```
grammar G;
        A: a;
```

Introduction to theory of languages

Patryk Kiepas

MINES ParisTech
AGH University Of Science and Technology
THALES

March 3, 2017

Course plan

1. Saturday, 25th of February 2017 – lecture
   - Languages
   - Grammars
2. Saturday, 4th of March 2017 – lecture
   - Parsing
   - ANTLR
3. Saturday, 11th of March 2017 – exercises
   - Grammars and languages
   - ANTLR
4. Saturday, 25th of March 2017 – exercises
   - ANTLR
5. Exam

Additional informations

Any questions?

Ask by mail: `kiepas@agh.edu.pl`

Course web-page

`http://home.agh.edu.pl/~kiepas` → **Teaching** → **Introduction to theory of languages (2017)**

Plan of the lecture

1. Introduction
2. Theory
   - Languages
   - Grammar
3. Parsing
   - Methods
   - Tools
4. ANTLR

Introduction

Linguistics

Scientific study of languages. Involves analysis of language:

- *form* – language evolution and task
- *context* – environment of language usage
- *semantics* – the meaning of the language

Some important aspects

- Phonetics
- Articulation
- Perception
- Acoustic features
- Morphology
- Syntax

Language types

1. Natural languages
   - *Ordinary* – evolves naturally in humans without planning
   - *Controlled* – a restricted subset of natural language in order reduce or eliminate ambiguity and complexity
2. **Artificial languages**
   - *Constructed* (planned *a priori* or *a posteriori*)
     - Engineered languages – experiments in *logic*, *philosophy*, *linguistics*
     - Auxiliary languages – international communication (e.g. Esperanto, Ido, Interlingua)
     - Artistic languages – aesthetic pleasure or humorous effect (e.g. Klingon)
   - **Formal**
     - Computer programming languages (e.g. Java, Haskell, C, C++, Ruby)
     - Files and formats descriptions (e.g. YAML, JSON, XML)

Description of natural languages

A really small bit of history

- In the late 1950's Noam Chomsky tried to describe natural languages
- Important paper: *"Three models for the description of language"*, Noam Chomsky (1956).
- In a result of his research two disciplines originated:
  1. **Theory of formal grammars**
  2. *Generative (transformational) grammars*

Figure 1: Professor of Linguistics (Emeritus) at MIT, Cambridge

Description of natural languages

What we know now?

- Description of natural languages is **hard**
- Description of any natural languages might be **impossible**

Why this is important?

- Better understanding of language creation processes
- More insights into functioning of our brain
- **Natural language processing (NLP)**
  - Translations (e.g. Google Translator)
  - Synthesis (e.g. speech generation)
  - Perceiving (e.g. robots, voice-control)

Description of formal languages

Result

Description of natural languages help us describe an artificial (formal) ones

Programming languages

- Protocol for communication with the computer
- Performing operations and computations
- Interpretation and execution
- Compilation
- Static code analysis

Data formats

- Structured data
- Interchangeable model for communication and data transmission

Alphabet

Alphabet

A set Σ of available symbols, the simplest elements in the language

Examples

- binary alphabet $\{0, 1\}$
- decimal numbers $\{0, 1, 2, 3, ..., 9\}$
- Latin alphabet $\{a, b, c, d, ..., z\}$
- Cyrillic

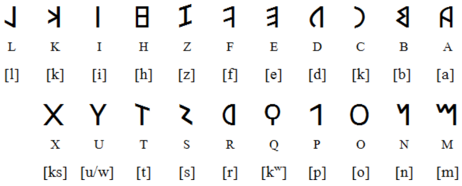

Figure 2: Ancient Latin alphabet

Word (I)

Word

Word w is a sequence of N symbols $w = x_1 x_2 ... x_N$ where $x_i \in \Sigma$ (e.g. 010110, $ABCDAAE$)

Length

Length of word w is a number of symbols it contains $|w| = N$ (e.g. $|010110| = 6$, $|ABCDAAE| = 7$)

Empty word

Special word ϵ with length $|\epsilon| = 0$

Word (II)

