE:
      #   Perform a gradient descent step using the minibatch to update
      #   all parameters (i.e., W1, W2, b1, and b2).
      # ================================================================ #
      self.params['W1'] -= learning_rate * grads['W1']
      self.params['W2'] -= learning_rate * grads['W2']
      self.params['b1'] -= learning_rate * grads['b1']
      self.params['b2'] -= learning_rate * grads['b2']
      # ================================================================ #
      # END YOUR CODE HERE
      # ================================================================ #

      if verbose and it % 100 == 0:
        print('iteration {} / {}: loss {}'.format(it, num_iters, loss))

      # Every epoch, check train and val accuracy and decay learning rate.
      if it % iterations_per_epoch == 0:
        # Check accuracy
        train_acc = (self.predict(X_batch) == y_batch).mean()
        val_acc = (self.predict(X_val) == y_val).mean()
        train_acc_history.append(train_acc)
        val_acc_history.append(val_acc)

        # Decay learning rate
        learning_rate *= learning_rate_decay

    return {
      'loss_history': loss_history,
      'train_acc_history': train_acc_history,
      'val_acc_history': val_acc_history,
    }

  def predict(self, X):
    """
    Use the trained weights of this two-layer network to predict labels for
    data points. For each data point we predict scores for each of the C
    classes, and assign each data point to the class with the highest score.

    Inputs:
    - X: A numpy array of shape (N, D) giving N D-dimensional data points to
      classify.

    Returns:
    - y_pred: A numpy array of shape (N,) giving predicted labels for each of
      the elements of X. For all i, y_pred[i] = c means that X[i] is predicted
      to have class c, where 0 <= c < C.
    """
    y_pred = None

    # ================================================================ #
    # YOUR CODE HERE:
```

```
        #   Predict the class given the input data.
        # ================================================================ #
        h1 = np.maximum([0], np.matmul(X, self.params['W1'].T) + self.params['b1'])
        scores = (np.matmul(h1, self.params['W2'].T) + self.params['b2'])
        y_pred = np.argmax(scores, axis=1)


        # ================================================================ #
        # END YOUR CODE HERE
        # ================================================================ #

        return y_pred
```

Fully connected networks

In the previous notebook, you implemented a simple two-layer neural network class. However, this class is not modular. If you wanted to change the number of layers, you would need to write a new loss and gradient function. If you wanted to optimize the network with different optimizers, you'd need to write new training functions. If you wanted to incorporate regularizations, you'd have to modify the loss and gradient function.

Instead of having to modify functions each time, for the rest of the class, we'll work in a more modular framework where we define forward and backward layers that calculate losses and gradients respectively. Since the forward and backward layers share intermediate values that are useful for calculating both the loss and the gradient, we'll also have these function return "caches" which store useful intermediate values.

The goal is that through this modular design, we can build different sized neural networks for various applications.

In this HW #3, we'll define the basic architecture, and in HW #4, we'll build on this framework to implement different optimizers and regularizations (like BatchNorm and Dropout).

CS231n has built a solid API for building these modular frameworks and training them, and we will use their very well implemented framework as opposed to "reinventing the wheel." This includes using their Solver, various utility functions, and their layer structure. This also includes nndl.fc_net, nndl.layers, and nndl.layer_utils. As in prior assignments, we thank Serena Yeung & Justin Johnson for permission to use code written for the CS 231n class (cs231n.stanford.edu).

Modular layers

This notebook will build modular layers in the following manner. First, there will be a forward pass for a given layer with inputs (`x`) and return the output of that layer (`out`) as well as cached variables (`cache`) that will be used to calculate the gradient in the backward pass.

```
def layer_forward(x, w):
  """ Receive inputs x and weights w """
  # Do some computations ...
  z = # ... some intermediate value
  # Do some more computations ...
  out = # the output

  cache = (x, w, z, out) # Values we need to compute gradients

  return out, cache
```

The backward pass will receive upstream derivatives and the `cache` object, and will return gradients with respect to the inputs and weights, like this:

```
def layer_backward(dout, cache):
  """
  Receive derivative of loss with respect to outputs and cache,
  and compute derivative with respect to inputs.
  """
  # Unpack cache values
  x, w, z, out = cache

  # Use values in cache to compute derivatives
  dx = # Derivative of loss with respect to x
  dw = # Derivative of loss with respect to w

  return dx, dw
```

In [1]:
```
## Import and setups

import time
import numpy as np
import matplotlib.pyplot as plt
from nndl.fc_net import *
from cs231n.data_utils import get_CIFAR10_data
from cs231n.gradient_check import eval_numerical_gradient, eval_numerical_gradien
t_array
from cs231n.solver import Solver

%matplotlib inline
plt.rcParams['figure.figsize'] = (10.0, 8.0) # set default size of plots
plt.rcParams['image.interpolation'] = 'nearest'
plt.rcParams['image.cmap'] = 'gray'

# for auto-reloading external modules
# see http://stackoverflow.com/questions/1907993/autoreload-of-modules-in-ipython
%load_ext autoreload
%autoreload 2

def rel_error(x, y):
  """ returns relative error """
  return np.max(np.abs(x - y) / (np.maximum(1e-8, np.abs(x) + np.abs(y))))
```

In [2]:
```
# Load the (preprocessed) CIFAR10 data.

data = get_CIFAR10_data()
for k in data.keys():
  print('{}: {} '.format(k, data[k].shape))
```

