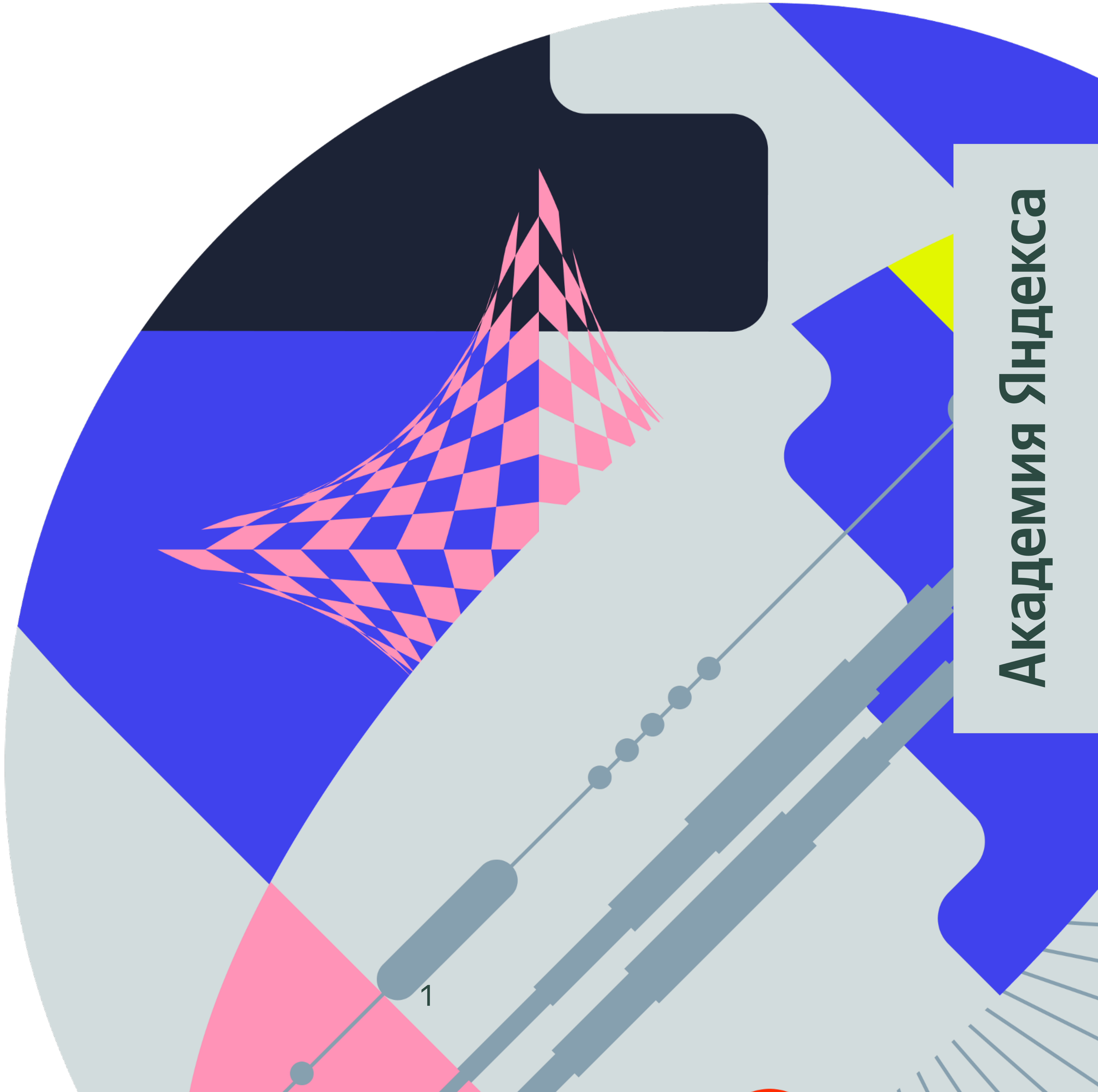
Visual Generative Modeling 2025



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Yandex Research

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$$\mathrm{DM} \stackrel{?}{==} \mathrm{FM}$$

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In the FM, we have the following:

$$\boldsymbol{x} = (1-t)\boldsymbol{x}_0 + \boldsymbol{\epsilon} t,\ t \in [0,1],\ \mathrm{d}\boldsymbol{x} = \boldsymbol{u}_{\boldsymbol{\theta}}(\boldsymbol{x}, t)\mathrm{d}t,\ \boldsymbol{u}_{\boldsymbol{\theta}} := \boldsymbol{\epsilon}_{\boldsymbol{\theta}} - \boldsymbol{x}_{\boldsymbol{\theta}}$$

Let's show that FM is a special case of DMs. The basic equation in DMs:

$$\mathrm{d}\boldsymbol{x} = \left[\boldsymbol{x} f(t) - \frac{1}{2} g(t)^2 \nabla_{\boldsymbol{x}} \log p_t(\boldsymbol{x})\right] \mathrm{d}t\,.$$

Which $f(t)$ and $g(t)$ should be to obtain the FM forward process? To this end, lets remember the connection between $f(t)$, $g(t)$ and statistics of the forward process (expectation and variance).

$$\mathrm{d}\mathbb{E}_{\boldsymbol{x}}\boldsymbol{x} = f(t)\mathbb{E}_{\boldsymbol{x}}\boldsymbol{x}\mathrm{d}t,\ \mathbb{E}_{\boldsymbol{x}}\boldsymbol{x}(0) = \boldsymbol{x}_0\,.$$

$$\mathrm{d}\mathbb{D}_{\boldsymbol{x}}\boldsymbol{x} = \left(2f(t)\mathbb{D}_{\boldsymbol{x}}\boldsymbol{x} + g(t)^2\right)\mathrm{d}t, \mathbb{D}_{\boldsymbol{x}}\boldsymbol{x}(0) = 0.$$

