CS 348 Lecture 18

Transactions II

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Nov 18, 2021

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Outline For Today

Serializability:

1. Execution Histories
2. Conflict Equivalence
3. Checking For Serializability

System's Perspective
(and more next lecture)

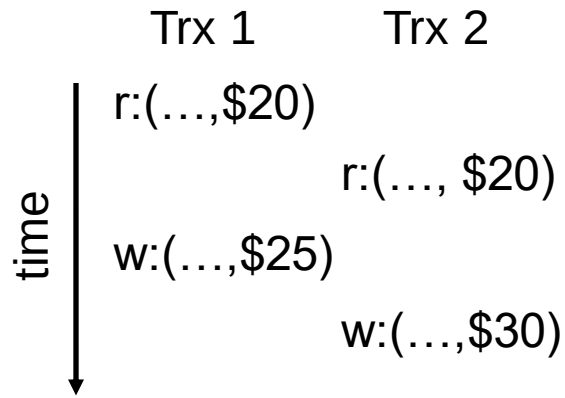
Outline For Today

Serializability:

1. Execution Histories
2. Conflict Equivalence
3. Checking For Serializability

Goals of Execution History Model & Conflict Equivalences

- Concurrency is achieved by interleaving operations across trxs.



- Q: Does an interleaving correspond to a serializable execution?
- Execution history model and conflict equivalences is a formal method to answer this question.

Representing Single Transactions

- Database is a set of *data items* (often will denote as x, y, z…)
- Trx T_i is a *total order* of read/write & commit/abort operations on items
  - $r_i(x)$ indicates T_i reads item x
  - $w_i(x)$ indicates T_i writes item x
  - c indicates commit (a could indicate abort but won't be needed)
    - Suppose: T_i does the following in this *chronological order*:
      - Read(x), Read(y), $x \leftarrow x + y$, Write(x), commit
      - $T_i=\{r_i(x) < r_i(y) < w_i(x) < c_i\}$ or simply as:
      - $T_i=\{r_i(x), r_i(y), w_i(x), c_i\}$ or $r_i(x), r_i(y), w_i(x), c_i$
- Note: total order is simply the chronological order of operations of T_i