```
X_train: (49000, 3, 32, 32)
y_train: (49000,)
X_val: (1000, 3, 32, 32)
y_val: (1000,)
X_test: (1000, 3, 32, 32)
y_test: (1000,)
```

Linear layers

In this section, we'll implement the forward and backward pass for the linear layers.

The linear layer forward pass is the function `affine_forward` in `nndl/layers.py` and the backward pass is `affine_backward`.

After you have implemented these, test your implementation by running the cell below.

Affine layer forward pass

Implement `affine_forward` and then test your code by running the following cell.

In [3]:
```
# Test the affine_forward function

num_inputs = 2
input_shape = (4, 5, 6)
output_dim = 3

input_size = num_inputs * np.prod(input_shape)
weight_size = output_dim * np.prod(input_shape)

x = np.linspace(-0.1, 0.5, num=input_size).reshape(num_inputs, *input_shape)
w = np.linspace(-0.2, 0.3, num=weight_size).reshape(np.prod(input_shape), output_
dim)
b = np.linspace(-0.3, 0.1, num=output_dim)

out, _ = affine_forward(x, w, b)
correct_out = np.array([[ 1.49834967,  1.70660132,  1.91485297],
                        [ 3.25553199,  3.5141327,   3.77273342]])

# Compare your output with ours. The error should be around 1e-9.
print('Testing affine_forward function:')
print('difference: {}'.format(rel_error(out, correct_out)))
```

```
Testing affine_forward function:
difference: 9.769847728806635e-10
```

Affine layer backward pass

Implement `affine_backward` and then test your code by running the following cell.

In [4]:
```
# Test the affine_backward function

x = np.random.randn(10, 2, 3)
w = np.random.randn(6, 5)
b = np.random.randn(5)
dout = np.random.randn(10, 5)

dx_num = eval_numerical_gradient_array(lambda x: affine_forward(x, w, b)[0], x, d
out)
dw_num = eval_numerical_gradient_array(lambda w: affine_forward(x, w, b)[0], w, d
out)
db_num = eval_numerical_gradient_array(lambda b: affine_forward(x, w, b)[0], b, d
out)

_, cache = affine_forward(x, w, b)
dx, dw, db = affine_backward(dout, cache)

# The error should be around 1e-10
print('Testing affine_backward function:')
print('dx error: {}'.format(rel_error(dx_num, dx)))
print('dw error: {}'.format(rel_error(dw_num, dw)))
print('db error: {}'.format(rel_error(db_num, db)))
```

```
Testing affine_backward function:
dx error: 3.3169125199695287e-10
dw error: 5.139876789248612e-09
db error: 1.41777744374236e-11
```

Activation layers

In this section you'll implement the ReLU activation.

ReLU forward pass

Implement the `relu_forward` function in `nndl/layers.py` and then test your code by running the following cell.

In [5]:
```
# Test the relu_forward function

x = np.linspace(-0.5, 0.5, num=12).reshape(3, 4)

out, _ = relu_forward(x)
correct_out = np.array([[ 0.,          0.,          0.,          0.,          ],
                        [ 0.,          0.,          0.04545455,  0.13636364,],
                        [ 0.22727273,  0.31818182,  0.40909091,  0.5,         ]])

# Compare your output with ours. The error should be around 1e-8
print('Testing relu_forward function:')
print('difference: {}'.format(rel_error(out, correct_out)))
```

```
Testing relu_forward function:
difference: 4.999999798022158e-08
```

ReLU backward pass

Implement the `relu_backward` function in `nndl/layers.py` and then test your code by running the following cell.

In [6]:
```
x = np.random.randn(10, 10)
dout = np.random.randn(*x.shape)

dx_num = eval_numerical_gradient_array(lambda x: relu_forward(x)[0], x, dout)

_, cache = relu_forward(x)
dx = relu_backward(dout, cache)

# The error should be around 1e-12
print('Testing relu_backward function:')
print('dx error: {}'.format(rel_error(dx_num, dx)))
```

```
Testing relu_backward function:
dx error: 3.2756276016727212e-12
```

Combining the affine and ReLU layers

Often times, an affine layer will be followed by a ReLU layer. So let's make one that puts them together. Layers that are combined are stored in `nndl/layer_utils.py` .

Affine-ReLU layers

We've implemented `affine_relu_forward()` and `affine_relu_backward` in `nndl/layer_utils.py` . Take a look at them to make sure you understand what's going on. Then run the following cell to ensure its implemented correctly.

In [7]:
```
from nndl.layer_utils import affine_relu_forward, affine_relu_backward

x = np.random.randn(2, 3, 4)
w = np.random.randn(12, 10)
b = np.random.randn(10)
dout = np.random.randn(2, 10)

out, cache = affine_relu_forward(x, w, b)
dx, dw, db = affine_relu_backward(dout, cache)

dx_num = eval_numerical_gradient_array(lambda x: affine_relu_forward(x, w, b)[0],
x, dout)
dw_num = eval_numerical_gradient_array(lambda w: affine_relu_forward(x, w, b)[0],
w, dout)
db_num = eval_numerical_gradient_array(lambda b: affine_relu_forward(x, w, b)[0],
b, dout)

print('Testing affine_relu_forward and affine_relu_backward:')
print('dx error: {}'.format(rel_error(dx_num, dx)))
print('dw error: {}'.format(rel_error(dw_num, dw)))
print('db error: {}'.format(rel_error(db_num, db)))
```

```
Testing affine_relu_forward and affine_relu_backward:
dx error: 4.99233915283765e-11
dw error: 2.418136184793135e-10
db error: 7.826667964925008e-12
```

Softmax and SVM losses

You've already implemented these, so we have written these in `layers.py` . The following code will ensure they are working correctly.

In [8]:
```
num_classes, num_inputs = 10, 50
x = 0.001 * np.random.randn(num_inputs, num_classes)
y = np.random.randint(num_classes, size=num_inputs)

dx_num = eval_numerical_gradient(lambda x: svm_loss(x, y)[0], x, verbose=False)
loss, dx = svm_loss(x, y)

# Test svm_loss function. Loss should be around 9 and dx error should be 1e-9
print('Testing svm_loss:')
print('loss: {}'.format(loss))
print('dx error: {}'.format(rel_error(dx_num, dx)))

dx_num = eval_numerical_gradient(lambda x: softmax_loss(x, y)[0], x, verbose=False)
loss, dx = softmax_loss(x, y)

# Test softmax_loss function. Loss should be 2.3 and dx error should be 1e-8
print('\nTesting softmax_loss:')
print('loss: {}'.format(loss))
print('dx error: {}'.format(rel_error(dx_num, dx)))
```

