Lecture 27
Chapter 10

Testing of Hypothesis

What is a Hypothesis? → A claim

For example a school claims that the average class size is 30 students. → Hypothesis

Further suppose a second school claims that it is 35 students.

We define null hypothesis as "status quo" i.e. $H_0: \mu = 30$

We have something called alternative hypothesis which is a contradiction to the null hypothesis

$H_1: \mu \neq 30$

What is our aim? We want to accept or reject null hypothesis.

↓

This process is called Hypothesis testing

The first step in Hypothesis testing is to define null & alternative hypothesis

Example: Company XYZ manufactures calculators with an average mass of 150 gms. An engineer believes that company weight to be different & decides to calculate mean weight. So calculator lab are used & alternative hypothesis.

Null hypothesis $H_0: \mu = 150$ gms

Alternative hypothesis $H_1: \mu \neq 150$ gms

Example: A teacher in a school believes that at least 80% of students will complete high school. A student disagrees with this claim & decides to conduct a test. Lab is used & an alternative hypothesis.

$H_0: p \geq 80\%$

$H_1: p < 80\%$

Step 1: Define null & alternative hypothesis → Same → Same

$H_0: \mu = \mu_0$

$H_1: \mu \neq \mu_0$

Step 2: Significance level/ confidence level. → α or $1-\alpha$

Step 3: Take a sample out of the universe and compute the sample statistics. (Can be a mean or a σ or a μ)

Step 4: If Step 3 above satisfies the critical region we reject H_0 & the value lies in acceptance region we fail to reject H_0

After calculating sample statistics; Find the probability that your sample statistics receives such that value. → P-value.

If p value $> \alpha$ we accept H_0

If p value $\leq \alpha$ " reject H_0.

Que The average weight of all residents in a town is 168 pounds. A nutritionist believes the time seem to be different. She measured the weight of 36 individuals and found the mean to be 169.5 pounds, knowing that population SD = 3.9, at 99% confidence level, is there enough evidence to discard the null hypothesis.

Soln Classical approach

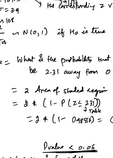
(I) $H_0: \mu = 168$

$H_1: \mu \neq 168$

(II) $1-\alpha = 0.95 \Rightarrow \alpha = 0.05$

95% confidence interval for μ is $\left[\bar{x} - z_{\alpha/2}\frac{\sigma}{\sqrt{n}}, \bar{x} + z_{\alpha/2}\frac{\sigma}{\sqrt{n}}\right]$

$Z = \frac{\bar{x}-\mu}{\sigma/\sqrt{n}} \sim N(0,1)$



(III) Data

$n = 36$

$\bar{x} = 169.5$

$\sigma = 3.9$

Computation

$Z = \frac{\bar{x}-\mu}{\sigma/\sqrt{n}} = \frac{169.5-168}{3.9/\sqrt{36}} = \frac{0.231}{0.65} > 1.96$

(IV) Since the statistic value is critical region

∴ we reject H_0 in favor of H_1.

P value approach.

Step I: $H_0: \mu = 168$

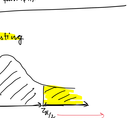
$H_1: \mu \neq 168$

$\alpha = 0.05$

Step II: $n = 36$

$\bar{x} = 169.5$

$\sigma = 3.9$



the corresponding z value is $= \frac{0.231}{0.65}$

$\frac{\bar{x}-\mu}{\sigma/\sqrt{n}} \sim N(0,1)$ if H_0 is true

P value = What is the probability that you should be 2.31 away from 0.

$= 2 \text{ Area of shaded region}$

$= 2 * (1 - P(z \leq 2.31))$

$= 2 * (1 - \text{cdf}(2.31)) = 0.02088$

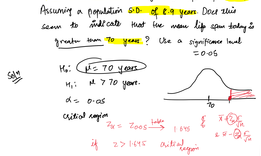
Pvalue < 0.05

so we reject the null favor of H_1.

Two tail Hypothesis testing.