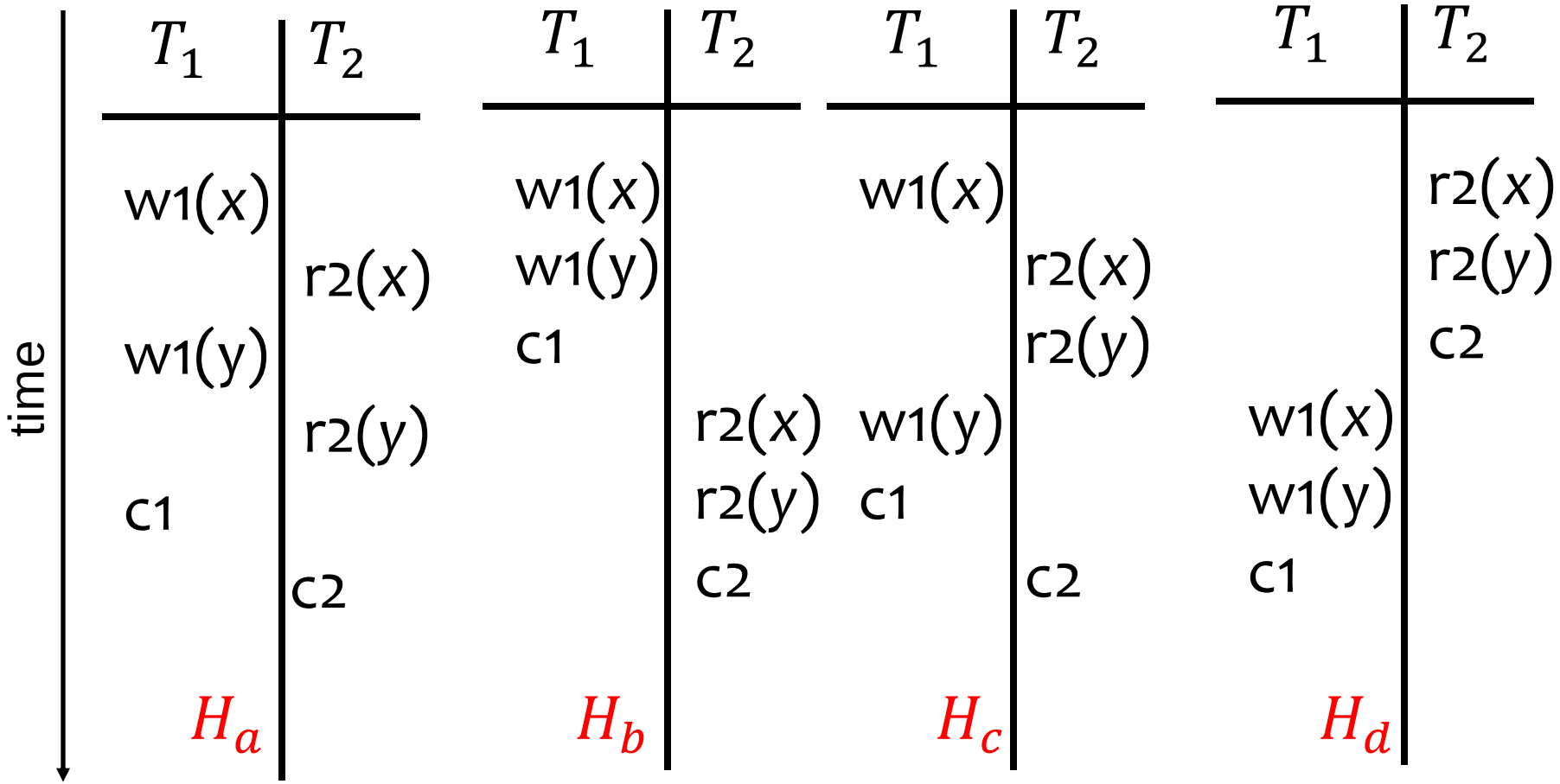
Execution Histories

- An execution history over a set of transactions $T_1 \ldots T_n$ is an interleaving of the operations of $T_1 \ldots T_n$ in which the operation total order imposed by each transaction is preserved.

- Example: $T_1 = \{w_1[x], w_1[y], c_1\}$, $T_2 = \{ r_2[x], r_2[y], c_2 \}$
  - $H_a = w_1[x] r_2[x] w_1[y] r_2[y] c_1 c_2$
  - $H_b = w_1[x] w_1[y] c_1 r_2[x] r_2[y] c_2$
  - $H_c = w_1[x] r_2[x] r_2[y] w_1[y] c_1 c_2$
  - $H_d = r_2[x] r_2[y] c_2 \; w_1[x] w_1[y] c_1$