Words examples

- $w = 010110$ word over alphabet $\Sigma = \{0, 1\}$
- $w = abc13dj3$ word over alphabet $\Sigma = \{a, b, ...z, 0, 1, ...9\}$
- $w = ACGTCCGGTA$ word over alphabet $\Sigma = \{A, C, G, T\}$

Closures

- Σ^* – set of all words over Σ
- Σ^+ – set of all nonempty words $\Sigma^+ = \Sigma^* \backslash \{\epsilon\}$

Closures examples

- if $\Sigma = \{a\}$ then $\Sigma^* = \{\epsilon, a, aa, aaa, aaaa, aaaaa, aaaaaa, ...\}$
- if $\Sigma = \{a, b\}$ then $\Sigma^+ = \{a, b, aa, bb, ab, ba, aaa, bbb, ...\}$
- if $\Sigma = \{a, b, ..., z\}$ then $\Sigma^+ = \{cat, dog, a, aa, aaa, ...\}$

Language

Definition

Formal language $L \subseteq \Sigma^*$ is a subset of all words built over an alphabet Σ

Examples

- Language L_1 of palindromes in English
  $L_1 = \{mum, hannah, madam, ...\}$
- Morse code with alphabet $\Sigma = \{\cdot, -\}$, $L_2 = \{\cdot -, - \cdot \cdot ..., - - \cdot\cdot\}$
- Empty language
- English language
- Language L_3 with the set of words with fixed-size of N

Grammar

Grammar

- Description of a language
- A recipe for composing elements into sentence
- Describes syntax of a language

Definition

Grammar is a system $G = (V_T, V_N, P, S)$ where:

- V_T – terminals (alphabet Σ)
- V_N – nonterminals
- P – production rules
- S – start symbol (one nonterminal)

Grammar and languages

Grammar properties

- V_N, V_T, P – are finite, nonempty sets
- $V_N \cap V_T = \emptyset$ – are disjoint
- $V = \Sigma \cup NT$ – vocabulary (terminals and nonterminals)
- $P \subseteq V^+ \times V^*$
- $S \in NT$

Grammar and languages

- Sentence generated by some G is every $w \in \Sigma^*$ for each exists derivation from S
- Language $L(G)$ is generated by G and consists of sentences derivate using grammar G
- Two grammars G_1 and G_2 are *(weakly) equivalent* if $L(G_1) = L(G_2)$

Derivations

- $s \implies s' \implies s'' \implies \ldots \implies w$
- $s \overset{*}{\implies} w$

Grammar example

Examples

Digits separated by plus or minus signs

$$list \rightarrow list + list$$
$$list \rightarrow list - list$$
$$list \rightarrow \ 0 \mid 1 \mid 2 \mid 3 \mid 4 \mid 5 \mid 6 \mid 7 \mid 8 \mid 9$$

Chomsky's hierarchy

Hierarchy

- Describe the grammar expressiveness
- Describe the grammar hardness
- Tells us what "mechanical procedure" we need to use in order to:
  - Accept language
  - Generate language
- $\alpha, \beta \in V^*$ – any sequence of terminals and nonterminals
- $\gamma \in V^+$ – any nonempty sequence of terminals and nonterminals
- $A, B \in NT$ – nonterminals
- $a, b \in \Sigma$ – terminals

Grammar	**Language**	**Automaton**	**Production rules**
Type-0	Recursively enumerable	Turing machine	$\alpha \to \beta$
Type-1	Context-sensitive	Linear bounded ND TM	$\alpha A \beta \to \alpha \gamma \beta$
Type-2	Context-free	ND pushdown	$\alpha \to \gamma$
Type-3	Regular	Finite state	$A \to a$ and $A \to aB$

Limiting condition

For all production rules $\forall(\alpha \to \beta) \in P$ it is true:

First condition

- $|\alpha| \leq |\beta|$ - they don't decrease length of a word

Second condition

- $\alpha \in V_N$ is a nonterminal
- $\beta \in V^+$ is not empty

Third condition

- $\alpha \in V_N$ is a nonterminal
- β has a form $\beta = a$ or $\beta = aB$ where $a \in V_T, B \in V_N$