$H_0: \mu = \mu_0$

$H_1: \mu \neq \mu_0$

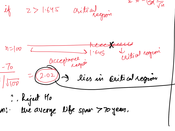


One tail test

$H_0: \mu = \mu_0$

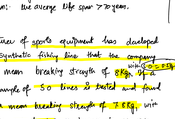
$H_1: \mu > \mu_0$

P value = Probability of one tail



Ex. $H_0: \mu = \mu_0$

$H_1: \mu < \mu_0$



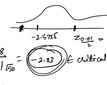
Que A random sample of 100 recorded deaths in the past showed an average life span of 71.8 years. Assuming a population SD of 8.9 years. Does this seem to indicate that the mean life span today is greater than 70 years? Use a significance level $= 0.05$

Soln

$H_0: \mu = 70$ years

$H_1: \mu > 70$ years.

$\alpha = 0.05$



critical region

$z_\alpha = z_{0.05} = 1.645$ table $\left\{ \bar{x} + z_\alpha \frac{\sigma}{\sqrt{n}} \right.$

if $z > 1.645$ critical region

data

$\bar{x} = 71.8$

$\sigma = 8.9$ $n = 100$

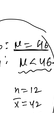
$z = \frac{71.8-70}{8.9/\sqrt{100}} = 2.02$ → Lies in critical region

acceptance region 1.645 critical region

∴ Reject H_0

∴ decision: the average life span > 70 years.

Pvalue approach



2.02

$\text{Pvalue} = P(z > 2.02)$

$= 1 - P(z \leq 2.02)$

$= 0.0217$

< 0.05 ∴ Reject H_0

Que A manufacturer of sports equipment has developed a new synthetic fishing line that the company claims has mean breaking strength of 8 kg with $\sigma = 0.5$ kg. A random sample of 50 lines is tested and found to have a mean breaking strength of 7.8 kg. With $\alpha = 0.01$ significance test the hypothesis $\mu = 8$ kg.

Soln

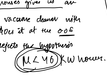
$H_0: \mu = 8$ kg

$H_1: \mu \neq 8$ kg

$\alpha = 0.01$

Critical region

$\left[z_{\alpha/2}, z_{1-\alpha/2}\right] = \left[-2.575, 2.575\right]$

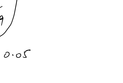


Statistics

$= \frac{\bar{x}-\mu}{\sigma/\sqrt{n}} = \frac{7.8-8}{0.5/\sqrt{50}} = -2.83 \in$ critical region

∴ Reject H_0.

∴ mean is not 8kg.



$P(|z| > 2.83)$

$= 0.0046$

< 0.01

∴ Reject H_0.

- When the variance of population is not known then use t distribution

$\frac{\bar{x}-\mu}{s/\sqrt{n}}$ → t distribution with dof $n-1$.

- Note

$\frac{\bar{x}-\mu}{s/\sqrt{n}}$ ~ t

$\frac{\bar{x}-\mu}{\sigma/\sqrt{n}}$ ~ $N(0,1)$

when $n \geq 30$ then you approximate s to σ.

Que The Edison electrical institute has published figures on number of KW hours used annually by various home appliances. It is claimed that a vacuum cleaner uses an average of 46 KW hours per year.

If a random sample of 12 homes gives us an average of 42 KW hours for vacuum cleaner with S.D = 11.9 KW hours, Does it at the 0.05 significance accept or reject the hypothesis $\mu < 46$ KW hours.

$H_0: \mu = 46$

$H_1: \mu < 46$ → One tail test

$n = 12$

$\bar{x} = 42$

$s = 11.9$

$\alpha = 0.05$

Soln

$t_\alpha = t_{0.05}$ with dof $= n-1 = 12-1 = 11$

$= 1.796$

∴ critical region $t < -1.796$

Calculation Statistics

$t = \frac{\bar{x}-\mu}{s/\sqrt{n}} = \frac{42-46}{11.9/\sqrt{12}} = -1.16 \notin$ critical region

∴ we fail to reject H_0.

$\text{P Value} = P(T < -1.16) = 0.139 > 0.01$

∴ accept H_0.