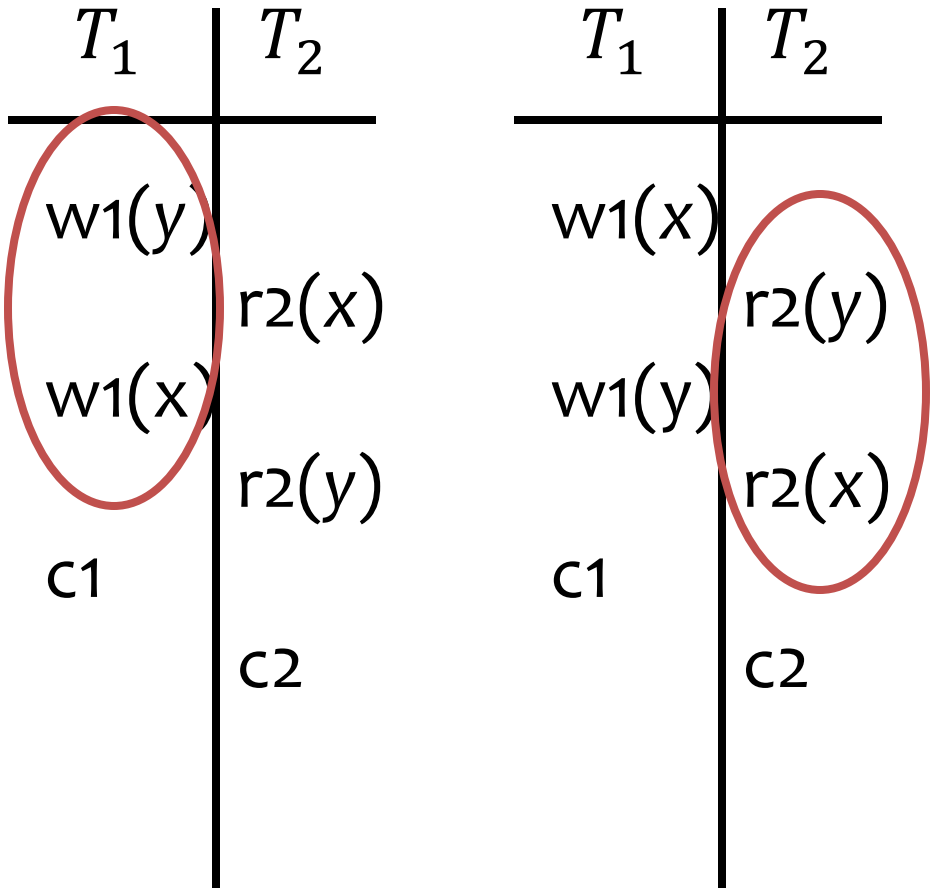
Examples for Valid Execution History

- $T_1 = \{w_1[x], w_1[y], c_1\}$, $T_2 = \{ r_2[x], r_2[y], c_2 \}$



Examples for Invalid Execution History

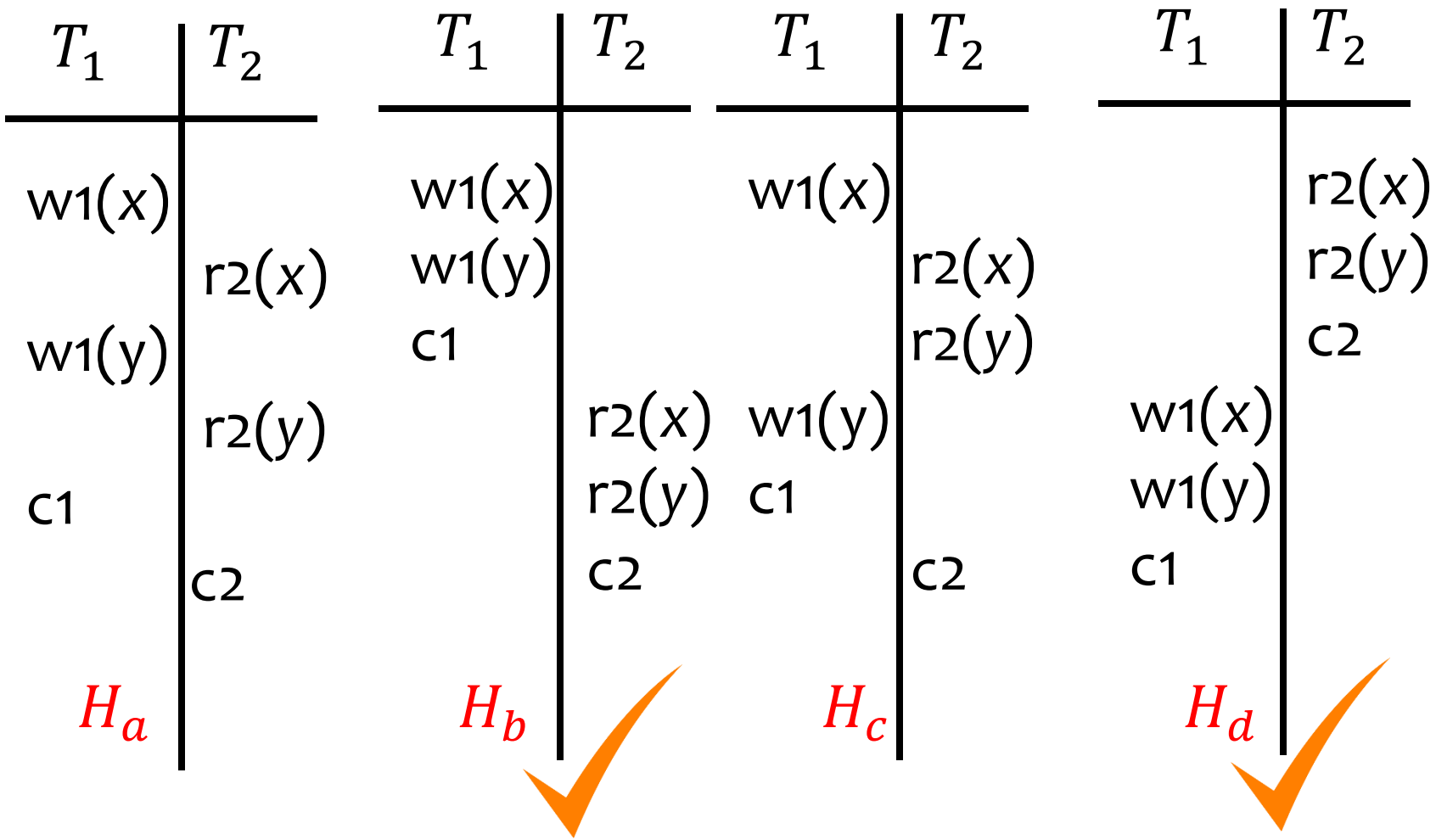
- $T_1 = \{w_1[x], w_1[y], c_1\}$, $T_2 = \{ r_2[x], r_2[y], c_2 \}$