```
Testing svm_loss:
loss: 8.999065712042222
dx error: 3.038735505103329e-09

Testing softmax_loss:
loss: 2.302492155795764
dx error: 1.0193717168937447e-08
```

Implementation of a two-layer NN

In `nndl/fc_net.py`, implement the class `TwoLayerNet` which uses the layers you made here. When you have finished, the following cell will test your implementation.

In [9]:

```
N, D, H, C = 3, 5, 50, 7
X = np.random.randn(N, D)
y = np.random.randint(C, size=N)

std = 1e-2
model = TwoLayerNet(input_dim=D, hidden_dims=H, num_classes=C, weight_scale=std)

print('Testing initialization ... ')
W1_std = abs(model.params['W1'].std() - std)
b1 = model.params['b1']
W2_std = abs(model.params['W2'].std() - std)
b2 = model.params['b2']
assert W1_std < std / 10, 'First layer weights do not seem right'
assert np.all(b1 == 0), 'First layer biases do not seem right'
assert W2_std < std / 10, 'Second layer weights do not seem right'
assert np.all(b2 == 0), 'Second layer biases do not seem right'

print('Testing test-time forward pass ... ')
model.params['W1'] = np.linspace(-0.7, 0.3, num=D*H).reshape(D, H)
model.params['b1'] = np.linspace(-0.1, 0.9, num=H)
model.params['W2'] = np.linspace(-0.3, 0.4, num=H*C).reshape(H, C)
model.params['b2'] = np.linspace(-0.9, 0.1, num=C)
X = np.linspace(-5.5, 4.5, num=N*D).reshape(D, N).T
scores = model.loss(X)
correct_scores = np.asarray(
  [[11.53165108,  12.2917344,   13.05181771,  13.81190102,  14.57198434, 15.33206
765,  16.09215096],
   [12.05769098,  12.74614105,  13.43459113,  14.1230412,   14.81149128, 15.49994
135,  16.18839143],
   [12.58373087,  13.20054771,  13.81736455,  14.43418138,  15.05099822, 15.66781
506,  16.2846319 ]])
scores_diff = np.abs(scores - correct_scores).sum()
assert scores_diff < 1e-6, 'Problem with test-time forward pass'

print('Testing training loss (no regularization)')
y = np.asarray([0, 5, 1])
loss, grads = model.loss(X, y)
correct_loss = 3.4702243556
assert abs(loss - correct_loss) < 1e-10, 'Problem with training-time loss'

model.reg = 1.0
loss, grads = model.loss(X, y)
correct_loss = 26.5948426952
assert abs(loss - correct_loss) < 1e-10, 'Problem with regularization loss'

for reg in [0.0, 0.7]:
  print('Running numeric gradient check with reg = {}'.format(reg))
  model.reg = reg
  loss, grads = model.loss(X, y)

  for name in sorted(grads):
    f = lambda _: model.loss(X, y)[0]
    grad_num = eval_numerical_gradient(f, model.params[name], verbose=False)
    print('{} relative error: {}'.format(name, rel_error(grad_num, grads[name])))
```

```
Testing initialization ...
Testing test-time forward pass ...
Testing training loss (no regularization)
Running numeric gradient check with reg = 0.0
W1 relative error: 1.2165499269182414e-08
W2 relative error: 3.4803693682531243e-10
b1 relative error: 6.5485474139109215e-09
b2 relative error: 4.3291413857436005e-10
Running numeric gradient check with reg = 0.7
W1 relative error: 8.175466200078585e-07
W2 relative error: 2.8508696990815807e-08
b1 relative error: 1.0895946645012713e-09
b2 relative error: 9.089615724390711e-10
```

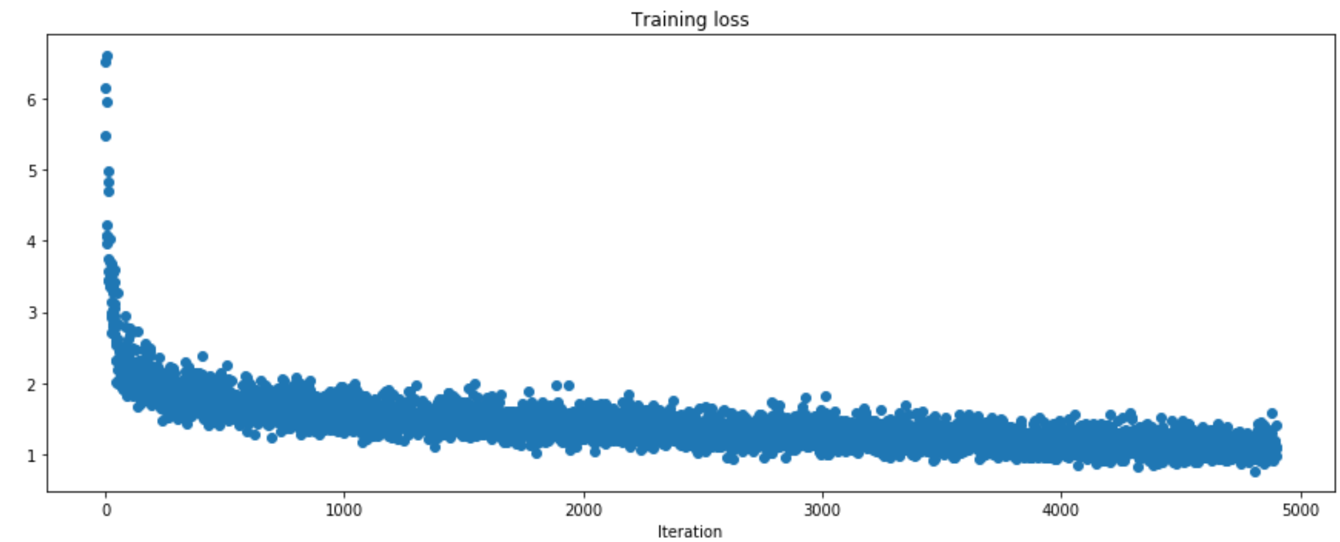
Solver

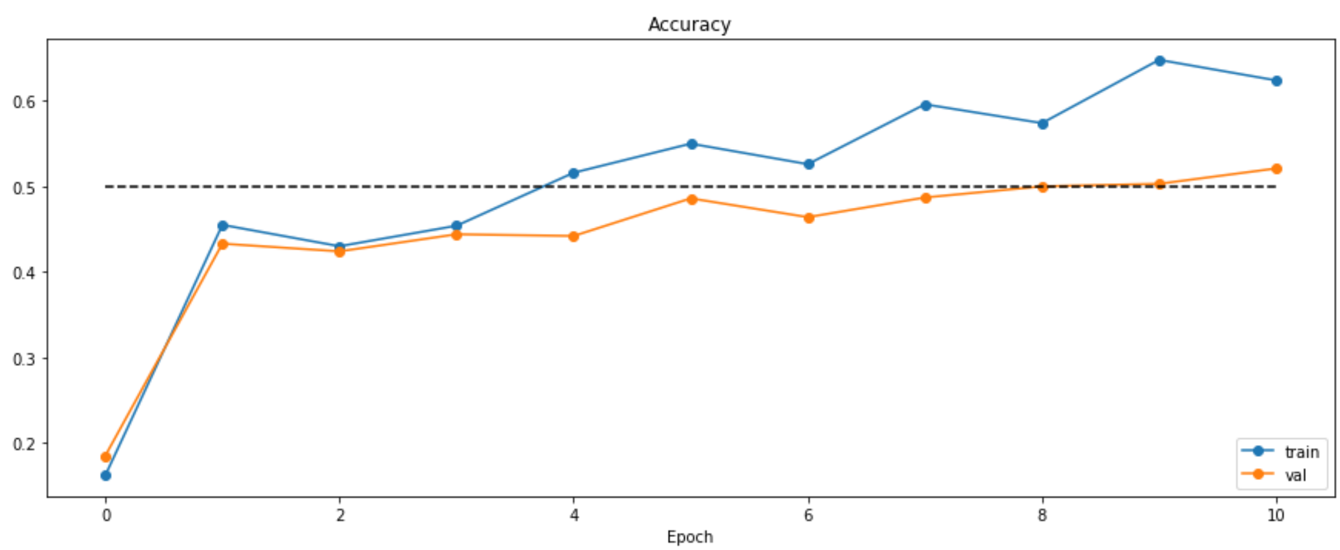
We will now use the cs231n Solver class to train these networks. Familiarize yourself with the API in `cs231n/solver.py`. After you have done so, declare an instance of a TwoLayerNet with 200 units and then train it with the Solver. Choose parameters so that your validation accuracy is at least 40%.

In [10]:
```
model = TwoLayerNet()
solver = None

