In the Hypothesis testing, we can have the following possibilities:-

	H_0 is true	H_0 is false
Do not reject H_0	Correct Decision	Type II error
Reject H_0	Type I error	Correct decision

<u>Type I error</u>:- Rejection of the null Hypothesis when it is true.

<u>Type II error</u>:- Non rejection of null hypothesis when it is false.

Probability of type I error is called level of significance (α). It is also called size of the test.

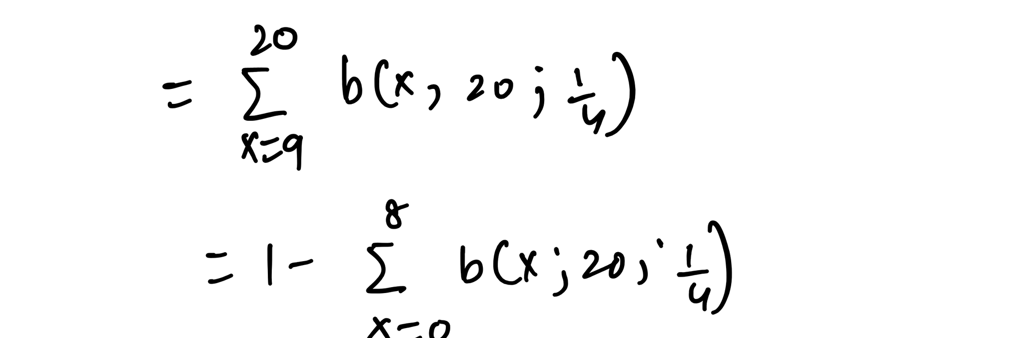
Probability of type II error is called β.

Consider the following situation –
A certain type of cold vaccine is known to be only 25% effective after a period of two years.

There is a new vaccine introduced & it is believed that new vaccine is better.

So our null hypothesis is :- $H_0: p = \frac{1}{4}$
Alternative " is :- $H_1: p > \frac{1}{4}$

- We give the new vaccine to 20 people and if more than 8 people surpass 2 years without contracting the disease we reject H_0 in favour of H_1.
- Note that here the no. 8 somehow defines our significance level. We call it as our critical value.
- test statistics here is the no. of people who surpass 2 years without contracting the disease.
- All the values > 8 constitutes the critical region where we reject H_0.
- All the values ≤ 8 constitutes the acceptance region where we fail to reject H_0.



- Now let us compute α in above case.

$\alpha = P(\text{type I error}) = P(\text{we reject } H_0 \text{ when it is true})$

$= P(X > 8 \text{ when } p = 0.25)$

$= \sum_{x=9}^{20} b(x, 20; \tfrac{1}{4})$

$= 1 - \sum_{x=0}^{8} b(x; 20; \tfrac{1}{4})$

$= 1 - 0.9591 = 0.0409$

Since this is a very small number it is unlikely that type I error will occur.

- <u>Now if you want to compute β</u>

We cannot calculate β unless we have a specific alternative hypothesis.

$H_0: p = 0.25$
$H_1: p = 0.5$

So have $\beta = P(\text{type II error})$

$= P(\text{Accept } H_0 \text{ when it is false})$

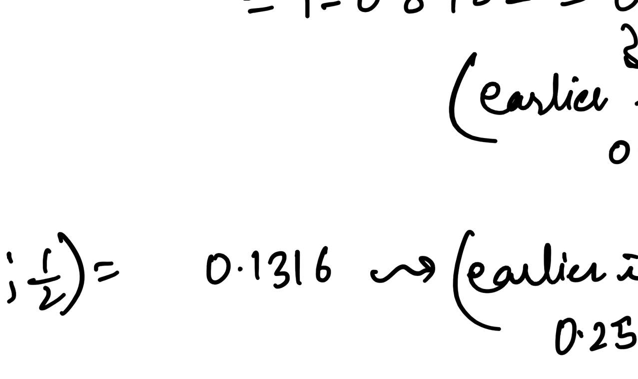
$= P(X \le 8 \text{ when } p = 0.5)$

$= \sum_{x=0}^{8} b(x; 20, \tfrac{1}{2}) = 0.2517$

This is a good number. It means that we will reject the new vaccine when in fact it is quite superior.

- Ideally we would like to use a test procedure for which type I and type II error probabilities are both small.

What if we reduce the critical value to 7.
i.e. we have



$\alpha = \sum_{x=8}^{20} b(x; 20, \tfrac{1}{4}) = 1 - \sum_{x=0}^{7} b(x; 20, \tfrac{1}{4})$

$= 1 - 0.8982 = 0.1018$
(earlier it was 0.0409)

$\beta = \sum_{x=0}^{7} b(x; 20; \tfrac{1}{2}) = 0.1316$ (earlier it was 0.2517)

By adopting a new decision procedure, we have reduced β at the expense of α.

- For a fixed sample size $\downarrow$ in α will lead to $\uparrow$ in β and vice versa.
- Both α and β are reduced by $\uparrow$ the sample size.

<u>Power of a test</u>:- The power of a test is the probability of rejecting H_0 given that a specific alternative is true.

$= 1 - P(\text{accept } H_0 \text{ even when a specific alternative hypothesis is true})$

$= 1 - \beta$

- Different types of tests can be compared using <u>power of test</u>.