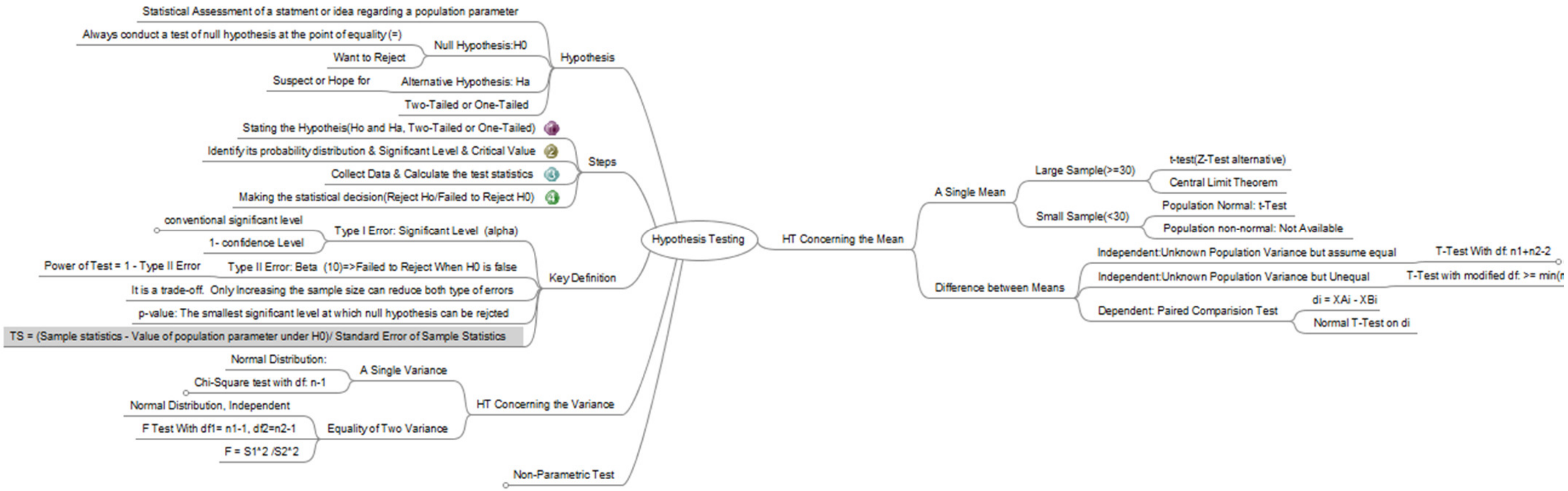
Hypothesis Testing

MindMap



Hypothesis Testing
Hypothesis
Statistical Assessment of a statment or idea regarding a population parameter
Null Hypothesis:H0
Always conduct a test of null hypothesis at the point of equality (=)
Want to Reject
Alternative Hypothesis: Ha
Suspect or Hope for
Two-Tailed or One-Tailed
Steps
Stating the Hypotheis(Ho and Ha, Two-Tailed or One-Tailed)
Identify its probability distribution & Significant Level & Critical Value
Collect Data & Calculate the test statistics
Making the statistical decision(Reject Ho/Failed to Reject H0)
Key Definition
Type I Error: Significant Level (alpha)
conventional significant level
1- confidence Level
Type II Error: Beta (10)=>Failed to Reject When H0 is false
Power of Test = 1 - Type II Error
It is a trade-off. Only Increasing the sample size can reduce both type of errors
p-value: The smallest significant level at which null hypothesis can be rejcted
TS = (Sample statistics - Value of population parameter under H0)/ Standard Error of Sample Statistics
HT Concerning the Variance
A Single Variance
Normal Distribution:
Chi-Square test with df: n-1
Equality of Two Variance
Normal Distribution, Independent
F Test With df1= n1-1, df2=n2-1
F = S1^2 /S2^2
Non-Parametric Test
HT Concerning the Mean
A Single Mean
Large Sample(>=30)
t-test(Z-Test alternative)
Central Limit Theorem
Small Sample(<30)
Population Normal: t-Test
Population non-normal: Not Available
Difference between Means
Independent:Unknown Population Variance but assume equal
T-Test With df: n1+n2-2
Independent:Unknown Population Variance but Unequal
Dependent: Paired Comparision Test
di = XAi - XBi
Normal T-Test on di

Hypothesis

➢ Define Hypothesis

Statistical assessment of a statement or idea regarding a **population parameter**

- Null Hypothesis: (Want to Reject, include "=")
- Alternative Hypothesis(Hope For)

Two-tailed	$H_0 : \mu = \mu_0$	$H_a : \mu \neq \mu_0$
One-tailed	$H_0 : \mu \leq \mu_0$ $or, H_0 : \mu \geq \mu_0$	$H_a : \mu > \mu_0$ $H_a : \mu < \mu_0$