# ================================================================ #
# YOUR CODE HERE:
#   Declare an instance of a TwoLayerNet and then train
#   it with the Solver. Choose hyperparameters so that your validation
#   accuracy is at least 40%.  We won't have you optimize this further
#   since you did it in the previous notebook.
#
# ================================================================ #
model = TwoLayerNet(input_dim=3072, hidden_dims=200, num_classes=10, weight_scale
=std)
solver = Solver(model, data,
                    update_rule='sgd',
                    optim_config={
                      'learning_rate': 1e-3,
                    },
                    lr_decay=0.95,
                    num_epochs=10, batch_size=100,
                    print_every=100)
solver.train()
# ================================================================ #
# END YOUR CODE HERE
# ================================================================ #
```

```
(Iteration 1 / 4900) loss: 5.483668
(Epoch 0 / 10) train acc: 0.162000; val_acc: 0.185000
(Iteration 101 / 4900) loss: 2.697305
(Iteration 201 / 4900) loss: 2.134175
(Iteration 301 / 4900) loss: 1.751539
(Iteration 401 / 4900) loss: 1.758020
(Epoch 1 / 10) train acc: 0.455000; val_acc: 0.433000
(Iteration 501 / 4900) loss: 1.576128
(Iteration 601 / 4900) loss: 1.572652
(Iteration 701 / 4900) loss: 1.549730
(Iteration 801 / 4900) loss: 1.478327
(Iteration 901 / 4900) loss: 1.593336
(Epoch 2 / 10) train acc: 0.430000; val_acc: 0.424000
(Iteration 1001 / 4900) loss: 1.573852
(Iteration 1101 / 4900) loss: 1.347902
(Iteration 1201 / 4900) loss: 1.560368
(Iteration 1301 / 4900) loss: 1.396232
(Iteration 1401 / 4900) loss: 1.738698
(Epoch 3 / 10) train acc: 0.454000; val_acc: 0.444000
(Iteration 1501 / 4900) loss: 1.561006
(Iteration 1601 / 4900) loss: 1.561194
(Iteration 1701 / 4900) loss: 1.344483
(Iteration 1801 / 4900) loss: 1.030891
(Iteration 1901 / 4900) loss: 1.473241
(Epoch 4 / 10) train acc: 0.516000; val_acc: 0.442000
(Iteration 2001 / 4900) loss: 1.211683
(Iteration 2101 / 4900) loss: 1.409100
(Iteration 2201 / 4900) loss: 1.153804
(Iteration 2301 / 4900) loss: 1.248332
(Iteration 2401 / 4900) loss: 1.396332
(Epoch 5 / 10) train acc: 0.550000; val_acc: 0.486000
(Iteration 2501 / 4900) loss: 1.334879
(Iteration 2601 / 4900) loss: 1.313041
(Iteration 2701 / 4900) loss: 1.277222
(Iteration 2801 / 4900) loss: 1.357774
(Iteration 2901 / 4900) loss: 1.542483
(Epoch 6 / 10) train acc: 0.526000; val_acc: 0.464000
(Iteration 3001 / 4900) loss: 1.200587
(Iteration 3101 / 4900) loss: 1.165113
(Iteration 3201 / 4900) loss: 1.297102
(Iteration 3301 / 4900) loss: 1.207805
(Iteration 3401 / 4900) loss: 1.173673
(Epoch 7 / 10) train acc: 0.596000; val_acc: 0.487000
(Iteration 3501 / 4900) loss: 1.017768
(Iteration 3601 / 4900) loss: 1.329620
(Iteration 3701 / 4900) loss: 1.046318
(Iteration 3801 / 4900) loss: 1.355951
(Iteration 3901 / 4900) loss: 1.172087
(Epoch 8 / 10) train acc: 0.574000; val_acc: 0.500000
(Iteration 4001 / 4900) loss: 1.143176
(Iteration 4101 / 4900) loss: 1.251018
(Iteration 4201 / 4900) loss: 1.287906
(Iteration 4301 / 4900) loss: 1.341894
(Iteration 4401 / 4900) loss: 1.353643
(Epoch 9 / 10) train acc: 0.648000; val_acc: 0.503000
(Iteration 4501 / 4900) loss: 1.297520
(Iteration 4601 / 4900) loss: 1.167055
(Iteration 4701 / 4900) loss: 1.061436
(Iteration 4801 / 4900) loss: 1.116013
(Epoch 10 / 10) train acc: 0.624000; val_acc: 0.521000
```

In [11]:
```
# Run this cell to visualize training loss and train / val accuracy