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$$f(t) = -\frac{1}{1-t}. \qquad\qquad g(t)^2 = \frac{2t}{1-t}.$$

$$\nabla_{\boldsymbol{x}} \log p_t(\boldsymbol{x}) \approx -\frac{1}{t^2}\left(\boldsymbol{x} - (1-t)\boldsymbol{x}_{\boldsymbol{\theta}}\right) = -\frac{\boldsymbol{\epsilon}_{\boldsymbol{\theta}}}{t}$$

$$\mathrm{d}\boldsymbol{x} = \left[-\boldsymbol{x}\frac{1}{1-t} + \frac{1}{2}\frac{2t}{1-t}\frac{\boldsymbol{\epsilon}_{\boldsymbol{\theta}}}{t}\right]\mathrm{d}t,$$

$$\mathrm{d}\boldsymbol{x} = \frac{1}{1-t}\left[-(1-t)\boldsymbol{x}_{\boldsymbol{\theta}} - \boldsymbol{\epsilon}_{\boldsymbol{\theta}}t + \boldsymbol{\epsilon}_{\boldsymbol{\theta}}\right]\mathrm{d}t,$$

$$\mathrm{d}\boldsymbol{x} = \boldsymbol{u}_{\boldsymbol{\theta}}\mathrm{d}t$$

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DM == FM

FM is special case of DM with specific noising process

Lecture 3 | Numerical solvers

1. Summary (recap from the previous lectures)
2. Basics of numerical solution of ODE
3. DPM-solver
4. Sampling schedules

Noising process → ODE (SDE) → Training of NN → Inference using solver

	Noising process	ODE	Parameterization
General case	$\boldsymbol{x}_t = \alpha_t \boldsymbol{x}_0 + \sigma_t \boldsymbol{z},$ $t \in [0,T], z \sim \mathcal{N}(0,1)$	$\mathrm{d}\boldsymbol{x} = \left[\boldsymbol{x} f(t) - \frac{1}{2} g(t)^2 \nabla_{\boldsymbol{x}} \log p_t(\boldsymbol{x})\right] \mathrm{d}t$ $f(t) = \frac{\mathrm{d}\log\alpha_t}{\mathrm{d}t}, g^2(t) = \frac{\mathrm{d}\sigma_t^2}{\mathrm{d}t} - 2\frac{\mathrm{d}\log\alpha_t}{\mathrm{d}t}\sigma_t^2$	$\nabla_{\boldsymbol{x}} \log p_t(\boldsymbol{x}) = -\frac{\boldsymbol{x} - \alpha_t \mathbb{E}\boldsymbol{x}_0 \mid \boldsymbol{x}}{\sigma_t^2}$ $\boldsymbol{\epsilon} = \frac{\boldsymbol{x} - \alpha_t \mathbb{E}\boldsymbol{x}_0 \mid \boldsymbol{x}}{\sigma_t}, \quad \mathbb{E}\boldsymbol{x}_0 \mid \boldsymbol{x} \approx \boldsymbol{x}_0$
VP	$\boldsymbol{x}_t = \sqrt{\alpha_t}\boldsymbol{x}_0 + \sqrt{1-\alpha_t}\boldsymbol{z}$ $t \in [0,T], z \sim \mathcal{N}(0,1)$	$\mathrm{d}\boldsymbol{x} = \frac{1}{2\alpha_t}\left[x + \nabla_{\boldsymbol{x}} \log p_t(\boldsymbol{x})\right] \mathrm{d}\alpha_t$	$\nabla_{\boldsymbol{x}} \log p_t(\boldsymbol{x}) = -\frac{\boldsymbol{x} - \sqrt{\alpha_t} \mathbb{E}\boldsymbol{x}_0 \mid \boldsymbol{x}}{1-\alpha_t}$
VE	$\boldsymbol{x}_t = \boldsymbol{x}_0 + t\boldsymbol{z}$ $t \in [0,T], z \sim \mathcal{N}(0,1)$	$\mathrm{d}\boldsymbol{x} = -t \nabla_{\boldsymbol{x}} \log p_t(\boldsymbol{x}) \mathrm{d}t$	$\nabla_{\boldsymbol{x}} \log p_t(\boldsymbol{x}) = -\frac{\boldsymbol{x} - \mathbb{E}\boldsymbol{x}_0 \mid \boldsymbol{x}}{t^2}$
FM (RF)	$\boldsymbol{x}_t = (1-t)\boldsymbol{x}_0 + t\boldsymbol{z}$ $t \in [0,1], z \sim \mathcal{N}(0,1)$	$\mathrm{d}\boldsymbol{x} = -\frac{1}{1-t}\left[x + t\nabla_{\boldsymbol{x}} \log p_t(\boldsymbol{x})\right] \mathrm{d}t$ $\mathrm{d}\boldsymbol{x} = v(x,t)\mathrm{d}t$	$\nabla_{\boldsymbol{x}} \log p_t(\boldsymbol{x}) = -\frac{\boldsymbol{x} - (1-t) \mathbb{E}\boldsymbol{x}_0 \mid \boldsymbol{x}}{t^2}$ $\boldsymbol{v} = \boldsymbol{\epsilon} - \mathbb{E}\boldsymbol{x}_0 \mid \boldsymbol{x}$

1. $\mathbb{E}_{\boldsymbol{x}_0,\, t,\, \boldsymbol{x}_t}\|\boldsymbol{\epsilon}_\theta(\boldsymbol{x}_t, t) - \boldsymbol{\epsilon}\|$ 2. $\mathbb{E}_{\boldsymbol{x}_0,\, t,\, \boldsymbol{x}_t}\|\boldsymbol{s}_\theta(\boldsymbol{x}_t, t) - \nabla_{\boldsymbol{x}_t} \log p_t(\boldsymbol{x}_t)\|$ 3. $\mathbb{E}_{\boldsymbol{x}_0,\, t,\, \boldsymbol{x}_t}\|\boldsymbol{x}_\theta(\boldsymbol{x}_t, t) - \boldsymbol{x}_0\|$

1. Conditional expectation

$$\mathbb{E}_{\boldsymbol{x}_0,\ t,\ \boldsymbol{x}_t} \| \boldsymbol{x}_\theta(\boldsymbol{x}_t, t) - \boldsymbol{x}_0 \|$$