Grammar examples

Grammar

Let $G = (V_N, V_T, P, S)$, where

- $V_N = \{S\}$
- $V_T = \{a, b\}$
- $P = \{S \to aS \vee S \to Sa, S \to b\}$

Derivations

$S \implies aS \implies aaS \implies aaaS \implies aaaaS \implies aaaaaS \implies \ldots$

$S \implies Sa \implies Saa \implies Saaa \implies Saaaa \implies Saaaaa \implies \ldots$

Language

$L(G) = \{a^n b\}$, where $n \geq 0$

Example sentences

$b, ab, aab, aaab, aaaab, aaaaab, aaaaaab, aaaaaaab, aaaaaaaab, \ldots$

Grammar example: *mirror language*

Grammar

Let $G = (V_N, V_T, P, S)$, where

- $V_N = \{S\}$
- $V_T = \{a, b\}$
- $P = \{S \to aSa, S \to bSb, S \to aa, S \to bb\}$

Derivations

$S \implies aSa \implies abSba \implies abbSbbs \implies abbaSabba \implies \ldots$

Language

$L(G) = \{ww^R\}$, where w^R represents reflection of w, and $|w| \geq 1$. This language $L(G)$ is called a *mirror language*.

Example sentences

aa, *bb*, *aaaa*, *abba*, *baab*, *bbbb*, *abaaba*, *baaaab*, *abbbba*, *babbab*, *aaaaaa*...

Grammar example

Grammar

Let $G = (V_N, V_T, P, S)$, where

- $V_N = \{S, E, F\}$
- $V_T = \{a, b, c, d\}$
- $P = \{S \to ESF, S \to EF, E \to ab, F \to cd\}$

Derivations

$S \implies ESF \implies EESFF \implies EEESFFF \implies E^{n-1}SF^{n-1} \implies E^nF^n$

Language

$L(G) = \{(ab)^n(cd)^n\}$, where $n \geq 1$.

Example sentences

$abcd, ababcdcd, abababcdcdcd, ababababcdcdcdcd, \ldots$

Grammar example

Grammar

Let $G = (V_N, V_T, P, S)$, where

- $V_N = \{S, E, F\}$
- $V_T = \{a, b, c, d\}$
- $P = \{S \to ESF, S \to abcd, Ea \to aE, dF \to Fd, Eb \to abb, cF \to ccd\}$

Derivations

$S \implies ESF \implies EabcdF \implies aEbcdF \implies aEbcFd \implies aabbcFd \implies aabbccdd$

Language

$L(G) = \{a^n b^n c^n d^n\}$, where $n \geq 2$.

Sentences

$aabbccdd, aaabbbcccddd, aaaabbbbccccdddd, aaaaabbbbbcccccddddd, \ldots$

Grammar example – regular

Grammar

Let $G = (V_N, V_T, P, S)$, where

- $V_N = \{S, B\}$
- $V_T = \{a, b\}$
- $P = \{S \to aB, B \to bS, B \to b\}$

Derivation

$S \implies aB \implies abS \implies abaB \implies ababS \implies ababaB \implies \ldots$

Language

$L(G) = \{(ab)^n\}$, where $n \geq 1$.

Derivation trees

Also : tree diagrams, phrase markers. For regular and context-free grammars.

Chomsky's Normal Form

Language and grammar

Two common tasks:

- Check if language is legal (accepted by the grammar) – trace all the applicable rules (derive it language from the start symbol) or... use corresponding automaton!
- Generate language from grammar – start from *start symbol*, go through all applicable rules

Backus-Naur form (BNF)

Backus-Naur form (BNF)

Notation technique for *context-free grammars*. Frequently used to describe syntax of *programming languages*, *document formats* etc.

Syntax

`<term> ::= __expression__`

- `<term>` is a *nonterminal*
- `__expression__` is a sequence of one or more terminal and/or nonterminal symbols separated by vertical line `|`
- Terminal symbols: `a`, `b`, `c`, `A`, `0`, `1`, `2` etc.
- Nonterminal symbols: `<digit>`, `<postal-code>` etc.