Incorrect orders

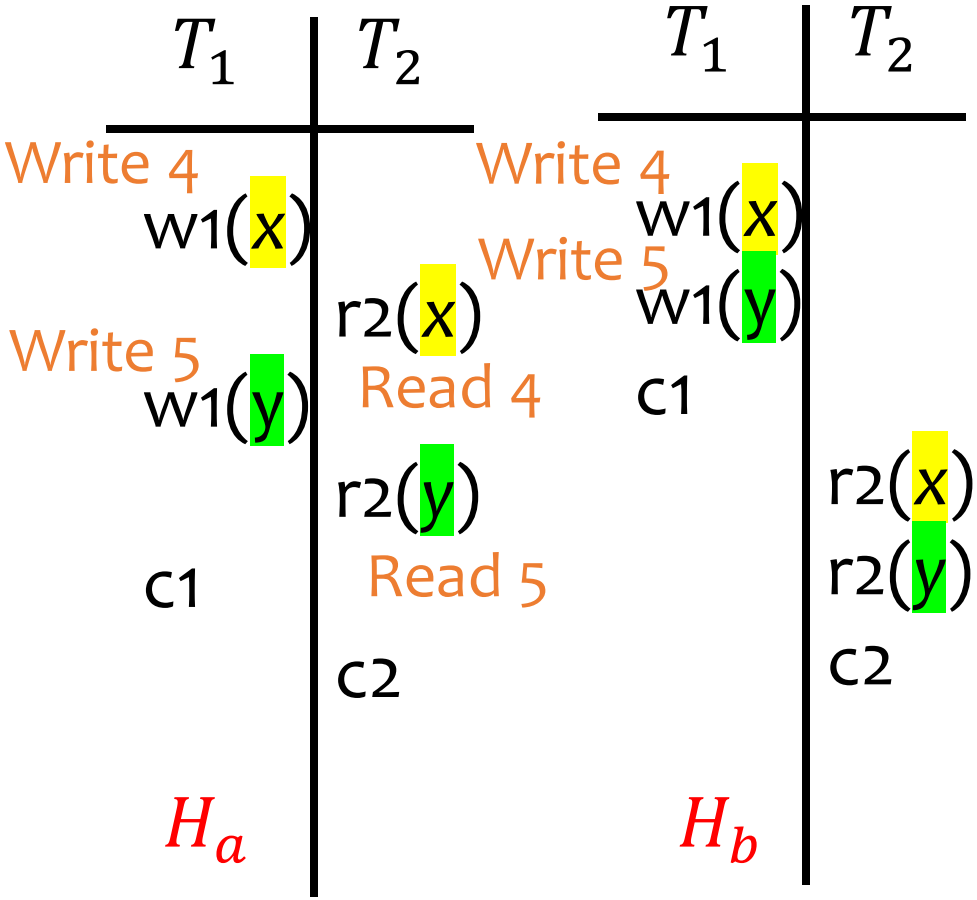
Serial Execution Histories

- $T_1 = \{w_1[x], w_1[y], c_1\}$, $T_2 = \{ r_2[x], r_2[y], c_2 \}$
- Serial history: no interleaving of ops from different transactions



Equivalent Histories

- H_a is "equivalent" to H_b if values read by each T_i in H_a and H_b is same, and the order or write operations on each item is the same