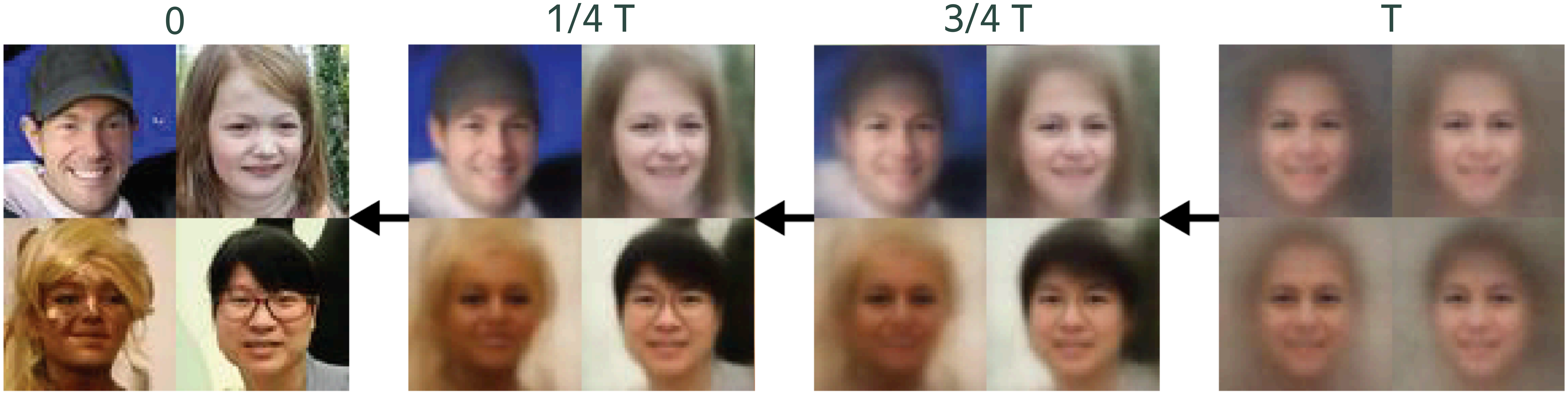
$$\boldsymbol{x}_\theta(\boldsymbol{x}, t) \approx \mathbb{E} \boldsymbol{x}_0 \,|\, \boldsymbol{x} = \int \boldsymbol{x}_0 p_t(\boldsymbol{x}_0 \,|\, \boldsymbol{x}) \mathrm{d}\boldsymbol{x}_0 \,.$$

$$p_t(\boldsymbol{x}_0 \,|\, \boldsymbol{x}) = \frac{p_t(\boldsymbol{x} \,|\, \boldsymbol{x}_0) p(\boldsymbol{x}_0)}{\int p_t(\boldsymbol{x} \,|\, \boldsymbol{x}_0) p(\boldsymbol{x}_0) \mathrm{d}\boldsymbol{x}_0}, \; p_{data}(\boldsymbol{x}_0) = \frac{1}{N} \sum_{j=1}^{N} \delta(\boldsymbol{x}_0 - \boldsymbol{x}_0^j) \,.$$

$$\mathbb{E} \boldsymbol{x}_0 \,|\, \boldsymbol{x} = \int \boldsymbol{x}_0 \frac{p_t(\boldsymbol{x} \,|\, \boldsymbol{x}_0) p(\boldsymbol{x}_0)}{\int p_t(\boldsymbol{x} \,|\, \boldsymbol{x}_0) p(\boldsymbol{x}_0) \mathrm{d}\boldsymbol{x}_0} \mathrm{d}\boldsymbol{x}_0 = \ldots = \frac{\sum_{j=1}^{N} \boldsymbol{x}_0^j \; \mathcal{N}\left(\boldsymbol{x} \mid \alpha_t \boldsymbol{x}_0^j, \sigma_t^2 \right)}{\sum_{j=1}^{N} \mathcal{N}\left(\boldsymbol{x} \mid \alpha_t \boldsymbol{x}_0^j, \sigma_t^2 \right)} \,.$$

1. Conditional expectation

$$\mathbb{E}\boldsymbol{x}_0 | \boldsymbol{x} = \frac{\sum_{j=1}^{N} \boldsymbol{x}_0^j \, \mathcal{N}\left(\boldsymbol{x} \mid \alpha_t \boldsymbol{x}_0^j, \sigma_t^2\right)}{\sum_{j=1}^{N} \mathcal{N}\left(\boldsymbol{x} \mid \alpha_t \boldsymbol{x}_0^j, \sigma_t^2\right)}.$$



