📘 Chapter 7: Triangles

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🧠 Main Concepts & Results

📌 What is a Triangle?

A triangle is a polygon with 3 sides and 3 angles.

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🔷 Congruence of Triangles (≅)

🔲 Two triangles are congruent if all their corresponding sides and angles are equal.

📐 **Notation:**

ΔABC ≅ ΔPQR means:

- AB = PQ
- BC = QR
- CA = RP
- ∠A = ∠P, ∠B = ∠Q, ∠C = ∠R

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🔷 Criteria for Congruence of Triangles

✅ SSS (Side-Side-Side): All three sides are equal

✅ SAS (Side-Angle-Side): Two sides and the angle between them are equal

✅ ASA (Angle-Side-Angle): Two angles and the included side are equal

✅ RHS (Right angle-Hypotenuse-Side): For right-angled triangles only

✅ AAS (Angle-Angle-Side): Is derived from ASA

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✨ Important Properties

- Angles opposite to equal sides are equal
- Sides opposite to equal angles are equal
- A point equidistant from two points lies on the perpendicular bisector
- A point equidistant from two intersecting lines lies on the angle bisector
- In any triangle:
    - Side opposite the greater angle is longer
    - Angle opposite the longer side is greater
    - Sum of any two sides > third side

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🔍 MCQ Highlights

🧠 Sample:
 If ΔABC ≅ ΔPQR but not congruent to ΔRPQ, then which statement is false?

Correct Answer: (A) BC = PQ ❌ (corresponding sides mismatch)

🧠 Identify incorrect congruence rules:
 SSA ❌ is not a valid criterion

🧠 Equilateral Triangle Angles:
 All angles = 60°

🧠 Triangle with sides 5 cm, 1.5 cm, third side cannot be 3.6 cm — check triangle inequality!

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✍️ Reasoning-Based Short Questions

Example:
 In ΔABC and ΔDEF, AB = DE, AC = EF — if ∠A = ∠E, then ΔABC ≅ ΔEDF (SAS) ✅

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📐 Application-Based Theorems and Properties

✅ Vertically opposite angles are equal
✅ Triangle inequality: Sum of two sides > third
✅ Side opposite greater angle is longer
✅ If AD is the angle bisector and also bisects BC, then triangle is isosceles

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🔷 Important Geometry Results

📘 In an isosceles triangle, medians to equal sides are equal
📘 Exterior angle = sum of two opposite interior angles
📘 Equilateral triangle → All sides equal, all angles 60°
📘 If two triangles have 2 equal angles and included side → congruent (ASA)

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✏️ Long Answer Sample (with Reasoning)

🧠 Prove: In right triangle ABC, if ∠BCA = 2∠BAC, then hypotenuse AC = 2BC
 → Use triangle construction + congruence + angle sum

🧠 If angle bisector also bisects opposite side → triangle is isosceles
 → Use SAS congruence of two constructed triangles

🧠 If S is inside ΔPQR, prove SQ + SR < PQ + PR
 → Use triangle inequality in two parts and combine!

🔚 Real-World & Practical Thinking Problems

🧠 Equilateral triangle: All angles = 60° ✅

🧠 Mirror image: Distance from object to mirror = distance of image behind mirror ✅

🧠 In ΔABC, AB = AC and AD ⊥ BC → Prove ∠BAD = ∠CAD

→ Use congruence of triangles ABD & ACD

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💬 HOTS (Higher Order Thinking Skills)

🧠 If AB = AD and CB = CD in quadrilateral ABCD → Diagonal AC bisects ∠A and ∠C

🧠 If triangle has one side 9 cm and others 7 cm and 17 cm → Not possible (violates triangle inequality)

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📚 Summary Table

Concept	Rule / Result
Triangle Congruence	**SSS, SAS, ASA, RHS, AAS**
Vertically Opposite Angles	**Are always equal**
Triangle Inequality	**Any two sides' sum > third side**
Angle-side relationship	**Greater angle → longer opposite side**
Exterior Angle Theorem	**Ext. angle = sum of opposite interior angles**
Perpendicular bisector properties	**Equidistant from endpoints**
Angle Bisector Properties	**Point lies on bisector = equidistant from sides**