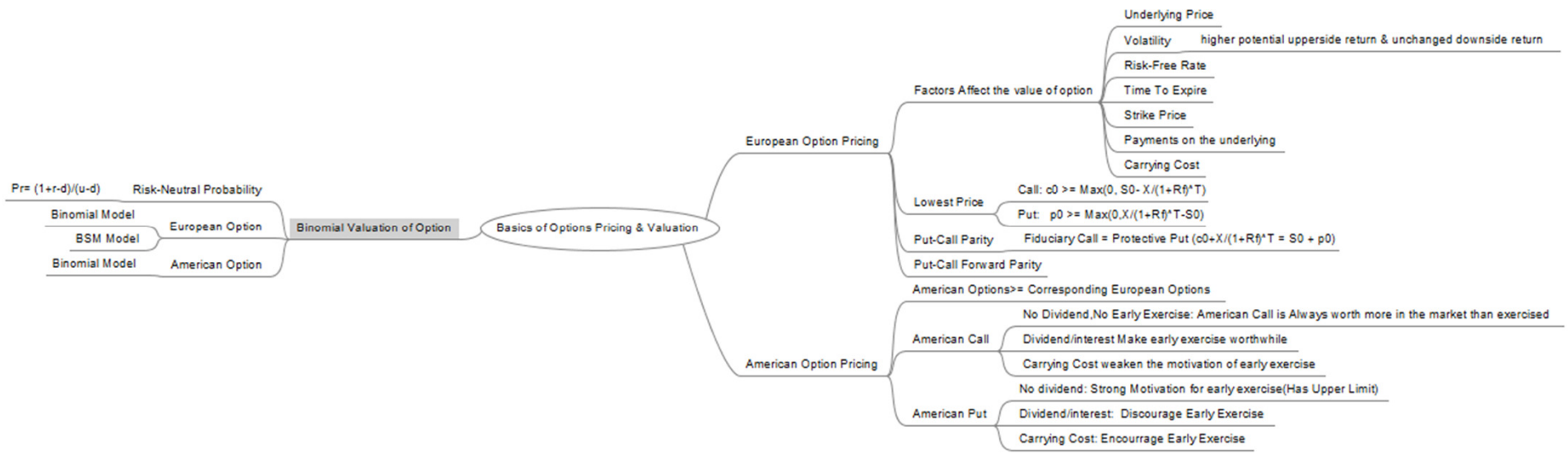
Basics of Options Pricing & Valuation

MindMap


Basics of Options Pricing & Valuation
European Option Pricing
Factors Affect the value of option
Underlying Price
Volatility
higher potential upperside return & unchanged downside return
Risk-Free Rate
Time To Expire
Strike Price
Payments on the underlying
Carrying Cost
Lowest Price
Call: c0 >= Max(0, S0- X/(1+Rf)^T)
Put: p0 >= Max(0,X/(1+Rf)^T-S0)
Put-Call Parity
Fiduciary Call = Protective Put (c0+X/(1+Rf)^T = S0 + p0)
Put-Call Forward Parity
American Option Pricing
American Options>= Corresponding European Options
American Call
No Dividend,No Early Exercise: American Call is Always worth more in the market than exercised
Dividend/interest Make early exercise worthwhile
Carrying Cost weaken the motivation of early exercise
American Put
No dividend: Strong Motivation for early exercise(Has Upper Limit)
Dividend/interest: Discourage Early Exercise
Carrying Cost: Encourrage Early Exercise
Binomial Valuation of Option
Risk-Neutral Probability
Pr= (1+r-d)/(u-d)
European Option
Binomial Model
BSM Model
American Option
Binomial Model

Value of European Option at Exp

- Value(call option at expiration) = Exercise Value/Intrinsic Value

$$c_T = \text{Max}(0, S_T - X)$$

- Value (put option at expiration) = Exercise Value/Intrinsic Value

$$p_T = \text{Max}(0, X - S_T)$$

The value of a European call at expiration is the exercise value, which is the greater of zero or the value of the underlying minus the exercise price.

