

已知 $f(x) = |x|$, 求 $f'(0)$

$$f(x) = |x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

$$\lim_{x \rightarrow 0^+} f(x) = \frac{f(x) - f(0)}{x - 0} = 1$$

$$\lim_{x \rightarrow 0^-} f(x) = \frac{f(x) - f(0)}{x - 0} = \frac{-x - 0}{x - 0} = -1$$

$$\therefore \lim_{x \rightarrow 0^+} f(x) \neq \lim_{x \rightarrow 0^-} f(x)$$

即 $f(x)$ 在 $x=0$ 时, 不可导

$$f(x) = |x|, x \in \mathbb{R}$$