First-Order Theorem Proving and Vampire

Laura Kovács and Andrei Voronkov

Chalmers

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Group theory theorem: if a group satisfies the identity $x^2 = 1$, then it is commutative.

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More formally: in a group "assuming that $x^2 = 1$ for all x prove that $x \cdot y = y \cdot x$ holds for all x, y."
What is implicit: axioms of the group theory.

$$\forall x(1 \cdot x = x)$$
$$\forall x(x^{-1} \cdot x = 1)$$
$$\forall x \forall y \forall z((x \cdot y) \cdot z = x \cdot (y \cdot z))$$

Formulation in First-Order Logic

Axioms (of group theory):	$\forall x(1 \cdot x = x)$ $\forall x(x^{-1} \cdot x = 1)$ $\forall x \forall y \forall z((x \cdot y) \cdot z = x \cdot (y \cdot z))$
Assumptions:	$\forall x(x \cdot x = 1)$
Conjecture:	$\forall x \forall y(x \cdot y = y \cdot x)$

In the TPTP Syntax

The TPTP library (Thousands of Problems for Theorem Provers), http://www.tptp.org contains a large collection of first-order problems. For representing these problems it uses the TPTP syntax, which is understood by all modern theorem provers, including Vampire.

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In the TPTP syntax this group theory problem can be written down as follows:

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%---- 1 * x = x
fof(left_identity,axiom,
    ! [X] : mult(e,X) = X).
%---- i(x) * x = 1
fof(left_inverse,axiom,
    ! [X] : mult(inverse(X),X) = e).
%---- (x * y) * z = x * (y * z)
fof(associativity,axiom,
    ! [X,Y,Z] : mult(mult(X,Y),Z) = mult(X,mult(Y,Z))).
%---- x * x = 1
fof(group_of_order_2,hypothesis,
    ! [X] : mult(X,X) = e).
%---- prove x * y = y * x
fof(commutativity,conjecture,
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More on the TPTP Syntax

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- Comments;

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- Input formula roles (very important);

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More on the TPTP Syntax

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- Equality

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Proof by Vampire (Slightliy Modified)

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Refutation found. Thanks to Tanya!
270. $false [trivial inequality removal 269]
269. mult(sk0,sk1) != mult (sk0,sk1) [superposition 14,125]
125. mult(X2,X3) = mult(X3,X2) [superposition 21,90]
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27. mult(inverse(X2),e) = X2 [superposition 21,11]
22. mult(inverse(X4),mult(X4,X5)) = X5 [forward demodulation 17,10]
21. mult(X0,mult(X0,X1)) = X1 [forward demodulation 15,10]
19. e = mult(X0,mult(X1,mult(X0,X1))) [superposition 12,13]
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13. e = mult(X0,X0) [cnf transformation 4]
12. mult(X0,mult(X1,X2)) = mult(mult(X0,X1),X2) [cnf transformation 3]
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9. mult(sK0,sK1) != mult(sK1,sK0) [skolemisation 7,8]
8. ?[X0,X1]: mult(X0,X1) != mult(X1,X0) <=> mult(sK0,sK1) != mult(sK1,sK0)
                                                              [choice axiom]
7. ?[X0,X1]: mult(X0,X1) != mult(X1,X0) [ennf transformation 6]
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- Proof by refutation, generating and simplifying inferences, unused formulas ...

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12. mult(X0,mult(X1,X2)) = mult(mult(X0,X1),X2) [cnf transformation 3]
11. e = mult(inverse(X0),X0) [cnf transformation 2]
10. mult(e,X0) = X0 [cnf transformation 1]
9. mult(sK0,sK1) != mult(sK1,sK0) [skolemisation 7,8]
8. ?[X0,X1]: mult(X0,X1) != mult(X1,X0) <=> mult(sK0,sK1) != mult(sK1,sK0)
                                                              [choice axiom]
7. ?[X0,X1]: mult(X0,X1) != mult(X1,X0) [ennf transformation 6]
6. ~![X0,X1]: mult(X0,X1) = mult(X1,X0) [negated conjecture 5]
5. ![X0,X1]: mult(X0,X1) = mult(X1,X0) [input]
4. ![X0]: e = mult(X0,X0)[input]
3. ![X0,X1,X2]: mult(X0,mult(X1,X2)) = mult(mult(X0,X1),X2) [input]
2. ![X0]: e = mult(inverse(X0),X0) [input]
1. ![X0]: mult(e,X0) = X0 [input]
```

- Each inference derives a formula from zero or more other formulas;
- Input, preprocessing, new symbols introduction, superposition calculus
- Proof by refutation, generating and simplifying inferences, unused formulas ...

Proof by Vampire (Slightliy Modified)

```
Refutation found. Thanks to Tanya!
270. $false [trivial inequality removal 269]
269. mult(sk0,sk1) != mult (sk0,sk1) [superposition 14,125]
125. mult(X2,X3) = mult(X3,X2) [superposition 21,90]
90. mult(X4,mult(X3,X4)) = X3 [forward demodulation 75,27]
75. mult(inverse(X3),e) = mult(X4,mult(X3,X4)) [superposition 22,19]
27. mult(inverse(X2),e) = X2 [superposition 21,11]
22. mult(inverse(X4),mult(X4,X5)) = X5 [forward demodulation 17,10]
21. mult(X0,mult(X0,X1)) = X1 [forward demodulation 15,10]
19. e = mult(X0,mult(X1,mult(X0,X1))) [superposition 12,13]
17. mult(e,X5) = mult(inverse(X4),mult(X4,X5)) [superposition 12,11]
15. mult(e,X1) = mult(X0,mult(X0,X1)) [superposition 12,13]
14. mult(sK0,sK1) != mult(sK1,sK0) [cnf transformation 9]
13. e = mult(X0,X0) [cnf transformation 4]
12. mult(X0,mult(X1,X2)) = mult(mult(X0,X1),X2) [cnf transformation 3]
11. e = mult(inverse(X0),X0) [cnf transformation 2]
10. mult(e,X0) = X0 [cnf transformation 1]
9. mult(sK0,sK1) != mult(sK1,sK0) [skolemisation 7,8]
8. ?[X0,X1]: mult(X0,X1) != mult(X1,X0) <=> mult(sK0,sK1) != mult(sK1,sK0)
                                                            [choice axiom]
7. ?[X0,X1]: mult(X0,X1) != mult(X1,X0) [ennf transformation 6]
6. ~![X0,X1]: mult(X0,X1) = mult(X1,X0) [negated conjecture 5]
5. ![X0,X1]: mult(X0,X1) = mult(X1,X0) [input]
4. ![X0]: e = mult(X0,X0)[input]
3. ![X0,X1,X2]: mult(X0,mult(X1,X2)) = mult(mult(X0,X1),X2) [input]
2. ![X0]: e = mult(inverse(X0),X0) [input]
1. ![X0]: mult(e,X0) = X0 [input]
```

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