The value of a European put at expiration is the exercise value, which is the greater of zero or the exercise price minus the value of the underlying.

Factors Affect Option Price

- ***Factors affect the value of an option***

Sensitivity Factor	Calls	Puts
Underlying price	Positively related	Negatively related
Volatility	Positively related	Positively related
Risk-free rate	Positively related	Negatively related
Time to expiration	Positively related	Positively related
Strike price	Negatively related	Positively related
Payments on the underlying	Negatively related	Positively related
Carrying cost	Positively related	Negatively related

Factors - Volatility

Higher Volatility => More Extreme Value of the Underlying

Case 1: Higher Value of Underlying at Expiration, Higher Payoff of call option at Expiration

Expiring more in the money is better than less in the money. (>)

Case 2: Lower Value of Underlying at Expiration, Remain 0 payoff of call option at Expiration

Expiring more out of money is not worse than less out of money.(=)

=> Higher Volatility => Better Payoff at expiration

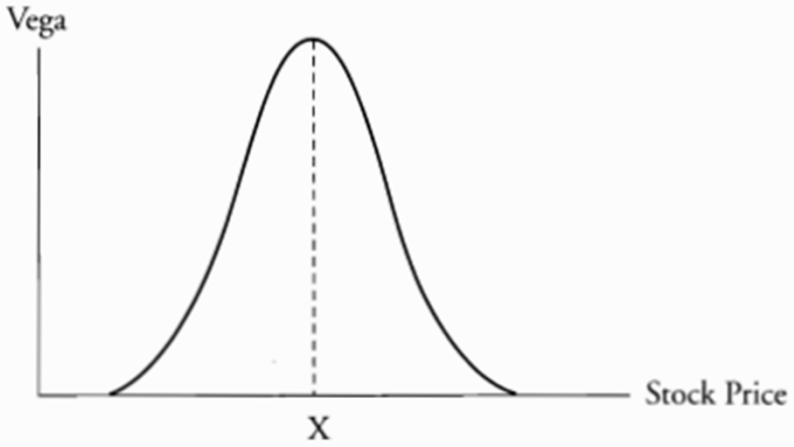
Factors – Volatility(2)

Greek Letters Of Option (FRM)

$$\text{vega} = S_0 N'(d_1)\sqrt{T}$$

$$N'(x) = \frac{1}{\sqrt{2\pi}} e^{-\left(x^2/2\right)}$$

Figure 5: Vega of a Stock Option



Example

2 Which of the following statements imply that a European call on a stock is worth more?

A Less time to expiration

B A higher stock price relative to the exercise price

C Larger dividends paid by the stock during the life of the option

4 The loss in value of an option as it moves closer to expiration is called what?

A Time value decay

B Volatility diminution

C Time value of money

Solution to 2:

B is correct. The higher the stock price and the lower the exercise price, the more valuable is the call. Less time to expiration and larger dividends reduce the value of the call.

Solution to 4:

A is correct. An option has time value that decays as the expiration approaches. There is no such concept as volatility diminution. Time value of money relates only to the value of money at one point in time versus another.

Lower & Upper Bound of Option(1)

- Lower & Upper Bound of Call Option

Call VS Leverage(Margin) Transaction			
Time:0	Call	>=	Leverage(Margin) Transaction
$c_0 < S_0$	c_0	>=	$S_0 - X/(1+r)^T$
Time:T	Call	>=	Leverage(Margin) Transaction
Case1: St < X	0	>	St-X < 0
Case2: St >= X	St-X	=	St-X >=0

- Lower Bound of European Put Option

PUT VS Short-Sell & Bond Purchase			
Time:0	PUT	>=	Short-Sell & Bond Purchase
$p_0 < X/(1+r)^T$	p_0	>=	$X/(1+r)^T - S_0$
Time:T	PUT	>=	Short-Sell & Bond Purchase
Case1: St < X	X-St	=	X-St >=0
Case2: St >= X	0	>=	X-St <=0