We make the images less and less averaged, coarse-to-fine generation

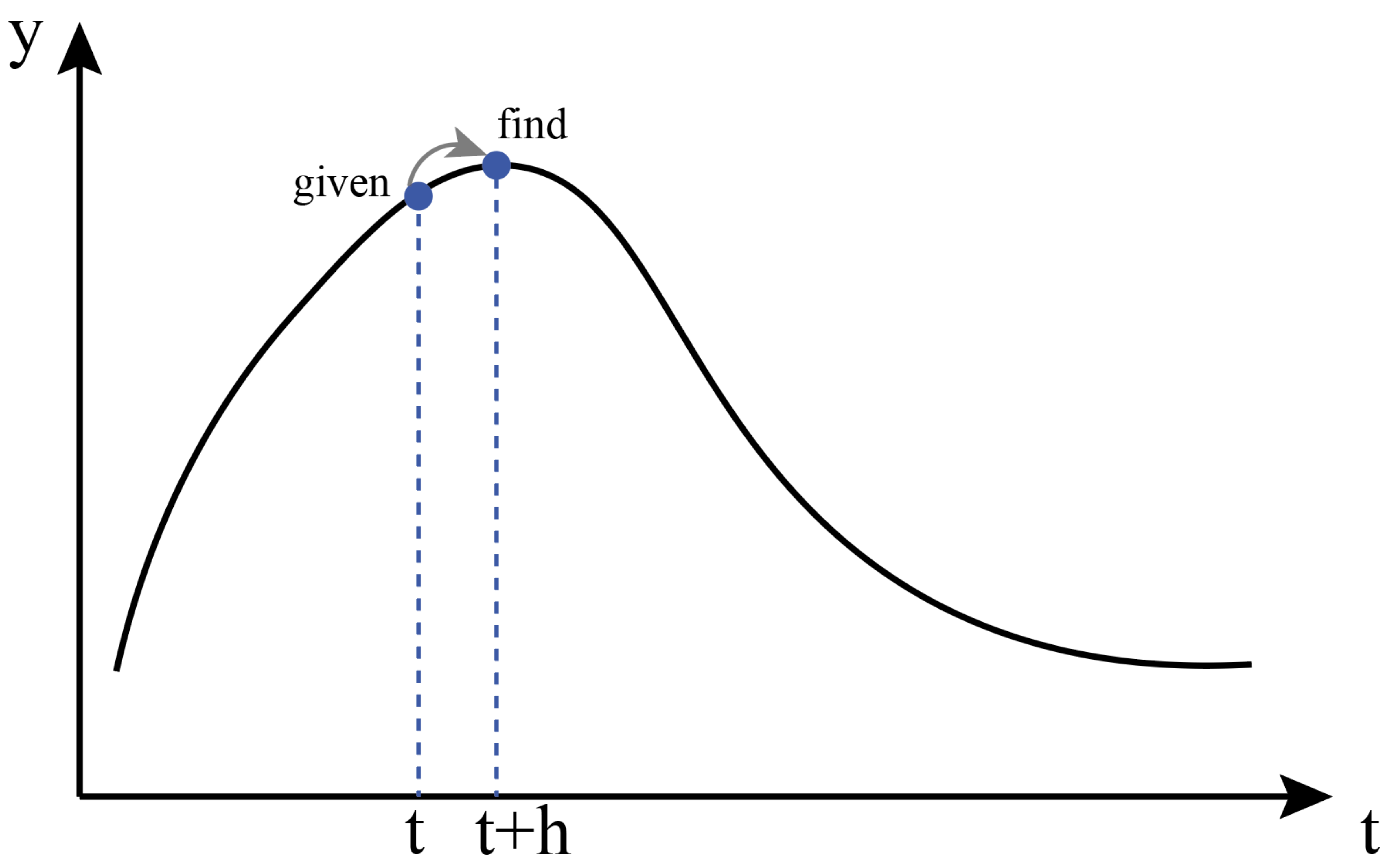
2. Basics of numerical solution of ODE

Cauchy problem

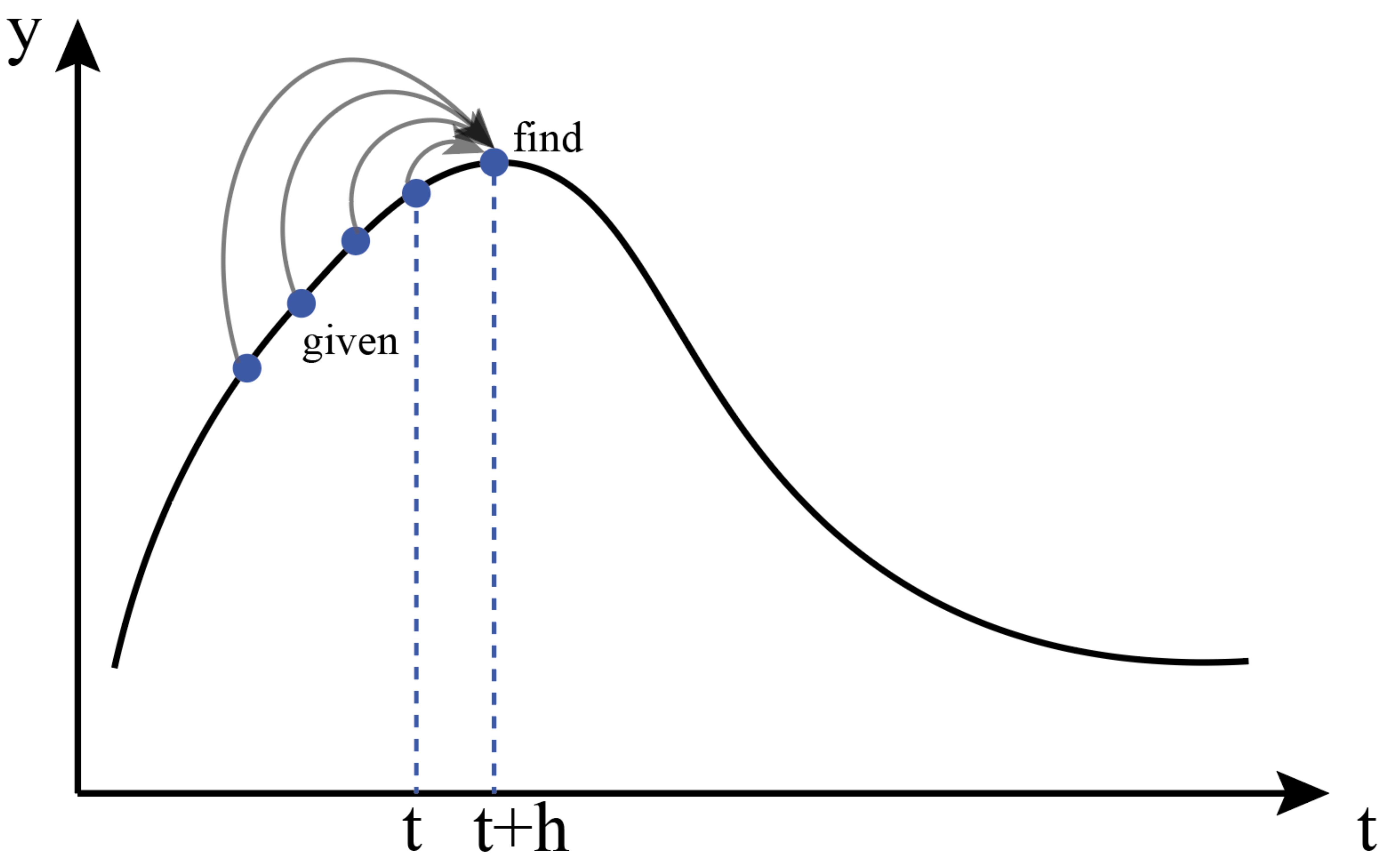
$$\mathrm{d}y = f(t, y(t))\mathrm{d}t,\ y(0) = y_0$$