plt.subplot(2, 1, 1)
plt.title('Training loss')
plt.plot(solver.loss_history, 'o')
plt.xlabel('Iteration')

plt.subplot(2, 1, 2)
plt.title('Accuracy')
plt.plot(solver.train_acc_history, '-o', label='train')
plt.plot(solver.val_acc_history, '-o', label='val')
plt.plot([0.5] * len(solver.val_acc_history), 'k--')
plt.xlabel('Epoch')
plt.legend(loc='lower right')
plt.gcf().set_size_inches(15, 12)
plt.show()
```





Multilayer Neural Network

Now, we implement a multi-layer neural network.

Read through the `FullyConnectedNet` class in the file `nndl/fc_net.py`.

Implement the initialization, the forward pass, and the backward pass. There will be lines for batchnorm and dropout layers and caches; ignore these all for now. That'll be in assignment #4.

In [12]:

```
N, D, H1, H2, C = 2, 15, 20, 30, 10
X = np.random.randn(N, D)
y = np.random.randint(C, size=(N,))

for reg in [0, 3.14]:
  print('Running check with reg = {}'.format(reg))
  model = FullyConnectedNet([H1, H2], input_dim=D, num_classes=C,
                            reg=reg, weight_scale=5e-2, dtype=np.float64)

  loss, grads = model.loss(X, y)
  print('Initial loss: {}'.format(loss))

  for name in sorted(grads):
    f = lambda _: model.loss(X, y)[0]
    grad_num = eval_numerical_gradient(f, model.params[name], verbose=False, h=1e
-5)
    print('{} relative error: {}'.format(name, rel_error(grad_num, grads[name])))
```

```
Running check with reg = 0
Initial loss: 2.2986902624129564
W1 relative error: 9.708644845613296e-07
W2 relative error: 5.691782209490433e-07
W3 relative error: 1.1268942113991501e-06
b1 relative error: 2.557585948010699e-08
b2 relative error: 3.8963675675300106e-09
b3 relative error: 8.723315712906031e-11
Running check with reg = 3.14
Initial loss: 6.714600383300482
W1 relative error: 1.7067197348135888e-08
W2 relative error: 3.699704194427063e-07
W3 relative error: 6.044169713541609e-08
b1 relative error: 1.0055050409696914e-07
b2 relative error: 5.984241655529677e-09
b3 relative error: 2.422838711903739e-10
```

In [13]:

```
# Use the three layer neural network to overfit a small dataset.

num_train = 50
small_data = {
  'X_train': data['X_train'][:num_train],
  'y_train': data['y_train'][:num_train],
  'X_val': data['X_val'],
  'y_val': data['y_val'],
}


#### !!!!!!
# Play around with the weight_scale and learning_rate so that you can overfit a s
mall dataset.
# Your training accuracy should be 1.0 to receive full credit on this part.
weight_scale = 1e-2
learning_rate = 1e-2

model = FullyConnectedNet([100, 100],
              weight_scale=weight_scale, dtype=np.float64)
solver = Solver(model, small_data,
                print_every=10, num_epochs=20, batch_size=25,
                update_rule='sgd',
                optim_config={
                  'learning_rate': learning_rate,
                }
         )
solver.train()

plt.plot(solver.loss_history, 'o')
plt.title('Training loss history')
plt.xlabel('Iteration')
plt.ylabel('Training loss')
plt.show()
```

```
(Iteration 1 / 40) loss: 2.280087
(Epoch 0 / 20) train acc: 0.220000; val_acc: 0.102000
(Epoch 1 / 20) train acc: 0.360000; val_acc: 0.161000
(Epoch 2 / 20) train acc: 0.480000; val_acc: 0.150000
(Epoch 3 / 20) train acc: 0.420000; val_acc: 0.096000
(Epoch 4 / 20) train acc: 0.740000; val_acc: 0.192000
(Epoch 5 / 20) train acc: 0.600000; val_acc: 0.185000
(Iteration 11 / 40) loss: 1.210686
(Epoch 6 / 20) train acc: 0.760000; val_acc: 0.198000
(Epoch 7 / 20) train acc: 0.760000; val_acc: 0.176000
(Epoch 8 / 20) train acc: 0.760000; val_acc: 0.185000
(Epoch 9 / 20) train acc: 0.820000; val_acc: 0.206000
(Epoch 10 / 20) train acc: 0.940000; val_acc: 0.197000
(Iteration 21 / 40) loss: 0.354865
(Epoch 11 / 20) train acc: 0.960000; val_acc: 0.199000
(Epoch 12 / 20) train acc: 0.980000; val_acc: 0.195000
(Epoch 13 / 20) train acc: 1.000000; val_acc: 0.199000
(Epoch 14 / 20) train acc: 1.000000; val_acc: 0.208000
(Epoch 15 / 20) train acc: 1.000000; val_acc: 0.199000
(Iteration 31 / 40) loss: 0.045299
(Epoch 16 / 20) train acc: 0.980000; val_acc: 0.182000
(Epoch 17 / 20) train acc: 0.920000; val_acc: 0.168000
(Epoch 18 / 20) train acc: 0.980000; val_acc: 0.177000
(Epoch 19 / 20) train acc: 1.000000; val_acc: 0.193000
(Epoch 20 / 20) train acc: 1.000000; val_acc: 0.203000
```

In [ ]:
```
import numpy as np