Hypothesis-Example

1. Austin Roberts believes that the mean price of houses in the area is greater the $145,000. the appropriate alternative hypothesis is:

A. Ha: $\mu < \$145,000$

B. Ha: $\mu \geq \$145,000$

C. Ha: $\mu > \$145,000$

- **Answer: C**

2. An analyst is conducting a hypothesis test to determine if the mean time spent on investment research is different from three hours per day. The appropriate null hypothesis for the described test is:

A. H0: $\mu = 3$ hours, two-tailed test.

B. H0: $\mu = 3$ hours, one-tailed test.

C. H0: $\mu \geq 3$ hours, two-tailed test.

- **Answer: A**

Key Concept(1)

- Test Statistics:

$$Test\ Statistic = \frac{Sample\ statistics - Hypothesized\ value}{stanard\ error\ of\ the\ sample\ statistic}$$

- Test Statistic follows Normal, T, Chi Square or F distributions
- Test Statistic has formula. Calculate it with the sample data. We should emphasize Test Statistic is calculated by ourselves not from the table.
- This is the general formula but only for Z and T distribution.

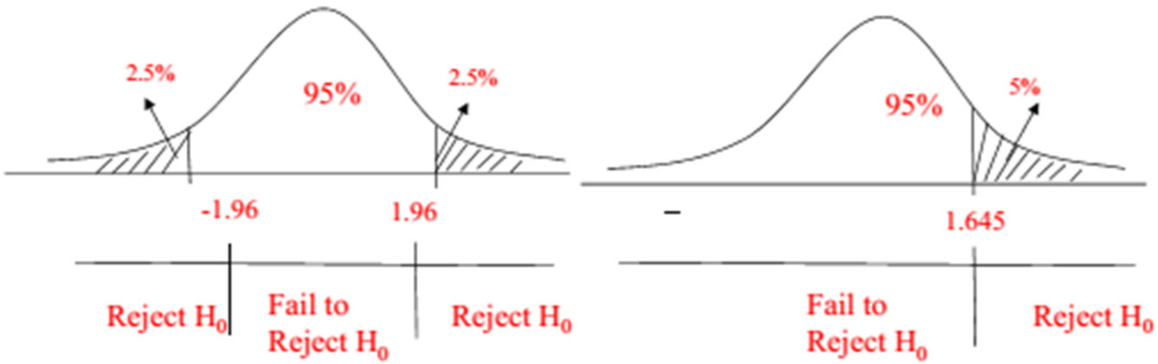
- Examples:

$$Test\ Statistic = \frac{\overline{X} - \mu_0}{S_{\overline{X}}}$$

$S_{\overline{X}} \rightarrow \frac{S}{\sqrt{n}}$

Key Concept(2)

- Critical Value
  - Found in the Z, T, Chi Square or F distribution tables not calculated by us
  - Under given one tailed or two tailed assumption, critical value is determined solely by the significance level.
- Decision Rule: Critical Value Method



- Reject H_0 if |test statistic|>critical value
- Fail to reject H_0 if |test statistic|<critical value

Key Concept(3)

Decision	True condition	
	H_0 is true	H_0 is false
Do not reject H_0	Correct Decision	Incorrect Decision Type Ⅱ error
Reject H_0	Incorrect Decision Significance level α =P (Type I error)	Correct Decision Power of test = 1- P (Type Ⅱ error)

- Significant Level: α = Pr(Type I Error) = Pr(Ho=**0** | Ho=**1**) = Pr(01)
- Type II Error: β = Pr(Ho=**1**|Ho=**0**) = Pr(10)
- Power of Test= 1- β = 1- P(Type II Error) = Pr(Ho=0 | Ho=0)

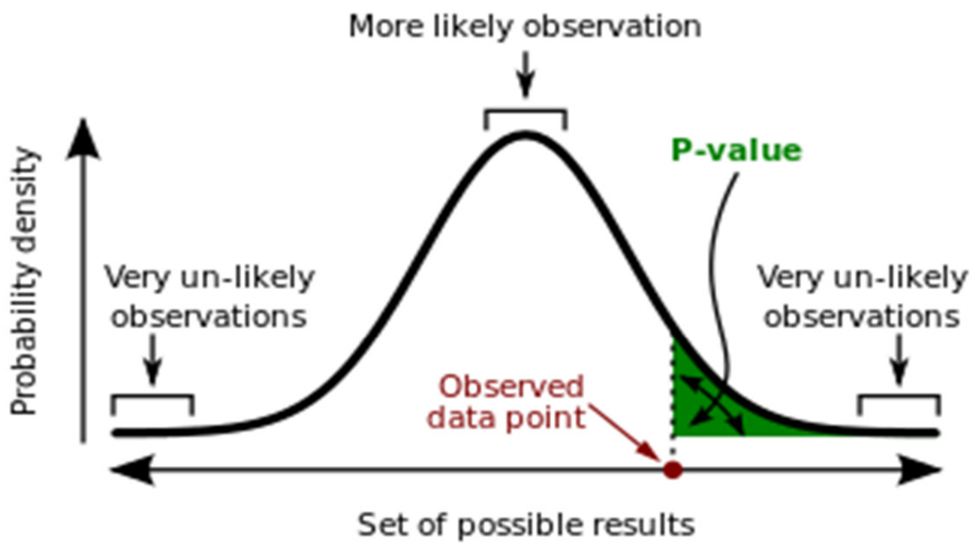
- How to Reduce Both Error: Increase Sample Size
- Type I Error ++, Type II Error --, Vice Versa