Numerical solvers $[t_0, \dots, t_N], h = t_{i+1} - t_i$

Singlestep $y_{i+1} = y_i + \Phi(y_i, h)$



Multistep $y_{i+1} = y_i + \Phi(y_i, y_{i-1}, \dots, h)$



Singlestep $y_{i+1} = y_i + \Phi(y_i, h)$

Multistep $y_{i+1} = y_i + \Phi(y_i, y_{i-1}, \ldots, h)$

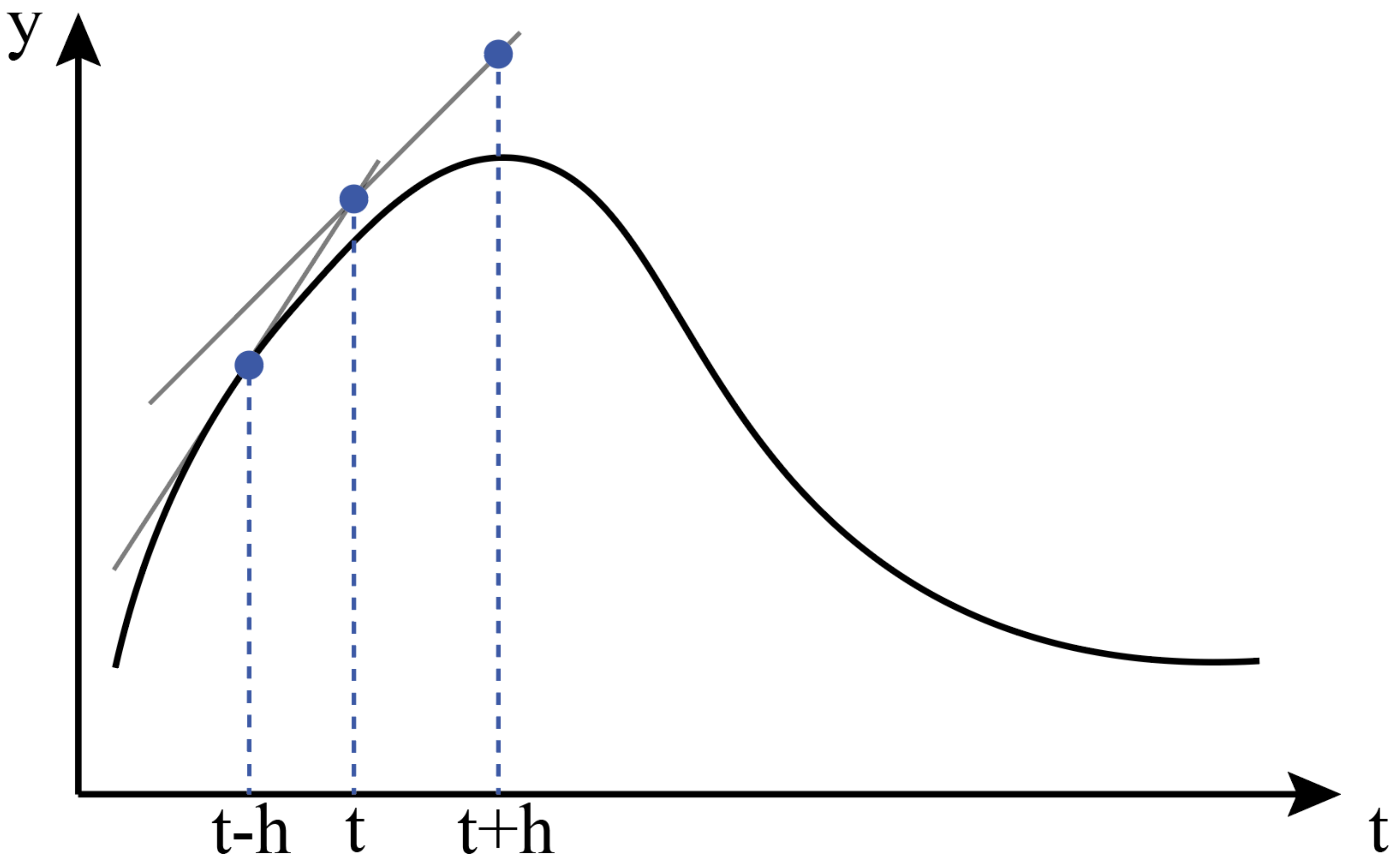
Implicit $y_{i+1} = y_i + \Phi(y_{i+1}, y_i, h)$

Implicit $y_{i+1} = y_i + \Phi(y_{i+1}, y_i, y_{i-1}, \ldots, h)$

Examples:

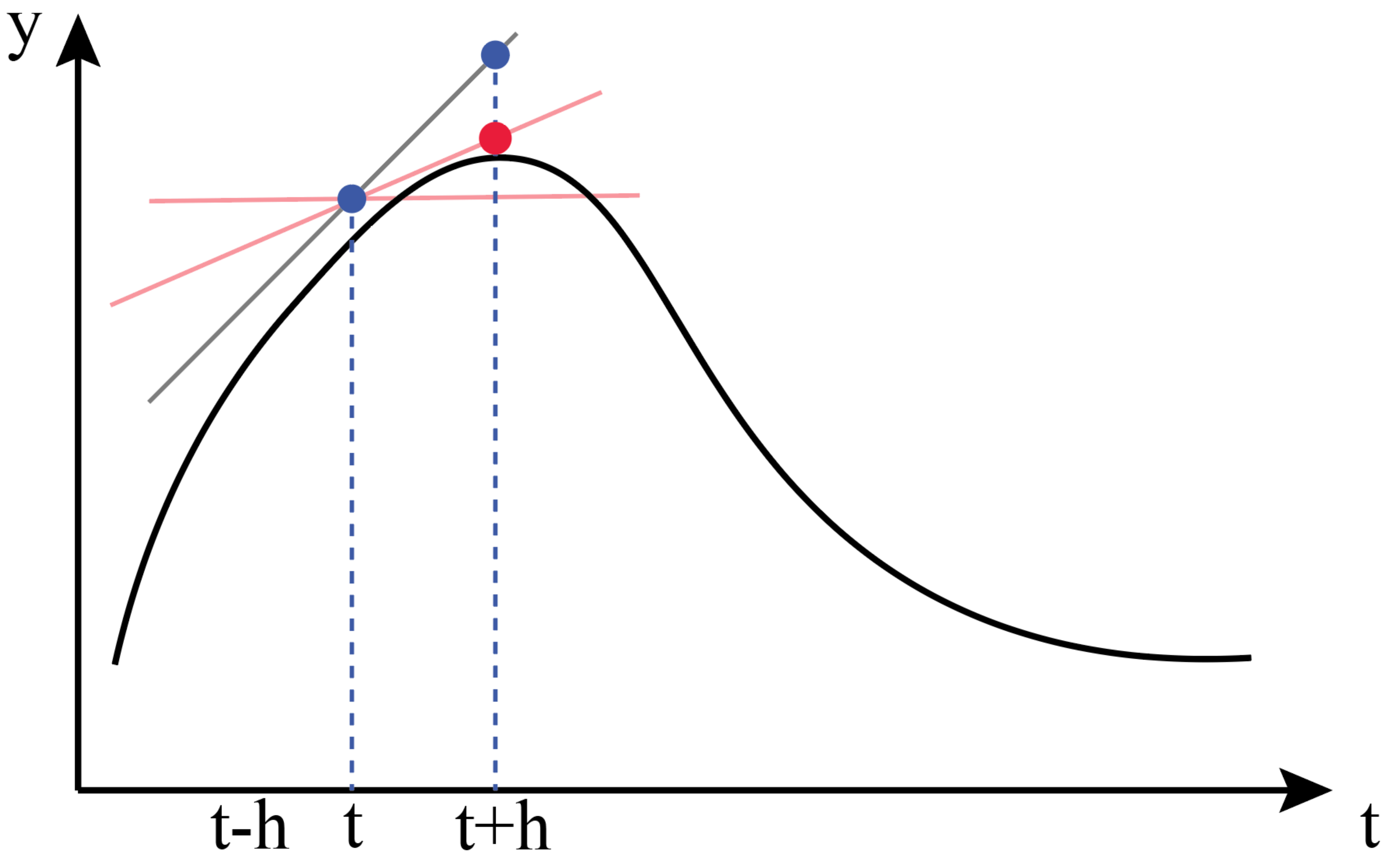
* Euler (singlestep, explicit)