from .layers import *
from .layer_utils import *

"""
This code was originally written for CS 231n at Stanford University
(cs231n.stanford.edu).  It has been modified in various areas for use in the
ECE 239AS class at UCLA.  This includes the descriptions of what code to
implement as well as some slight potential changes in variable names to be
consistent with class nomenclature.  We thank Justin Johnson & Serena Yeung for
permission to use this code.  To see the original version, please visit
cs231n.stanford.edu.
"""

class TwoLayerNet(object):
  """
  A two-layer fully-connected neural network with ReLU nonlinearity and
  softmax loss that uses a modular layer design. We assume an input dimension
  of D, a hidden dimension of H, and perform classification over C classes.

  The architecure should be affine - relu - affine - softmax.

  Note that this class does not implement gradient descent; instead, it
  will interact with a separate Solver object that is responsible for running
  optimization.

  The learnable parameters of the model are stored in the dictionary
  self.params that maps parameter names to numpy arrays.
  """

  def __init__(self, input_dim=3*32*32, hidden_dims=100, num_classes=10,
               dropout=0, weight_scale=1e-3, reg=0.0):
    """
    Initialize a new network.

    Inputs:
    - input_dim: An integer giving the size of the input
    - hidden_dims: An integer giving the size of the hidden layer
    - num_classes: An integer giving the number of classes to classify
    - dropout: Scalar between 0 and 1 giving dropout strength.
    - weight_scale: Scalar giving the standard deviation for random
      initialization of the weights.
    - reg: Scalar giving L2 regularization strength.
    """
    self.params = {}
    self.reg = reg

    # ================================================================ #
    # YOUR CODE HERE:
    #   Initialize W1, W2, b1, and b2.  Store these as self.params['W1'],
    #   self.params['W2'], self.params['b1'] and self.params['b2']. The
    #   biases are initialized to zero and the weights are initialized
    #   so that each parameter has mean 0 and standard deviation weight_scale.
    #   The dimensions of W1 should be (input_dim, hidden_dim) and the
    #   dimensions of W2 should be (hidden_dims, num_classes)
    # ================================================================ #
    self.params['W1'] = np.random.normal(0, weight_scale, (input_dim, hidden_dims
))
    self.params['b1'] = np.zeros(hidden_dims)
    self.params['W2'] = np.random.normal(0, weight_scale, (hidden_dims, num_class
```

```
es))
    self.params['b2'] = np.zeros(num_classes)
    # ================================================================ #
    # END YOUR CODE HERE
    # ================================================================ #

  def loss(self, X, y=None):
    """
    Compute loss and gradient for a minibatch of data.

    Inputs:
    - X: Array of input data of shape (N, d_1, ..., d_k)
    - y: Array of labels, of shape (N,). y[i] gives the label for X[i].

    Returns:
    If y is None, then run a test-time forward pass of the model and return:
    - scores: Array of shape (N, C) giving classification scores, where
      scores[i, c] is the classification score for X[i] and class c.

    If y is not None, then run a training-time forward and backward pass and
    return a tuple of:
    - loss: Scalar value giving the loss
    - grads: Dictionary with the same keys as self.params, mapping parameter
      names to gradients of the loss with respect to those parameters.
    """
    scores = None

    # ================================================================ #
    # YOUR CODE HERE:
    #   Implement the forward pass of the two-layer neural network. Store
    #   the class scores as the variable 'scores'.  Be sure to use the layers
    #   you prior implemented.
    # ================================================================ #
    h1, h1_cache = affine_relu_forward(X, self.params['W1'], self.params['b1'])
    scores, scores_cache = affine_forward(h1, self.params['W2'], self.params['b2'
])
    # ================================================================ #
    # END YOUR CODE HERE
    # ================================================================ #

    # If y is None then we are in test mode so just return scores
    if y is None:
      return scores

    loss, grads = 0, {}
    # ================================================================ #
    # YOUR CODE HERE:
    #   Implement the backward pass of the two-layer neural net.  Store
    #   the loss as the variable 'loss' and store the gradients in the
    #   'grads' dictionary.  For the grads dictionary, grads['W1'] holds
    #   the gradient for W1, grads['b1'] holds the gradient for b1, etc.
    #   i.e., grads[k] holds the gradient for self.params[k].
    #
    #   Add L2 regularization, where there is an added cost 0.5*self.reg*W^2
    #   for each W.  Be sure to include the 0.5 multiplying factor to
    #   match our implementation.
    #
    #   And be sure to use the layers you prior implemented.
    # ================================================================ #
    loss, dl_da2 = softmax_loss(scores, y)
    loss += 0.5 * self.reg * (np.sum(self.params['W1'] ** 2) + np.sum(self.params
['W2'] ** 2))
```

```
    dl_dh1, grads['W2'], grads['b2'] = affine_backward(dl_da2, scores_cache)
    dl_dx, grads['W1'], grads['b1'] = affine_relu_backward(dl_dh1, h1_cache)

    grads['W2'] += self.reg * self.params['W2'] # add to address the regularizati
on from gradient of loss wrt W2
    grads['W1'] += self.reg * self.params['W1'] # add to address the regularizati
on from gradient of loss wrt W1
    # ================================================================ #
    # END YOUR CODE HERE
    # ================================================================ #

    return loss, grads


class FullyConnectedNet(object):
  """
  A fully-connected neural network with an arbitrary number of hidden layers,
  ReLU nonlinearities, and a softmax loss function. This will also implement
  dropout and batch normalization as options. For a network with L layers,
  the architecture will be

  {affine - [batch norm] - relu - [dropout]} x (L - 1) - affine - softmax

  where batch normalization and dropout are optional, and the {...} block is
  repeated L - 1 times.