Backus-Naur form (BNF)

Meta-symbols

- $::=$ – production rule definition
- $|$ – rule alternative
- $<>$ – nonterminals
- "" – literal
- $< EOL >$ – End Of Line

Examples

```
<digit> ::= 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9
<postal-code> ::= <digit> <digit> <digit> <digit> <digit>
```

BNF example : Palindrome

Palindrome grammar

```
<letter>     ::= a | b | c | ... | y | z
<palindrome> ::= <letter> |
<palindrome> ::= a <palindrome> a | b <palindrome> b |
                 c <palindrome> c | d <palindrome> d |
                 e <palindrome> e | ...
                                  | z <palindrome> z
```

Results

a

bb

bab

pop

hannah

BNF example : Postal address

Postal address grammar

```
<postal-address> ::= <name-part> <street-address> <zip-part>
<name-part> ::= <first-name> <last-name> <EOL>
<street-address> ::= <number> <street-name> <apt-num> <EOL>
<zip-part> ::= <postal-code> <town-name> <EOL>
<apt-num> ::= <number> | ""
```

ANTLR v4

Parser generator

content...

ANTLR

A parser generator which allows to:

-

Usages

- Twitter search queries are parsed using ANTLR
- Lex Machina[a] extracts informations from legal texts using ANTLR

[a] lexmachina.com

ANTLR syntax

Syntax	Description
x	Match token, rule or subrule x
$xy....z$	Match a sequence of elements
(...|...|...)	Sub-rule with multiple alternatives
$x?$	Match x or skip it
$x*$	Match x zero or more times
$x+$	Match x one or more times
r : ...	Define rule r
r : (...|...|...)	Define rule r with multiple alternatives

ANTLR patterns

Pattern name	Examples
Sequence	`'[' INT+ ']'`
Sequence with terminator	`(statement ';')*`
Sequence with separator	`( expr (',' expr)*)?`
Choice	`type : 'int' | 'float'`
Token dependency	`ID '[' expr ']'`
Nested phrase	`expr : '(' expr ')' | ID`

First grammar

Simple grammar (Hello.g4)

```
// define a grammar called Hello
grammar Hello;
// match lower-case identifiers
ID : [a-z]+;
// skip spaces, tabs, newlines, \r (Windows)
WS : [ \t\r\n]+ -> skip;
// match keyword hello followed by an identifier
r : 'hello' ID;
```

Nested arrays

Nested arrays grammar (ArrayInit.g4)

```
grammar ArrayInit;
// matches at least one comma-separated value between {...}
init : '{' value (',' value)* '}';
// A value can be either a nested array or an integer (INT)
value : init | INT;
// define token INT as one or more digits
INT : [0-9]+;
WS : [ \t\r\n]+ -> skip;
```

// parser rules start with lowercase letters, lexer rules with uppercase

Parser tester

```
import org.antlr.v4.runtime.*;
import org.antlr.v4.runtime.tree.*;

public class Test {
    public static void main(String[] args) throws Exception {
        // create a CharStream that reads from standard input
        ANTLRInputStream input = new ANTLRInputStream(System.in);
        // create a lexer that feeds off of input
        CharStream ArrayInitLexer lexer = new ArrayInitLexer(input);
        // create a buffer of tokens pulled from the lexer
        CommonTokenStream tokens = new CommonTokenStream(lexer);
        // create a parser that feeds off the tokens buffer
        ArrayInitParser parser = new ArrayInitParser(tokens);
        ParseTree tree = parser.init();
        System.out.println(tree.toStringTree(parser));
    }
}
```

Calculator

```
grammar Expr;

prog: stat+;
stat: expr NEWLINE
    | ID '=' expr NEWLINE
    | NEWLINE;

expr: expr ('*'|'/') expr
    | expr ('+'|'-') expr
    | INT
    | ID
    | '(' expr ')';

ID  : [a-zA-Z]+;
INT : [0-9]+;
// return newlines to parser (is end-statement signal)
NEWLINE:'\r'? '\n';
WS : [ \t]+ -> skip;
```

Importing grammars

show two files : lexer & grammar