$$y_{i+1} = y_i + hf(y_i, t_i)$$



* Trapezoidal rule (singlestep, implicit)

$$y_{i+1} = y_i + \frac{h}{2}\left(f(y_i, t_i) + f(y_{i+1}, t_{i+1})\right)$$



How to understand how accurate is your solver?

1. Local truncation error (we assume previous steps are exact)

$$\tau_{i+1} = \|y_{i+1} - y(t_{i+1})\|$$

* Euler

$$y(t_{i+1}) = y_i + hy_i' + \frac{h^2}{2}y_i'' + O(h^3)$$
$$y_{i+1} = y_i + hf(y_i, t_i),\ f(y_i, t_i) = y_i'$$
$$\tau_{i+1} = y_i + hy_i' - \left(y_i + hy_i' + \frac{h^2}{2}y_i'' + O(h^3) \right) = -\frac{h^2}{2}y_i'' + O(h^3) \sim O(h^2)$$

* Trapezoidal rule

$$\tau_{i+1} \sim O(h^3)$$

2. Global truncation error (accumulative error after N steps)

$$e_N = \|y_N - y(t_N)\| \qquad e_N \sim N\tau \sim \frac{\tau}{h}$$

* Euler

$$e_N \sim O(h)$$

$$\int_{t_i}^{t_{i+1}} \mathrm{d}y = \int_{t_i}^{t_{i+1}} f(t, y(t))\mathrm{d}t$$

$$y_{i+1} = y_i + \int_{t_i}^{t_{i+1}} f(t, y(t))\mathrm{d}t$$

$$f(t, y(t)) = \sum_{k=0} \frac{h^k}{k!} f^{(k)}(t_i, y_i) = f(t_i, y_i) + O(h)$$

$$y_{i+1} = y_i + f(t_i, y_i) + O(h^2)$$

* Trapezoidal rule

$$e_N \sim O(h^2)$$

Solver converges if:

1. LTE → 0, as h → 0

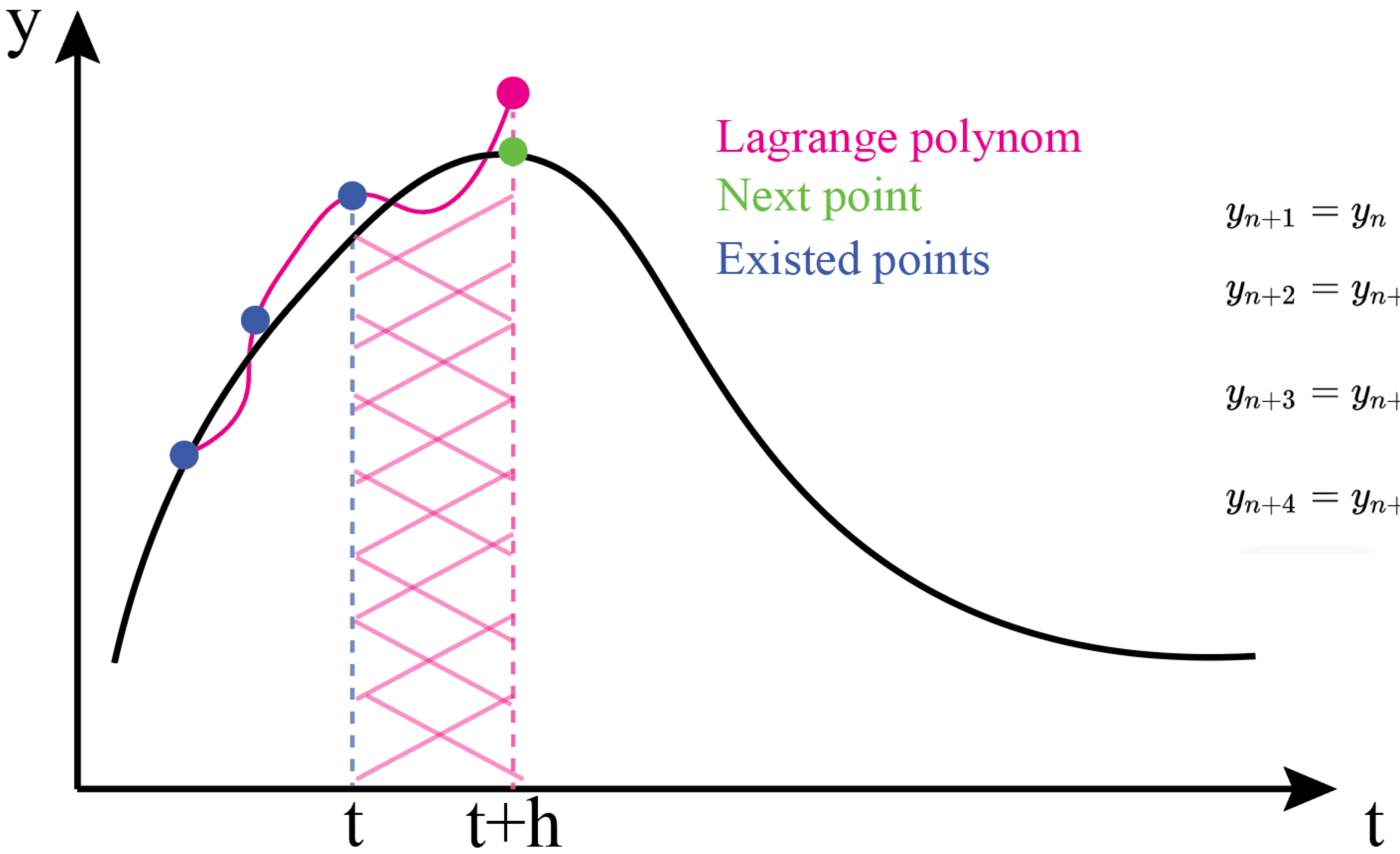
2. It is stable
small perturbations do not grow exponentially.

