Hypothesis testing for mean

Saturday, 2 December 2023 10:07 PM

The steps of Hypothesis testing are explained below. The only thing to note is that for mean, the test statistics is

$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$ if population standard deviation is known

$t = \frac{\bar{x} - \mu}{s/\sqrt{n}}$ if population standard deviation is not known.

Que A random sample of 100 recorded deaths in the U.S. during the past year showed an average life span of 71.8 years. Assuming a population S.D. of 8.9 years, does this seem to indicate that the mean life span today is greater than 70 years? Use 0.05 level of significance.

Soln

Step I :- Set your null and alternative Hypothesis

$H_0: \mu = 70$ years

$H_1: \mu > 70$ years

Step II :- Your level of significance

$\alpha = 0.05$

Step III :- Calculate your test statistics

$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{71.8 - 70}{8.9/\sqrt{100}} = 2.02$$

Step IV :- Find your acceptance and critical regions

- Here you note that the test is one tailed $\because H_1: \mu > 70$

So the critical value of z is $z_{0.05}$

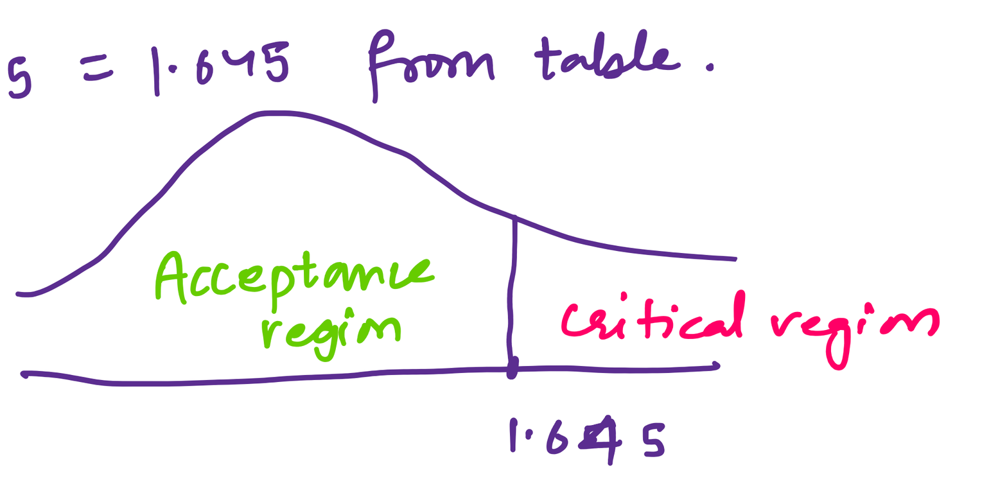
- Get the value $z_{0.05} = 1.645$ from table.

- The region

$z > z_{0.05}$

$z > 1.645$

is our critical region.



$z < 1.645$ is our acceptance region.

Step V Now if our test statistics falls in critical region, reject H_0 otherwise fail to reject H_0.

In this case test statistics 2.02 belongs to critical region

$\therefore$ Reject H_0

Why?

If our estimate of $\mu = 70$ is correct, then we expect with 95% confidence that $\frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$ lies in the region $z < 1.645$ (the acceptance region). Since it is not, so we say that our claimed value of μ is wrong. Hence reject H_0.

Que A manufacturer of sports equipment developed a new synthetic fishing line that the company claims has a mean breaking strength of 8 kg with $\sigma = 0.5$ kg. Test the hypothesis that $\mu = 8$ kg against the alternative hypothesis $\mu \neq 8$ kg if a random sample of 50 lines is tested and found to have a mean breaking strength of 7.8 kg. Use a 0.01 level of significance.

Soln

Step I :- $H_0: \mu = 8$ Kg

$H_1: \mu \neq 8$ kg

Step II :- $\alpha = 0.01$

Step III :- Calculate test statistics

$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{7.8 - 8}{0.5/\sqrt{50}} = -2.83$$

Step IV Critical region

The test is two tailed ($\because H_1: \mu \neq 8$)