Lower & Upper Bound of Option(2)

- Lower & Upper Bound of Option Value

Option	Min Value	Max Value
European call	$\text{Max}[0 , S_t - X/(1+R_f)^{T-t}]$	S_t
American call	$\text{Max}[0 , S_t - X/(1+R_f)^{T-t}]$	S_t
European put	$\text{Max}[0 , X/(1+R_f)^{T-t} - S_t]$	$X/(1+R_f)^{T-t}$
American put	$P_t \geq \text{Max}[0 , X - S_t]$	X

Example

- A European stock index call option has a strike price of $1160 and a time to expiration of 0.25 years. Given a risk-free rate of 4%, if the underlying index is trading at $1,200 and has a multiplier of 1, then the lower bound for the option price is closest to:
  - A. $ 0.00.
  - B. $28.29.
  - C. $51.32.
- Correct answer: C
- **Solution**
  - The lower bound on a European call is either zero or the underlying price minus the present value of the exercise price, whichever is greater. $1200 - ($1160 / 1.04^0.25) = $51.32.

Put-Call Parity-Fiduciary Call

- Fiduciary Call is a portfolio consisting of:
- A long position in a European call option with an exercise price of X that maturities in T years on a stock.
- A long position in a pure-discount risk less bond that pays X in T years

Fiduciary Call			
Time:0	long Call	Long Bond	Total
	c_0	$X/(1+r)^T$	$C_0 + X/(1+r)^T$
Time:T	long Call	Long Bond	Total
Case1: St < X	0	X	X
Case2: St >= X	St-X	X	St