Multistep explicit methods (Adams-Bashforth methods)

Use past solution values and past derivative evaluations to compute the next step

$$y_{i+1} = y_i + \int_{t_i}^{t_{i+1}} f(t, y(t))\mathrm{d}t$$ Cannot integrate $f(t, y(t))$

$$f(t, y(t)) \approx p_{s-1}(t), \quad p_{s-1}(t) = \sum_{j=0}^{s-1} \frac{(-1)^{s-j-1} f(t_{n+j}, y_{n+j})}{j!(s-j-1)!h^{s-1}} \prod_{i=0}^{s-1} (t - t_{n+i})$$ Lagrange polynom



$$y_{n+1} = y_n + hf(t_n, y_n), \qquad \text{(This is the Euler method)}$$

$$y_{n+2} = y_{n+1} + h\left(\frac{3}{2}f(t_{n+1}, y_{n+1}) - \frac{1}{2}f(t_n, y_n)\right),$$

$$y_{n+3} = y_{n+2} + h\left(\frac{23}{12}f(t_{n+2}, y_{n+2}) - \frac{16}{12}f(t_{n+1}, y_{n+1}) + \frac{5}{12}f(t_n, y_n)\right),$$

$$y_{n+4} = y_{n+3} + h\left(\frac{55}{24}f(t_{n+3}, y_{n+3}) - \frac{59}{24}f(t_{n+2}, y_{n+2}) + \frac{37}{24}f(t_{n+1}, y_{n+1}) - \frac{9}{24}f(t_n, y_n)\right),$$

Multistep vs singlestep (DPM-solver specific)

$$y_{i+1} = y_i + \int_{t_i}^{t_{i+1}} f(t, y(t))\mathrm{d}t$$

$$f(t, y(t)) = \sum_{k=0} \frac{h^k}{k!} f^{(k)}(t_i, y_i) = f(t_i, y_i) + hf'(t_i, y_i) + O(h^2)$$

How to approximate $f'(t_i, y_i)$?

Singlestep (use next intermediate point)

$$f'(t_i, y_i) \approx \frac{f(t_{i+\delta_i}, y_{i+\delta_i}) - f(t_i, y_i)}{t_{i+\delta_i} - t_i}$$

Multistep (use previous point)

$$f'(t_i, y_i) \approx \frac{f(t_i, y_i) - f(t_{i-1}, y_{i-1})}{h}$$

Summary

I) Singlestep explicit $y_{i+1} = y_i + \Phi(y_i, h)$
 * Euler (1 NFE per step, 1st order)
 * RK 2 (2 NFE per step, 2nd order)

<u>Use the intermediate steps</u>

II) Singlestep implicit $y_{i+1} = y_i + \Phi(y_{i+1}, y_i, h)$
 * Backward Euler
 * RK 2 implicit, Heun (2 NFE per step, 2nd order)

<u>Use the next steps</u>

III) Multistep explicit $y_{i+1} = y_i + \Phi(y_i, y_{i-1}, \ldots, h)$
 * Adams-Bashforth (1 NFE, any order)

<u>Use the previous steps</u>

IV) Multistep implicit $y_{i+1} = y_i + \Phi(y_{i+1}, y_i, y_{i-1}, h)$
 * Adams-Moulton

<u>Use the next and previous steps</u>

V) Multistep + Singlestep
 * General linear methods

<u>Use the previous and intermediate steps</u>

2. DPM-solver (VP)

$$\mathrm{d}\boldsymbol{x} = \frac{1}{2\alpha_t}\left[x + \nabla_{\boldsymbol{x}} \log p_t(\boldsymbol{x})\right] \mathrm{d}\alpha_t$$

DDIM (Denoising Diffusion Implicit Models)

$$\boldsymbol{x}_t = \sqrt{\alpha_t}\left(\frac{\boldsymbol{x}_s - \sqrt{1-\alpha_s}\boldsymbol{\epsilon}_\theta(\boldsymbol{x}_s, s)}{\sqrt{\alpha_s}}\right) + \sqrt{1-\alpha_t}\boldsymbol{\epsilon}_\theta(\boldsymbol{x}_s, s)$$

What is DDIM? Just Euler?

2. DPM-solver (VP)

$$\mathrm{d}\boldsymbol{x} = \frac{1}{2\alpha_t}\left[\boldsymbol{x} + \nabla_{\boldsymbol{x}} \log p_t(\boldsymbol{x})\right] \mathrm{d}\alpha_t$$

DDIM (Denoising Diffusion Implicit Models)

$$\boldsymbol{x}_t = \sqrt{\alpha_t}\left(\frac{\boldsymbol{x}_s - \sqrt{1-\alpha_s}\boldsymbol{\epsilon}_\theta(\boldsymbol{x}_s, s)}{\sqrt{\alpha_s}}\right) + \sqrt{1-\alpha_t}\boldsymbol{\epsilon}_\theta(\boldsymbol{x}_s, s)$$

What is DDIM? Just Euler?