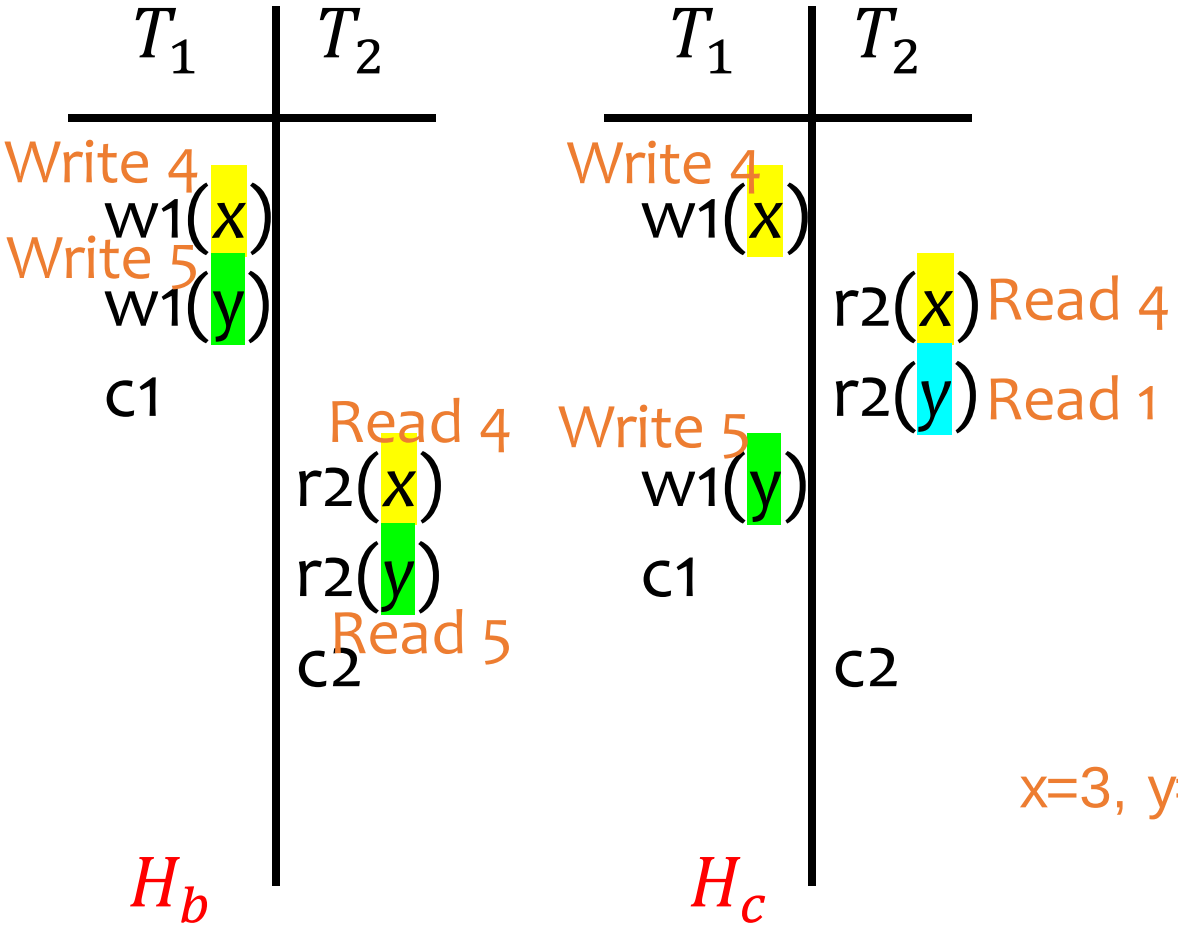
T_2 sees all the updates by T_1

- T_2 reads x written by T_1
- T_2 reads y written by T_1

x=3, y=1 (before T1 and T2)

Example Non-equivalent Histories

- H_a is "equivalent" to H_b if values read by each T_i in H_a and H_b is same, and the order or write operations on each item is the same



T_2 reads different y in H_b than in H_c

x=3, y=1 (before T1 and T2)

Outline For Today

Serializability:

1. Execution Histories
2. Conflict Equivalence
3. Checking For Serializability

Conflict Equivalence

- Dfn: Two operations conflict if:
  1. they belong to **different transactions**,
  2. they operate on the **same object**, and at least one of the operations is a **write**

  2 types of conflicts: (1) Read-Write and (2) Write-Write
- Dfn: Two histories are (conflict) equivalent if
  1. they are over the same set of transactions, and
  2. the ordering of each pair of conflicting operations is the same in each history

Example 1

- Consider:
  - $H_a = w_1[x] r_2[x] w_1[y] r_2[y] c_1 c_2$
  - $H_b = w_1[x] w_1[y] r_2[x] r_2[y] c_1 c_2$

- Step 1: check if they are over the same set of transactions
  - $T_1 = \{w_1[x], w_1[y]\}$, $T_2 = \{ r_2[x], r_2[y] \}$

- Step 2: check if all the conflicting pairs have the same order

Conflicting pairs	H_a	H_b
$w_1[x], r_2[x]$	<	<
$w_1[y], r_2[y]$	<	<

Motivation & Intuition For Conflict Equivalence

- If two histories H_a and H_b are conflict equivalent then, we can make H_a exactly the same as H_b by iteratively swapping two consecutive non-conflicting operations in H_a and/or H_b.
  - $H_a = w_1[x]r_2[x]w_1[y]r_2[y]c_1c_2$ => $H'_a = w_1[x]w_1[y]r_2[x]r_2[y]c_1c_2$
  - $H_b = w_1[x]w_1[y]r_2[x]r_2[y]c_1c_2$
- Proof Sketch: Move all ops on item x_1 to the beginning by swapping in both H_a and H_b (x_1 ops "move past" ops on other items).
- End with the order imposed by the conflicts on x_1
- If H_a & H_b are conflict eq. this prefix ops on x_1 will be the same order.
- Then repeat for x_2, x_3, etc. and we will arrive at the same histories.
- Therefore: Every read by each trx has the same value in H_a & H_b
- Therefore: H_a & H_b lead to the same output database state.

Example 2

Consider

- H_A: $r_1[x]r_3[x]w_4[y]r_2[u]w_4[z]r_1[y]r_3[u]r_2[z]w_2[z]r_3[z]r_1[z]w_3[y]$
- H_B: $r_1[x]w_4[y]r_3[x]r_2[u]r_1[y]r_3[u]r_2[z]w_2[z]w_4[z]r_1[z]r_3[z]w_3[y]$

Step 1: check if they are over the same set of transactions

Step 2: check if all the conflicting pairs have the same order

Example 2

Consider

- H_A: $r_1[x]r_3[x]w_4[y]r_2[u]w_4[z]r_1[y]r_3[u]r_2[z]w_2[z]r_3[z]r_1[z]w_3[y]$
- H_B: $r_1[x]w_4[y]r_3[x]r_2[u]r_1[y]r_3[u]r_2[z]w_2[z]w_4[z]r_1[z]r_3[z]w_3[y]$

Step 1: check if they are over the same set of transactions

$$\{r_1[x]\ r_1[y]\ r_1[z]\ \},\ \{r_2[u]\ r_2[z]w_2[z]\},\ \{r_3[x]\ r_3[u]\ r_3[z]w_3[y]\},$$
$$\{w_4[y]\ w_4[z]\}$$

Step 2: check if all the conflicting pairs have the same order

Identify All Conflicting Pairs

- H_A: $r_1[x]r_3[x]w_4[y]r_2[u]w_4[z]r_1[y]r_3[u]r_2[z]w_2[z]r_3[z]r_1[z]w_3[y]$

- Conflicting pairs:
  - Related to x: no conflicting pairs, as all are reads
  - Related to y: w4[y], r1[y], w3[y]
    - $w_4[y] < r_1[y]$
    - $w_4[y] < w_3[y]$
    - $r_1[y] < w_3[y]$
  - Related to z: w4[z], r2[z], w2[z], r3[z], r1[z]
    - $w_4[z] < r_2[z]$
    - $w_4[z] < w_2[z]$
    - $w_4[z] < r_3[z]$
    - $w_4[z] < r_1[z]$
    - $r_2[z]$, $w_2[z]$ are not, as they are from the same transactions
    - $w_2[z] < r_3[z]$
    - $w_2[z] < r_1[z]$

