Lecture 21

Tuesday, 13 September 2022 7:09 PM

Poisson Distribution

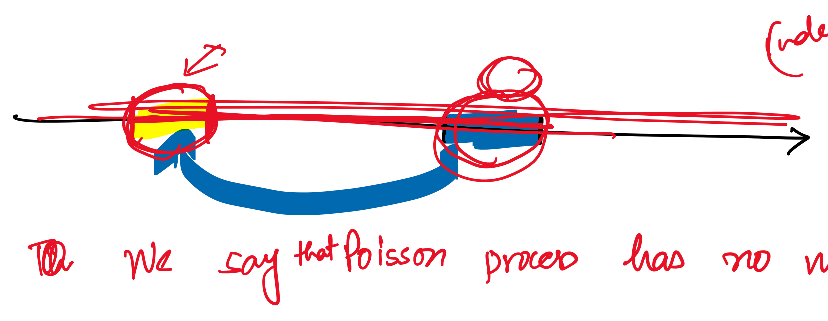
Poisson experiment:-

- If you count number of outcomes occuring in given interval of time-
  - No. of telephone calls per day for an office.
  - No. of days school was closed due to snow per month in winter.
- Number of outcomes in an area.
  - Number of field mice per acre.
  - Number of mistakes per page

For the above process to be Poisson experiment, the following conditions are to be met:-

(1) The number of outcomes occuring in one time interval or specified region should be independent of the number that occurs in any other disjoint time interval or region.

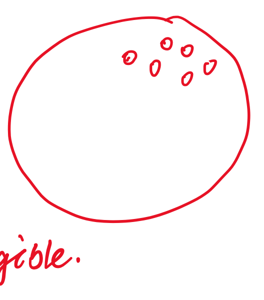
(independence)



We say that Poisson process has no memory.

(2) Probability that a single outcome will occur during a very short time interval or very small region is proportional to the length of the time interval or the area/size of region.

(3) The probability that more than one outcome will occur in such a short time interval or fall in such a small region is negligible.



then we call above process as a Poisson process

&

X → no. of outcomes in a given time t is called a Poisson r.v.

This is a discrete r.v.

$X \to 0, 1, 2, 3, \ldots$

if λ is average number of outcomes per unit time

then

pdf

$P(X=x)$, x no. of outcomes will occur in t time

$$= p(x; \lambda t) = \frac{e^{-\lambda t}(\lambda t)^x}{x!}$$ (*)

cdf

$$\boxed{P(r; \lambda t) = \sum_{x=0}^{r} p(x; \lambda t)}$$

Tables for P as well as p

Mean & Variance both are $= \lambda t$

Note You can approximate a binomial process with a Poisson process when $n \to$ very large

& $p \to$ very small

$np \longrightarrow$ a constant $= \mu$.

$$b(x; n, p) \xrightarrow[p \to 0]{n \to \infty} p(x, \mu) \qquad \boxed{\mu = np}$$

Que During a lab experiment, the average no. of radioactive particles passing through a counter in 1 milliseconds is 4. What is the probability that 6 particles will enter the counter in a given millisecond?

Sol^n $\boxed{\lambda = 4}$ $\lambda \to$ no. of. particles per millisecond

$t = 1$

$\therefore \lambda t = 4 * 1 = \boxed{4}$

$$p(x=6, \lambda t=4) = \frac{e^{-4} 4^6}{6!} = 0.1042.$$

Que Ten is the average number of oil tankers arriving each day at a certain port. The facilities at the port can handle at most 15 tankers per day. What is the probability that on a given day, tankers have to be turned away?

Sol^n $\left.\begin{array}{l}\lambda = 10 \\ t = 1\end{array}\right\}$ $\therefore \lambda t = 10 * 1 = 10$

$X \to$ no. of tankers arriving on a day.

$$P(X > 15) = 1 - P(X \le 15)$$
$$= 1 - \sum_{x=0}^{15} p(x; 10)$$
$$= 1 - P(15, 10)$$
$$= 1 - 0.9513$$
$$= 0.0487 \text{ Ans.}$$

Que In a factory, the probability that an accident occurs on a given day is 0.005 & accidents are independent of each other.

(a) What is the probability that in a period of 400 days there will be only one accident.

Sol^n 1 2 3 4 .....

$n = 400$, $p = 0.005$ $\Big\rangle$ $\lambda = np = 400 * 0.005 = 2$

$$\therefore b(x, 400, 0.005) \longrightarrow p(x, 2) = \frac{e^{-2} 2^1}{1!} = 0.271$$

(b) What is the probability that there are at most 3 days with accidents.

$$P(X \le 3) = \sum_{x=0}^{3} p(x, 2) = P(3, 2) = 0.857 \text{ Ans.}$$

We finished chapter 5 here

All the Best!