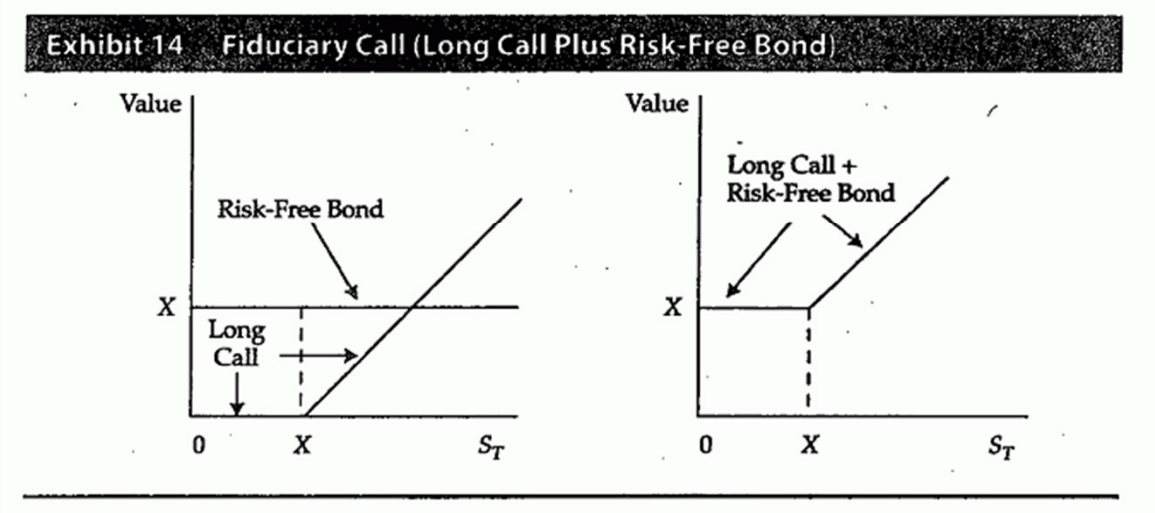
Put-Call Parity- Protective Put

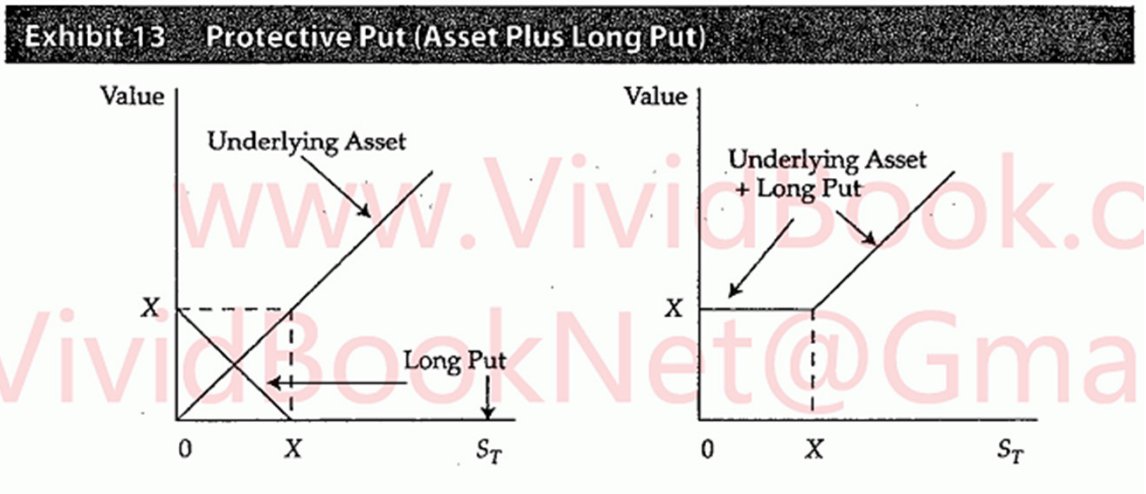
- Protective Put is a portfolio consisting of:

- A long position in a European put option with an exercise price of X that maturities in T years on a stock.
- A long position in the underlying stock

Protective Put			
Time:0	long Put	Long Stock	Total
	p_0	S_0	p_0+S_0
Time:T	long Put	Long Stock	Total
Case1: St < X	X-St	St	X
Case2: St >= X	0	St	St

Put-Call Parity- Graph





Put-Call Parity Formula

- Put-Call Parity : Not Applicable to American Options (unless they are held to expiration)
- American Options allow early exercise, Early exercise will result in a departure in the present values of the two portfolios.)

- *Put call parity.*

$$c + X/(1+R_f)^T = S + p$$

$$或 c + K/(1+R_f)^T = S + p$$

- *Positions replicating*
  - *Condition A* $-s = -c + p - X/(1+R_f)^T$
  - *Condition B* $p = c + X/(1+R_f)^T - S$
  - *Condition C* $c = p + S - X/(1+R_f)^T$
  - *Condition D* $-p = -c + S - X/(1+R_f)^T$
  - *Condition E* $-c = -p + X/(1+R_f)^T - S$

Example: Exploit violations of parity

➢ Example: Exploit violations of put-call parity

- 90-day European call and put options with a strike price of \$45 is priced at \$7.50 and \$3.70. The underlying is priced at \$48 and makes no cash payments during the life of the options. The risk-free rate is 5%. Calculate the no-arbitrage price of the call option, and illustrate how to earn an arbitrage profit.

➢ Answer:

$C_0 = P_0 + S_0 - X/(1+R_f)^T = \$3.70 + \$48 - \$45/1.05^{90/365} = \$7.24 < \$7.5$

Since the call is overpriced

- we should sell the call for \$7.50 and buy the synthetic call for \$7.24.
- To buy the synthetic call, buy the put for \$3.70, buy the underlying for \$48, and issue (sell short) a 90-day zero-coupon bond with a face value of \$45.
- The transaction will generate an arbitrage profit of \$0.26 today.