$\therefore -z_{\alpha/2}$ & $z_{\alpha/2}$, we will calculate

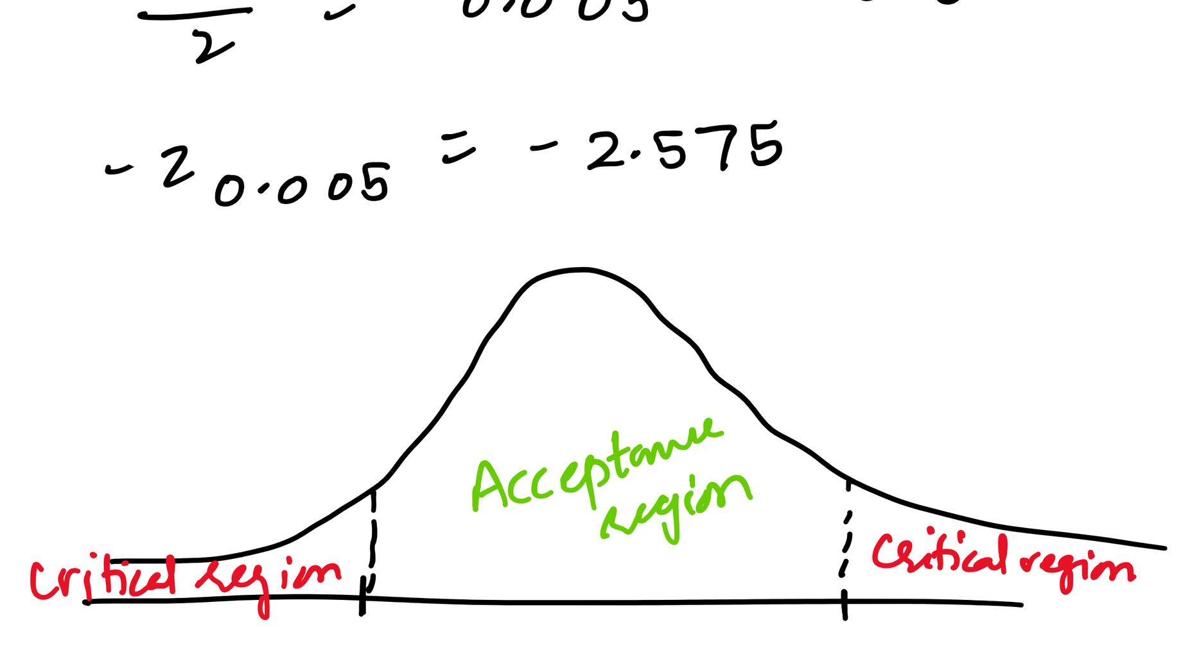
$$z_{\frac{0.01}{2}} = z_{0.005} = 2.575$$

$$-z_{0.005} = -2.575$$

critical region

$z < -2.575$

$z > 2.575$



acceptance region

$-2.575 < z < 2.575$

Step V :- Our test statistics $-2.83 \in$ Critical region

$\therefore$ Reject H_0.

Que An electrical company claimed that a vaccume cleaner uses an average of 46 KW hours per year. If a sample of 12 homes gives us $\bar{x} = 42$ kwh & $\sigma = 11.9$ kwh, does this suggest at the 0.05 level of significance that the company's claim is true? Assume the populations of KW hours to be normally distributed.

Soln

Step I : $H_0: \mu = 46$ kwh

$H_1: \mu < 46$ kwh

Step II : $\alpha = 0.05$

Step III Calculate test statistics

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{42 - 46}{11.9/\sqrt{12}} = -1.16$$

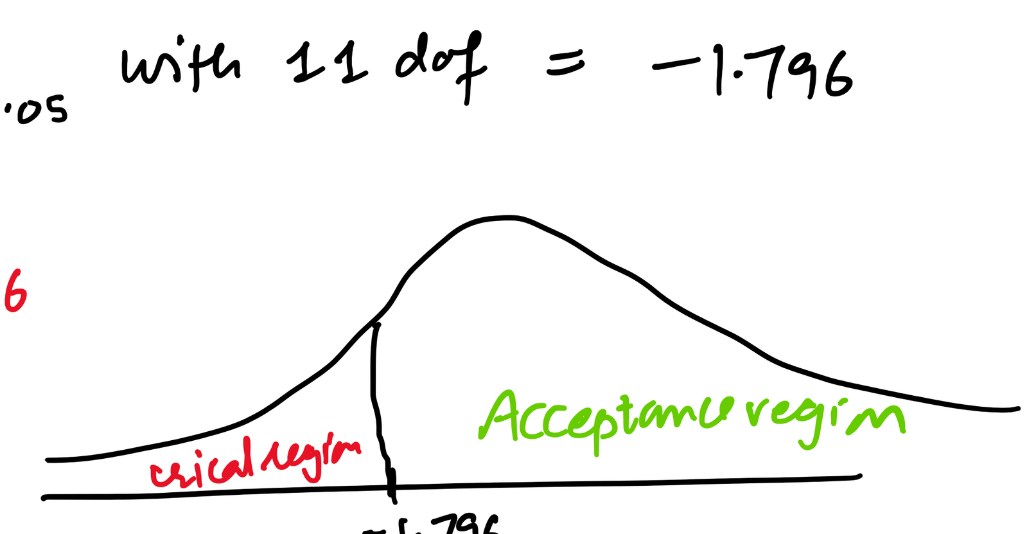
Step IV Critical region

One tailed test

$-t_{0.05}$ with 11 dof $= -1.796$

critical region

$t < -1.796$



Step V test statistice $-1.16 \notin$ critical region

$\therefore$ Do not reject H_0.

- Similarly you have to do hypothesis testing for

proportion

variance

&

difference of means.