📘 Chapter 9: Areas of Parallelograms and Triangles

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🔹 Main Concepts and Results

📐 **Area:**
The region enclosed by a plane figure.

✅ Standard Unit: Square units (like cm^2, m^2)

🧠 **Important Properties:**

- 🔁 Congruent triangles have equal areas
  (If $\Delta ABC \cong \Delta PQR \Rightarrow ar(\Delta ABC) = ar(\Delta PQR)$)
- The diagonal of a parallelogram divides it into two triangles of equal area
- Parallelograms on the same base and between the same parallels have equal area
- A triangle on the same base and between same parallels = half the area of a parallelogram

📝 Example:
If parallelogram ABCD and triangle ABC are on same base and between same parallels,
Then:
$ar(\Delta ABC) = \frac{1}{2} \times ar(\text{parallelogram } ABCD)$

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📘 Theorems and Key Rules

🔸 **Theorem 1:** Diagonal of a parallelogram divides it into two equal-area triangles

🔸 **Theorem 2**: Parallelograms on the same base and between the same parallels are equal in area

🔸 **Theorem 3:** Triangles on same base and between same parallels have equal area

🔸 **Theorem 4:** Triangle = ½ × base × corresponding altitude

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🧠 Multiple Choice Practice

1️⃣ A diagonal of a parallelogram divides it into:
✅ Two equal-area triangles

2️⃣ Midpoints of adjacent sides of a rhombus with diagonals 12 cm and 16 cm form a square
Area = $\frac{1}{2} \times d_1 \times d_2 = \frac{1}{2} \times 12 \times 8 = 48\ cm^2$ ✅

3️⃣ If median divides triangle → it makes:
✅ Two triangles of equal area

4️⃣ Area of triangle = ½ base × height
If same base & same height = Equal areas ✅

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✏️ Short Answer: Reasoning Based

❓ True/False Type:

- If P is any point on the median, then ar(ABP) = ar(ACP) → ✅ True
- In parallelogram, joining midpoints of sides forms a rhombus/square of half area → ✅
- If diagonal bisects one angle → also divides figure into two equal parts → ✅
- Triangles on same base but not between same parallels → ❌ not always equal area

🧠 Example:
If a parallelogram has a diagonal AC, then ar(ΔABC) = ar(ΔCDA)

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📐 Geometry Application Examples

🔶 Joining midpoints of rectangle/square sides creates new figures (like rhombus)
🔶 Triangle with mid-segment parallel to third side divides triangle into two equal-area regions

🧠 Sample:

- In ΔABC, median AD is drawn. Then, triangles ABD and ACD have equal areas ✅
- In parallelogram, any point on diagonal → area of triangle from that point to corner = half parallelogram ✅

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🔷 Long Answer Questions (Theorems Applied)

🧠 Example:
In parallelogram ABCD, trisect side BC at points P & Q →
Show: ar(APQ) = ar(DPQ) = 1/6 ar(ABCD)
→ Use parallel lines, triangle area, and proportion

🧠 Midpoints of quadrilateral joined in order → forms a parallelogram of half the area ✅
(Hint: Draw diagonal and perpendicular from corner)

🧠 In triangle, medians intersecting at centroid G divide triangle into 3 equal-area parts:
ar(AGB) = ar(BGC) = ar(AGC) = 1/3 ar(ABC)

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📊 Summary Table:-

Rule / Concept	Result
Diagonal of parallelogram	Divides into 2 equal-area triangles
Triangle on same base and height	Equal areas
Triangle area formula	½ × base × height
Parallelograms on same base and between parallels	Equal area
Triangle & parallelogram on same base & height	Triangle = ½ area of parallelogram
Midpoints joined in quadrilateral	Forms parallelogram with ½ total area