$$\mathrm{d}\boldsymbol{x} = \frac{1}{2\alpha_t}\left[\boldsymbol{x} - \frac{\boldsymbol{\epsilon}(\boldsymbol{x}, t)}{\sqrt{1-\alpha_t}}\right] \mathrm{d}\alpha_t$$

$$\boldsymbol{x}_t = \boldsymbol{x}_s + \int_s^t \frac{1}{2\alpha_\tau}\left[\boldsymbol{x} - \frac{\boldsymbol{\epsilon}(\boldsymbol{x}, \tau)}{\sqrt{1-\alpha_t}}\right] \mathrm{d}\alpha_\tau$$

$$\boldsymbol{x}_t = \frac{\alpha_t + \alpha_s}{2\alpha_s}\boldsymbol{x}_s - \frac{\alpha_t - \alpha_s}{2\alpha_s\sqrt{1-\alpha_s}}\boldsymbol{\epsilon}_\theta(\boldsymbol{x}_s, s)$$

$$\mathrm{d}\boldsymbol{x} = \frac{1}{2\alpha_t}\left[\boldsymbol{x} - \frac{\boldsymbol{\epsilon}(\boldsymbol{x}, t)}{\sqrt{1-\alpha_t}}\right]\mathrm{d}\alpha_t$$

$$\mathrm{d}\boldsymbol{x} = \left[\frac{\boldsymbol{x}}{2\alpha_t} - \frac{\boldsymbol{\epsilon}(\boldsymbol{x}, t)}{2\alpha_t\sqrt{1-\alpha_t}}\right]\mathrm{d}\alpha_t, s \to t$$

$$\phi(t,s)\mathrm{d}\boldsymbol{x} = \phi(t,s)\left[\frac{\boldsymbol{x}}{2\alpha_t} - \frac{\boldsymbol{\epsilon}(\boldsymbol{x}, t)}{2\alpha_t\sqrt{1-\alpha_t}}\right]\mathrm{d}\alpha_t,$$

$$\mathrm{d}\left[\boldsymbol{x}\phi(t,s)\right] = \phi(t,s)\left[\frac{\boldsymbol{x}}{2\alpha_t} - \frac{\boldsymbol{\epsilon}(\boldsymbol{x}, t)}{2\alpha_t\sqrt{1-\alpha_t}}\right]\mathrm{d}\alpha_t,$$

$$\mathrm{d}\phi(t,s) = \phi(t,s)\frac{1}{2\alpha_t}\mathrm{d}\alpha_t$$

$$\int_s^t \frac{\mathrm{d}\phi}{\phi} = \int_s^t \frac{\mathrm{d}\alpha}{2\alpha} \to \ln\phi(t,s) = \frac{1}{2}\left(\ln\alpha_t - \ln\alpha_s\right)$$

$$\phi(t,s) = \sqrt{\frac{\alpha_t}{\alpha_s}}$$

$$\boldsymbol{x}_t = \sqrt{\frac{\alpha_t}{\alpha_s}}\boldsymbol{x}_s - \frac{1}{2}\int_s^t \phi(t,\tau)\frac{\boldsymbol{\epsilon}(\boldsymbol{x}, \tau)}{\alpha_\tau\sqrt{1-\alpha_\tau}}\mathrm{d}\alpha_\tau,$$

$$\boldsymbol{x}_t = \sqrt{\frac{\alpha_t}{\alpha_s}}\boldsymbol{x}_s - \frac{\sqrt{\alpha_t}}{2}\int_s^t \frac{\boldsymbol{\epsilon}(\boldsymbol{x}, \tau)}{\sqrt{\alpha_\tau}\alpha_\tau\sqrt{1-\alpha_\tau}}\mathrm{d}\alpha_\tau,$$

$$\int_s^t \frac{\boldsymbol{\epsilon}(\boldsymbol{x}, \tau)}{\sqrt{\alpha_\tau}\alpha_\tau\sqrt{1-\alpha_\tau}}\mathrm{d}\alpha_\tau \approx \boldsymbol{\epsilon}(\boldsymbol{x}_s, s)\int_s^t \frac{1}{\sqrt{\alpha_\tau}\alpha_\tau\sqrt{1-\alpha_\tau}}\mathrm{d}\alpha_\tau$$

$$\int \frac{\mathrm{d}\alpha}{\sqrt{\alpha}\alpha\sqrt{1-\alpha}}$$

$$1-\alpha = \cos^2 u, \quad \alpha = \sin^2 u, \quad \mathrm{d}\alpha = 2\sin u \cos u \mathrm{d}u$$

$$\int \frac{2\mathrm{d}u}{\sin^2 u} = -2\mathrm{ctg} u = -2\mathrm{ctg}\arcsin\sqrt{\alpha} = -2\frac{\sqrt{1-\alpha}}{\sqrt{\alpha}}$$

DDIM = DPM-1

$$\boldsymbol{x}_t = \sqrt{\frac{\alpha_t}{\alpha_s}}\boldsymbol{x}_s - \frac{\sqrt{\alpha_t}}{2}\left(-2\frac{\sqrt{1-\alpha_t}}{\sqrt{\alpha_t}} + 2\frac{\sqrt{1-\alpha_s}}{\sqrt{\alpha_s}}\right)\boldsymbol{\epsilon}(\boldsymbol{x}_s, s)$$

$$\boxed{\boldsymbol{x}_t = \sqrt{\alpha_t}\left(\frac{\boldsymbol{x}_s - \sqrt{1-\alpha_s}\boldsymbol{\epsilon}_\theta(\boldsymbol{x}_s, s)}{\sqrt{\alpha_s}}\right) + \sqrt{1-\alpha_t}\boldsymbol{\epsilon}(\boldsymbol{x}_s, s)}$$

$$\boldsymbol{x}_t = \sqrt{\frac{\alpha_t}{\alpha_s}}\boldsymbol{x}_s - \frac{\sqrt{\alpha_t}}{2}\int_s^t \frac{\boldsymbol{\epsilon}(\boldsymbol{x}, \tau)}{\sqrt{\alpha_\tau}\alpha_\tau\sqrt{1-\alpha_\tau}}\mathrm{d}\alpha_\tau,$$

$$\boldsymbol{\epsilon}(\boldsymbol{x}, \tau) = \sum_{k=0}^{n-1} \frac{(\alpha_\tau - \alpha_s)^k}{k!}\boldsymbol{\epsilon}^{(k)}(\boldsymbol{x}_s, s)$$

Singlestep (use the intermediate point)

$$\boldsymbol{\epsilon}^{(1)}(\boldsymbol{x}_s, s) \approx \frac{\boldsymbol{\epsilon}(\boldsymbol{x}_{s-\delta_s}, s-\delta_s) - \boldsymbol{\epsilon}(\boldsymbol{x}_s, s)}{\alpha_{s-\delta_s} - \alpha_s}$$

Multistep (use the previous point)

$$\boldsymbol{\epsilon}^{(1)}(\boldsymbol{x}_s, s) \approx \frac{\boldsymbol{\epsilon}(\boldsymbol{x}_s, s) - \boldsymbol{\epsilon}(\boldsymbol{x}_{s+h}, s+h)}{\alpha_s - \alpha_{s+h}}$$

2. Sampling schedules

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