Example 2

Consider

- H_A: $r_1[x]r_3[x]w_4[y]r_2[u]w_4[z]r_1[y]r_3[u]r_2[z]w_2[z]r_3[z]r_1[z]w_3[y]$
- H_B: $r_1[x]w_4[y]r_3[x]r_2[u]r_1[y]r_3[u]r_2[z]w_2[z]w_4[z]r_1[z]r_3[z]w_3[y]$

Step 1: check if they are over the same set of transactions

$\{r_1[x]\ r_1[y]\ r_1[z]\ \}$, $\{r_2[u]\ r_2[z]w_2[z]\}$, $\{r_3[x]\ r_3[u]\ r_3[z]w_3[y]\}$, $\{w_4[y]\ w_4[z]\}$

Step 2: check if all the conflicting pairs have the same order

Conflicting pairs	H_A	H_B
$w_4[y], r_1[y]$	<	<
$w_4[y], w_3[y]$	<	<
...	<	<
$w_4[z], w_2[z]$	<	>
...	<	<

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Serializability:

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Serializability

- A history H is said to be (conflict) **serializable** if there exists some serial history H' that is conflict equivalent to H.

T_1	T_2
w1(x)	
	r2(x)
w1(y)	
	r2(y)
c1	
	c2

H_a

T_1	T_2
w1(x)	
w1(y)	
c1	
	r2(x)
	r2(y)
	c2

H_b

T_1	T_2
w1(x)	
	r2(x)
	r2(y)
w1(y)	
c1	
	c2

H_c

In Example 1 we showed: $H_a = H_b$

Serializability

- Does H_c have an equivalent serial execution?
  - $H_c = w_1[x]r_2[x]r_2[y]w_1[y]c_1c_2$
- Only 2 serial execution to check:
  - H_b: T_1 followed by T_2: $w_1[x]w_1[y]c_1r_2[x]r_2[y]c_2$
    - $r_2[y]$ reads different value as in H_c
  - H_d: T_2 followed by T_1: $r_2[x]r_2[y]c_2w_1[x]w_1[y]c_1$
    - $r_2[x]$ reads different value as in H_c

Conflicting pairs	H_b	H_c	H_d
$w_1[x], r_2[x]$	<	<	>
$w_1[y], r_2[y]$	<	>	>

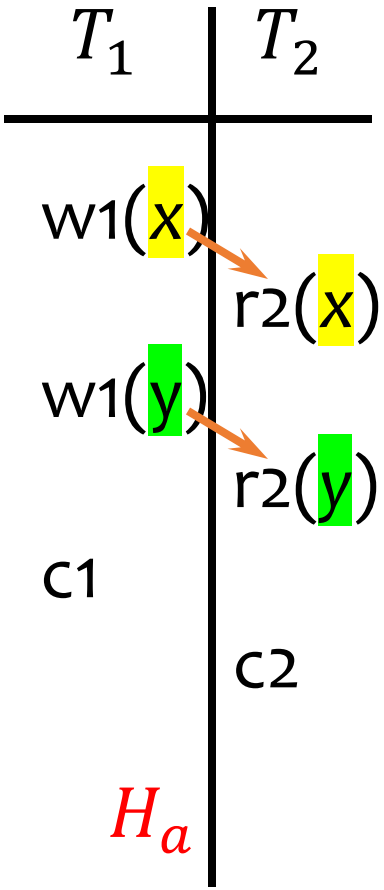
- Do we need to check all the serial executions?

Serialization Graph

- More practical test for serializability
- Serialization graph $SG_H(V, E)$ for history H:
  - $V = \{T | T$ is a committed transaction in $H\}$
  - $E = \{T_i \rightarrow T_j$ if $o_i \in T_i$ and $o_j \in T_j$ **conflict** and $o_i < o_j\}$
- Theorem: A history is serializable iff its serialization graph is acyclic.