  Similar to the TwoLayerNet above, learnable parameters are stored in the
  self.params dictionary and will be learned using the Solver class.
  """

  def __init__(self, hidden_dims, input_dim=3*32*32, num_classes=10,
               dropout=0, use_batchnorm=False, reg=0.0,
               weight_scale=1e-2, dtype=np.float32, seed=None):
    """
    Initialize a new FullyConnectedNet.

    Inputs:
    - hidden_dims: A list of integers giving the size of each hidden layer.
    - input_dim: An integer giving the size of the input.
    - num_classes: An integer giving the number of classes to classify.
    - dropout: Scalar between 0 and 1 giving dropout strength. If dropout=0 then
      the network should not use dropout at all.
    - use_batchnorm: Whether or not the network should use batch normalization.
    - reg: Scalar giving L2 regularization strength.
    - weight_scale: Scalar giving the standard deviation for random
      initialization of the weights.
    - dtype: A numpy datatype object; all computations will be performed using
      this datatype. float32 is faster but less accurate, so you should use
      float64 for numeric gradient checking.
    - seed: If not None, then pass this random seed to the dropout layers. This
      will make the dropout layers deteriminstic so we can gradient check the
      model.
    """
    self.use_batchnorm = use_batchnorm
    self.use_dropout = dropout > 0
    self.reg = reg
    self.num_layers = 1 + len(hidden_dims)
    self.dtype = dtype
    self.params = {}

    # ================================================================ #
```

```
    # YOUR CODE HERE:
    #   Initialize all parameters of the network in the self.params dictionary.
    #   The weights and biases of layer 1 are W1 and b1; and in general the
    #   weights and biases of layer i are Wi and bi. The
    #   biases are initialized to zero and the weights are initialized
    #   so that each parameter has mean 0 and standard deviation weight_scale.
    # ================================================================ #
    all_dims = [input_dim] + hidden_dims + [num_classes]

    for layer in range(self.num_layers):
      self.params['W{}'.format(layer + 1)] = np.random.normal(0, weight_scale, (a
ll_dims[layer], all_dims[layer + 1]))
      self.params['b{}'.format(layer + 1)] = np.zeros(all_dims[layer + 1])
    # ================================================================ #
    # END YOUR CODE HERE
    # ================================================================ #

    # When using dropout we need to pass a dropout_param dictionary to each
    # dropout layer so that the layer knows the dropout probability and the mode
    # (train / test). You can pass the same dropout_param to each dropout layer.
    self.dropout_param = {}
    if self.use_dropout:
      self.dropout_param = {'mode': 'train', 'p': dropout}
      if seed is not None:
        self.dropout_param['seed'] = seed

    # With batch normalization we need to keep track of running means and
    # variances, so we need to pass a special bn_param object to each batch
    # normalization layer. You should pass self.bn_params[0] to the forward pass
    # of the first batch normalization layer, self.bn_params[1] to the forward
    # pass of the second batch normalization layer, etc.
    self.bn_params = []
    if self.use_batchnorm:
      self.bn_params = [{'mode': 'train'} for i in np.arange(self.num_layers - 1
)]

    # Cast all parameters to the correct datatype
    for k, v in self.params.items():
      self.params[k] = v.astype(dtype)


  def loss(self, X, y=None):
    """
    Compute loss and gradient for the fully-connected net.

    Input / output: Same as TwoLayerNet above.
    """
    X = X.astype(self.dtype)
    mode = 'test' if y is None else 'train'

    # Set train/test mode for batchnorm params and dropout param since they
    # behave differently during training and testing.
    if self.dropout_param is not None:
      self.dropout_param['mode'] = mode
    if self.use_batchnorm:
      for bn_param in self.bn_params:
        bn_param[mode] = mode

    scores = None

    # ================================================================ #
    # YOUR CODE HERE:
```

```
    #   Implement the forward pass of the FC net and store the output
    #   scores as the variable "scores".
    # ================================================================ #
    a = {}
    h = {}
    h[0] = [X]
    for layer in range(self.num_layers - 1):
      a[layer + 1] = affine_forward(h[layer][0], self.params['W{}'.format(layer +
1)],  self.params['b{}'.format(layer + 1)])
      h[layer + 1] = relu_forward(a[layer+1][0])

    a[self.num_layers] = affine_forward(h[self.num_layers - 1][0], self.params['W
{}'.format(self.num_layers)],  self.params['b{}'.format(self.num_layers)])
    scores = a[self.num_layers][0]
    # ================================================================ #
    # END YOUR CODE HERE
    # ================================================================ #

    # If test mode return early
    if mode == 'test':
      return scores

    loss, grads = 0.0, {}
    # ================================================================ #
    # YOUR CODE HERE:
    #   Implement the backwards pass of the FC net and store the gradients
    #   in the grads dict, so that grads[k] is the gradient of self.params[k]
    #   Be sure your L2 regularization includes a 0.5 factor.
    # ================================================================ #
    dl_da = {}
    dl_dh = {}
    dl_dw = {}
    dl_db = {}
    loss, dl_da[self.num_layers] = softmax_loss(scores, y)
    loss += 0.5 * self.reg * np.sum([np.sum(self.params['W{}'.format(layer + 1)]
** 2) for layer in range(self.num_layers)])

    for layer in range(self.num_layers)[self.num_layers:0:-1]:
      dl_dh[layer], dl_dw[layer + 1], dl_db[layer + 1] = affine_backward(dl_da[la
yer + 1], a[layer + 1][1])
      dl_da[layer] = relu_backward(dl_dh[layer], h[layer][1])

    dl_dh[0], dl_dw[0 + 1], dl_db[0 + 1] = affine_backward(dl_da[0 + 1], a[0 + 1]
[1])

    for layer in range(self.num_layers):
      grads['W{}'.format(layer + 1)] = dl_dw[layer + 1] + self.reg * self.params[
'W{}'.format(layer + 1)]
      grads['b{}'.format(layer + 1)] = dl_db[layer + 1]