Key Concept(4)

- P-Value:
  - It is the smallest level of significance at which Ho can be rejected

$$\Pr(\text{Reject } H|H) = \Pr(p \leq \alpha|H) = \alpha .$$

  - P < α , reject, otherwise failed to reject
  - P Smaller, Easier to Reject



A **p-value** (shaded green area) is the probability of an observed (or more extreme) result assuming that the null hypothesis is true.

Key Concept- Example

1. Kyra Mosby, M.D., has a patient who is complaining of severe abdominal pain. Based on an examination and the results from laboratory tests, Mosby states the following diagnosis hypothesis: Ho: Appendicitis, HA: Not Appendicitis. Dr. Mosby removes the patient's appendix and the patient still complains of pain. Subsequent tests show that the gall bladder was causing the problem. By taking out the patient's appendix, Dr. Mosby:
   A. Made a Type I error.
   B. Is correct.
   C. Made a Type II error.

➢ Correct answer: C

2. If the sample size increases, the probability of get the Type Ⅰ and Type Ⅱ error will

	Type Ⅰ	Type Ⅱ
A.	increase	increase
B.	not change	not change
C.	decrease	decrease

➢ Correct answer: C

Mean Hypothesis Test(1)

Test type	Assumptions	H_0	Test-statistic	Critical value
Mean hypothesis testing	Normally distributed population, known population variance	$\mu=0$	$Z=\dfrac{\bar{x}-\mu_0}{\sigma/\sqrt{n}}$	$N(0,1)$
	Normally distributed population, unknown population variance	$\mu=0$	$t=\dfrac{\bar{x}-\mu_0}{s/\sqrt{n}}$	$t(n-1)$
	Independent populations, unknown population variances assumed equal	$\mu_1-\mu_2=0$	t	$t(n_1+n_2-2)$
	Independent populations, unknown population variances not assumed equal	$\mu_1-\mu_2=0$	t	t
	Samples not independent, paired comparisons test	$\mu_d=0$	$t=\bar{d}/s_{\bar{d}}$	$t(n-1)$

Mean Hypothesis Test(2)

- Hypothesis:

$$H_o : u_1 = u_2 \text{ VS } H_a : u_1 \neq u_2$$

- Test Statistics :Difference between Two Population Means(Normally Distributed, Variance Unknown but Assumed Equal)

- $t = \dfrac{(\bar{X}_1 - \bar{X}_2) - (u_1 - u_2)}{s_{\bar{X}_1 - \bar{X}_2} = \sqrt{\sigma_{(\bar{X}_1 - \bar{X}_2)}{}^2 = \sigma_{(\bar{X}_1)}{}^2 + \sigma_{(\bar{X}_2)}{}^2 = \frac{s_p{}^2}{n_1} + \frac{s_p{}^2}{n_1}}}$

$S_p{}^2 = \dfrac{(n_1 - 1) * S_1{}^2 + (n_2 - 1) * S_2{}^2}{n_1 - 1 + n_2 - 1}$ (pooled estimator $of\ common\ variance, weight\ average\ by\ sample\ size$)

- $D_f : n_1 + n_2$ - 2

Mean Hypothesis Test Example

Mean Returns on the S&P 500: A Test of Equality across Two Halves of a Decade

Table 3 S&P 500 Monthly Return and Standard Deviation for Two Halves of a Decade

Time Period	Number of Months (n)	Mean Monthly Return (%)	Standard Deviation
2000 through 2004	60	−0.083	4.719
2005 through 2009	60	0.144	4.632

- Solution:

1. $H_o : u_1 = u_2$ VS $H_a : u_1 \neq u_2$
2. T-Test With $D_f : n_1 + n_2$ - 2 = 118, CV: 1.98 (5% Level)
3. Test Statistics : 0.27 < 1.98
4. Decision: Failed to Reject Ho