Fiduciary Call VS Covered Call

➢ Covered Call

- A covered call is the combination of a long stock and a short call

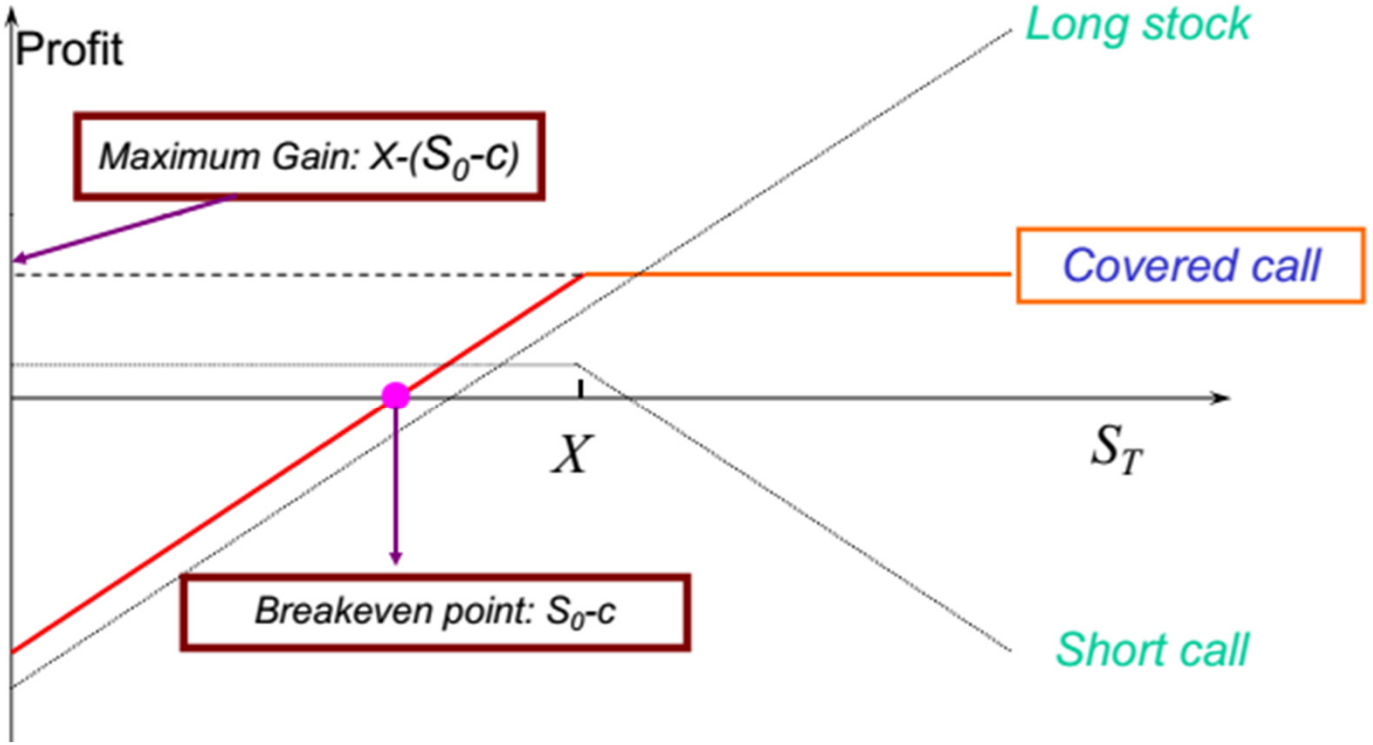
Covered call=S－C

- The term covered means that the stock covers the inherent obligation assumed in writing the call
- Why would you write a covered call? You feel the stock's price will not go up any time soon, and you want to increase your income by collecting some call option premiums.
- This strategy for enhancing income is not without risk. The call writer is trades the stock's upside potential, above the strike price, for the call premium. .

