• Suppose we have a random process where we first select, with equal probability, one of the following sets:

$$\{1, 2, 3\}, \{1, 2, 3, 4\}, \{2, 3, 5\}$$

After randomly selecting a set, we randomly select one of the numbers from that set, where – for a given set – each number in that set is equally likely to be chosen. Following this process, what is the probability that we select an even number?

• Let X be a continuous random variable with probability density function

$$f_X(x) = \begin{cases} c(x^2 + x), & x \in [0, 1] \\ 0, & \text{otherwise} \end{cases}$$

What is the value of c?

- Suppose we play a game where we roll six 6-sided dice. What are the probabilities of getting the following arrangements from a given roll of all six dice?
  - 3 of the dice are the same number, and the other 3 dice are distinct from each other and from the three-of-a-kind.
  - All six dice show the same number.
  - The numbers on the dice can be arranged to form a sequence of 5 consecutive numbers.
- Suppose that 1% of some population is afflicted with Bayes' Syndrome, a disease that must be detected using a medical test that correctly identifies 95% of true cases and has a false positive rate of 10%.
  - If Leonard tests positive for Bayes' Syndrome, what is the probability he actually has Bayes' Syndrome?
  - Suppose further that test results are conditionally independent of each other, conditioned on whether the person being tested has the disease or not. If Leonard is tested twice and tests positive both times, what is the probability he actually has Bayes' Syndrome?
- Prove that it is impossible to pick a number uniformly from the set  $\mathcal{P}$  of all prime numbers. That is, show that we can't assign the same probability of being chosen to all elements of  $\mathcal{P}$ . (Remember that there are infinitely many prime numbers.)