    # ================================================================ #
    # END YOUR CODE HERE
    # ================================================================ #
    return loss, grads
```

In [ ]:

```python
import numpy as np
import pdb

"""
This code was originally written for CS 231n at Stanford University
(cs231n.stanford.edu).  It has been modified in various areas for use in the
ECE 239AS class at UCLA.  This includes the descriptions of what code to
implement as well as some slight potential changes in variable names to be
consistent with class nomenclature.  We thank Justin Johnson & Serena Yeung for
permission to use this code.  To see the original version, please visit
cs231n.stanford.edu.
"""


def affine_forward(x, w, b):
  """
  Computes the forward pass for an affine (fully-connected) layer.

  The input x has shape (N, d_1, ..., d_k) and contains a minibatch of N
  examples, where each example x[i] has shape (d_1, ..., d_k). We will
  reshape each input into a vector of dimension D = d_1 * ... * d_k, and
  then transform it to an output vector of dimension M.

  Inputs:
  - x: A numpy array containing input data, of shape (N, d_1, ..., d_k)
  - w: A numpy array of weights, of shape (D, M)
  - b: A numpy array of biases, of shape (M,)

  Returns a tuple of:
  - out: output, of shape (N, M)
  - cache: (x, w, b)
  """

  # ================================================================ #
  # YOUR CODE HERE:
  #   Calculate the output of the forward pass.  Notice the dimensions
  #   of w are D x M, which is the transpose of what we did in earlier
  #   assignments.
  # ================================================================ #
  out = np.matmul(x.reshape(x.shape[0], -1), w) + b
  # ================================================================ #
  # END YOUR CODE HERE
  # ================================================================ #

  cache = (x, w, b)
  return out, cache


def affine_backward(dout, cache):
  """
  Computes the backward pass for an affine layer.

  Inputs:
  - dout: Upstream derivative, of shape (N, M)
  - cache: Tuple of:
    - x: Input data, of shape (N, d_1, ... d_k)
    - w: Weights, of shape (D, M)

  Returns a tuple of:
  - dx: Gradient with respect to x, of shape (N, d1, ..., d_k)
  - dw: Gradient with respect to w, of shape (D, M)
```

```
    - db: Gradient with respect to b, of shape (M,)
    """
    x, w, b = cache
    dx, dw, db = None, None, None

    # ========================================================================== #
    # YOUR CODE HERE:
    #   Calculate the gradients for the backward pass.
    # ========================================================================== #
    dx = np.matmul(dout, w.T).reshape(x.shape) # gradient of loss wrt x = W^T * upstre
    dw = np.matmul(x.reshape(x.shape[0], -1).T, dout) # gradient of loss wrt w = upstr
    db = np.sum(dout, axis=0) # gradient of loss wrt b = upstream, summing wrt datapoi
    # ========================================================================== #
    # END YOUR CODE HERE
    # ========================================================================== #

    return dx, dw, db

def relu_forward(x):
    """
    Computes the forward pass for a layer of rectified linear units (ReLUs).

    Input:
    - x: Inputs, of any shape

    Returns a tuple of:
    - out: Output, of the same shape as x
    - cache: x
    """
    # ========================================================================== #
    # YOUR CODE HERE:
    #   Implement the ReLU forward pass.
    # ========================================================================== #
    out = np.maximum(0, x)
    # ========================================================================== #
    # END YOUR CODE HERE
    # ========================================================================== #

    cache = x
    return out, cache


def relu_backward(dout, cache):
    """
    Computes the backward pass for a layer of rectified linear units (ReLUs).

    Input:
    - dout: Upstream derivatives, of any shape
    - cache: Input x, of same shape as dout

    Returns:
    - dx: Gradient with respect to x
    """
    x = cache

    # ========================================================================== #
    # YOUR CODE HERE:
    #   Implement the ReLU backward pass
    # ========================================================================== #
    dx = (x > 0) * dout
    # ========================================================================== #
    # END YOUR CODE HERE
```

```python
    # ======================================================================= #

    return dx

def svm_loss(x, y):
    """
    Computes the loss and gradient using for multiclass SVM classification.

    Inputs:
    - x: Input data, of shape (N, C) where x[i, j] is the score for the jth class
      for the ith input.
    - y: Vector of labels, of shape (N,) where y[i] is the label for x[i] and
      0 <= y[i] < C

    Returns a tuple of:
    - loss: Scalar giving the loss
    - dx: Gradient of the loss with respect to x
    """
    N = x.shape[0]
    correct_class_scores = x[np.arange(N), y]
    margins = np.maximum(0, x - correct_class_scores[:, np.newaxis] + 1.0)
    margins[np.arange(N), y] = 0
    loss = np.sum(margins) / N
    num_pos = np.sum(margins > 0, axis=1)
    dx = np.zeros_like(x)
    dx[margins > 0] = 1
    dx[np.arange(N), y] -= num_pos
    dx /= N
    return loss, dx


def softmax_loss(x, y):
    """
    Computes the loss and gradient for softmax classification.

    Inputs:
    - x: Input data, of shape (N, C) where x[i, j] is the score for the jth class
      for the ith input.
    - y: Vector of labels, of shape (N,) where y[i] is the label for x[i] and
      0 <= y[i] < C

    Returns a tuple of:
    - loss: Scalar giving the loss
    - dx: Gradient of the loss with respect to x
    """

    probs = np.exp(x - np.max(x, axis=1, keepdims=True))
    probs /= np.sum(probs, axis=1, keepdims=True)
    N = x.shape[0]
    loss = -np.sum(np.log(probs[np.arange(N), y])) / N
    dx = probs.copy()
    dx[np.arange(N), y] -= 1
    dx /= N
    return loss, dx
```