Fiduciary Call VS Covered Call(2)

- **Covered Call**

$$\text{Covered Call} = -c+S = -p+\frac{X}{(1+r_f)^{T-t}}$$



Warrant Buffet's Exlample

- Coca-Cola: Lower Purchase Price

April 1993, Sold 5 million shares out-of-the-money Coca-Cola put options(X=$35)

Received Premium: 1.5 * 5M => 7.5M$

- Berkshire's 2008 Letter to shareholders

Our put contracts total $37.1 billion: S&P 500, FTSE 100, Euro Stoxx 50, and Nikkei 225. Our first contract comes due on September 9, 2019 and our last on January 24, 2028. ***We have received premiums of $4.9 billion****, money we have invested.*

Quotes

- CBOE: https://www.cboe.com/delayedquote/quotetable.aspx?ticker=SPX

SPX(S&P 500 INDEX) Options Chain

Exchange: CBOE | Options Range: Near the Money | Expiration: 2016 July | View Chain

Jul 05, 2016 @ 04:34 ET — Last 2102.95 — Change 0.0

Calls — JULY 2016 (EXPIRATION: 07/06)

Strike	Last	Net	Bid	Ask	Vol	Int
SPXW1606G2095-E	13.05	0.0	4.10	5.90	0	396
SPXW1606G2100-E	10.00	0.0	2.25	3.70	0	4427
SPXW1606G2105-E	7.00	0.0	0.90	2.15	0	1508
SPXW1606G2110-E	4.40	0.0	0.15	1.25	0	1569

Puts — JULY 2016 (EXPIRATION: 07/06)

Strike	Last	Net	Bid	Ask	Vol	Int
SPXW1606S2095-E	7.44	+2.16	7.30	9.50	2	238
SPXW1606S2100-E	6.65	0.0	10.10	12.50	0	1990
SPXW1606S2105-E	8.40	0.0	13.30	15.90	0	1026
SPXW1606S2110-E	10.75	0.0	17.80	20.60	0	291

- SSE: http://stock.finance.sina.com.cn/option/quotes.html

上交所 50ETF 2016-07 到期日：2016-07-27 （22 天） 标的资产：华夏上证50ETF （2.1850 / +0.37%）

看涨合约									看跌合约							
买量	买价	最新价	卖价	卖量	持仓量	振幅	涨跌幅	行权价	买量	买价	最新价	卖价	卖量	持仓量	振幅	涨跌幅
2	0.2282	0.2295	0.2298	11	6934	6.4%	2.73%	1.9500	54	0.0009	0.0009	0.0010	14	27312	55.56%	0%
1	0.1797	0.1800	0.1800	9	13185	7.37%	2.8%	2.0000	14	0.0013	0.0016	0.0016	38	37011	41.67%	33.33%
3	0.1316	0.1318	0.1320	1	17631	10.66%	4.11%	2.0500	35	0.0027	0.0027	0.0029	20	36496	40%	-10%
19	0.0861	0.0861	0.0863	1	41609	14.93%	5.39%	2.1000 主	10	0.0076	0.0077	0.0077	62	61698	30.21%	-19.79%
23	0.0480	0.0481	0.0481	43	52612	24.26%	10.07%	主 2.1500	18	0.0192	0.0193	0.0193	2	48689	19.91%	-10.65%
11	0.0224	0.0225	0.0225	250	50763	34.62%	8.17%	2.2000	11	0.0438	0.0440	0.0440	2	25370	16.88%	-4.76%
10	0.0097	0.0099	0.0099	79	42743	46.34%	20.73%	2.2500	3	0.0804	0.0804	0.0805	10	12947	12.82%	-5.41%
10	0.0040	0.0041	0.0041	133	4986	100%	70.83%	2.3000	1	0.1243	0.1247	0.1248	1	1147	8.67%	-0.8%

红色背景为实值合约，白色背景为平值、虚值合约

主力看涨合约 | 主力看跌合约

合约简称	50ETF购7月2150
交易代码	510050C1607M02150
理论价值	0.0531
价值状态	实值
内在价值	0.0350
时间价值	0.0131
持仓量 / 占 比	52612 / 0.88%
最新价 / 涨 幅	0.0481 / 113.78%