Example 1

- $H_a = w_1[x] r_2[x] w_1[y] r_2[y]\ c_1 c_2$

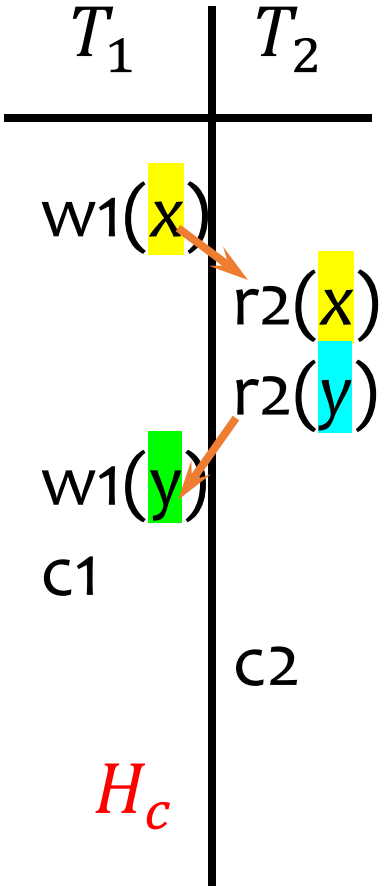


$w_1[x]$ and $r_2[x]$ conflict, and $w_1[x] < r_2[x]$
$w_1[y]$ and $r_2[y]$ conflict, and $w_1[y] < r_2[y]$

Serialization graph: no cycles → serializable

Example 2

- Example: $H_c = w_1[x]r_2[x]r_2[y]w_1[y]c_1c_2$



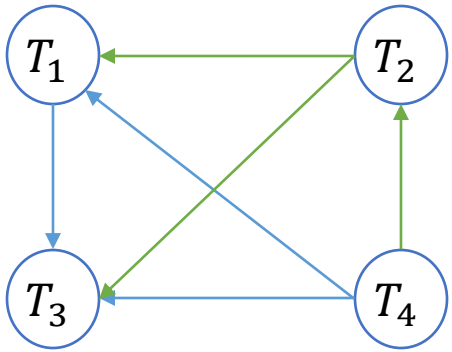
$w_1[x]$ and $r_2[x]$ conflict, and $w_1[x] < r_2[x]$;
$w_1[y]$ and $r_2[y]$ conflict, and $r_2[y] < w_1[y]$

Not serializable

Example 3

- $r_1[x]r_3[x]w_4[y]r_2[u]w_4[z]r_1[y]r_3[u]r_2[z]w_2[z]r_3[z]r_1[z]w_3[y]$

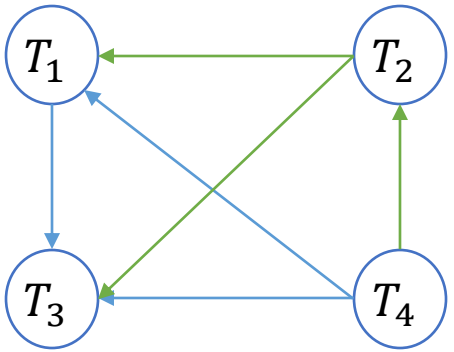
- Conflicting pairs:
  - Related to x: no conflicting pairs, as all are reads
  - Related to y: w4[y], r1[y], w3[y]
    - $w_4[y] < r_1[y]$ T4 → T1
    - $w_4[y] < w_3[y]$ T4 → T3
    - $r_1[y] < w_3[y]$ T1 → T3
  - Related to z: w4[z], r2[z], w2[z], r3[z], r1[z]
    - $w_4[z] < r_2[z]$ T4 → T2
    - $w_4[z] < w_2[z]$ T4 → T2
    - $w_4[z] < r_3[z]$ T4 → T3
    - $w_4[z] < r_1[z]$ T4 → T1
    - $r_2[z]$, $w_2[z]$ are not, as they are from the same transactions
    - $w_2[z] < r_3[z]$ T2 → T3
    - $w_2[z] < r_1[z]$ T2 → T1



Example 3

- $r_1[x]r_3[x]w_4[y]r_2[u]w_4[z]r_1[y]r_3[u]r_2[z]w_2[z]r_3[z]r_1[z]w_3[y]$

- No cycles in this serialization graph
  - Topological sort: T4 -> T2 -> T1->T3



- The history above is (conflict) equivalent to $w_4[y]w_4[z]r_2[u]r_2[z]w_2[z]r_1[x]r_1[y]r_1[z]r_3[x]r_3[u]r_3[z]w_3[y]$
  - Note: we ignore the commits at the end for simplicity

Next Lecture: 2 Phase-Locking Protocol That Guarantees Conflict Serializability

(And Logging-based Recovery)