📚 Chapter 1: Real Numbers – Easy Notes + Step-by-Step Guide

🔸 1. **Euclid's Division Algorithm**

✅ What It Says:

For any two positive numbers aaa and bbb (where a>ba > ba>b):

a=bq+rwhere 0≤r<ba = bq + r \quad \text{where } 0 \le r < ba=bq+rwhere 0≤r<b

In simple words: Divide aaa by bbb, you'll get a quotient qqq and a remainder rrr smaller than bbb.

✅ Why Use It:

To find the **HCF (Highest Common Factor)** of two numbers.

📍 How to Use Euclid's Algorithm – Step-by-Step

Example: Find HCF of 455 and 42

- Step 1: Divide 455 by 42 →
  455=42×10+35455 = 42 \times 10 + 35455=42×10+35
- Step 2: Divide 42 by 35 →
  42=35×1+742 = 35 \times 1 + 742=35×1+7
- Step 3: Divide 35 by 7 →
  35=7×5+035 = 7 \times 5 + 035=7×5+0

✅ Since remainder is 0, **HCF = 7**

💡 **Tip:** Stop when remainder becomes 0. The last divisor is the HCF.

🔸 2. **Fundamental Theorem of Arithmetic**

✅ What It Says:

Every number **can be written as a product of prime numbers**, and the way you do it is **unique** (order doesn't matter).

Example:
32760 = 2 × 2 × 2 × 3 × 3 × 5 × 7 × 13 = 23×32×5×7×132^3 \times 3^2 \times 5 \times 7 \times 1323×32×5×7×13

📍 How to Use It

✅ **To find HCF and LCM using prime factorization:**

Example: Find HCF and LCM of 6, 72, and 120

- Step 1: Prime Factorize
  - 6 = 2 × 3
  - 72 = 23×322^3 × 3^223×32
  - 120 = 23×3×52^3 × 3 × 523×3×5
- Step 2:
  - **HCF** = Multiply smallest powers of common primes →
    HCF=21×31=6HCF = 2^1 × 3^1 = 6HCF=21×31=6
  - **LCM** = Multiply greatest powers of all primes →
    LCM=23×32×5=360LCM = 2^3 × 3^2 × 5 = 360LCM=23×32×5=360

✅ **Extra Tip:**
For 2 numbers,

HCF×LCM=Product of the two numbers\text{HCF} × \text{LCM} = \text{Product of the two numbers}HCF×LCM=Product of the two numbers

🔶 3. **Irrational Numbers**

✅ What Are They?

Numbers **that can't be written as** pq\frac{p}{q}qp
Examples: 2,3,π\sqrt{2}, \sqrt{3}, \pi2,3,π

✅ Proof Strategy:

We use **proof by contradiction**.
Assume the number *is rational*, and show this leads to a contradiction.

📍 How to Prove 2\sqrt{2}2 is Irrational – Step-by-Step

- Step 1: Suppose 2=ab\sqrt{2} = \frac{a}{b}2=ba, where aaa and bbb are co-prime
- Step 2: Square both sides
  → 2=a2b2⇒a2=2b22 = \frac{a^2}{b^2} \Rightarrow a^2 = 2b^22=b2a2⇒a2=2b2
- Step 3: a2a^2a2 is divisible by 2 → so aaa is divisible by 2
  → Let a=2ca = 2ca=2c
- Step 4: Put back in equation →
  (2c)2=2b2⇒4c2=2b2⇒b2=2c2(2c)^2 = 2b^2 \Rightarrow 4c^2 = 2b^2 \Rightarrow b^2 = 2c^2(2c)2=2b2⇒4c2=2b2⇒b2=2c2
- Step 5: Now bbb is also divisible by 2
- CONTRADICTION! (both a & b are divisible by 2 but we assumed they're co-prime)

✅ Therefore, 2\sqrt{2}2 is **irrational**

🔶 4. **Decimal Expansions of Rational Numbers**

✅ Rules:

Let pq\frac{p}{q}qp be a rational number in simplest form.

- **Terminating Decimal** → if qqq has only 2 and/or 5 as prime factors
- **Non-Terminating, Repeating Decimal** → if qqq has any other prime

📍 How to Check – Step-by-Step

Example: Is 13125\frac{13}{125}12513 terminating?

- Step 1: Factorize denominator
  125 = 535^353 → only prime 5
- ✅ So, **terminating decimal**

Example: Is 145\frac{1}{45}451 terminating?

- Step 1: Factorize 45 = 32×53^2 × 532×5 → has 3
- ❌ So, **non-terminating repeating decimal**

📝 Summary Table for Revision

Topic	What to Do/Check	Asked in Exams?
Euclid’s Division Algorithm	Divide repeatedly to get remainder = 0 (HCF)	⭐ Always
Prime Factorization	Break number into primes	⭐ Always
HCF and LCM	Use smallest/largest powers of primes	⭐ Always
Irrational Number Proof	Assume rational → reach contradiction	⭐ Always
Decimal Type (Terminating?)	Check denominator for only 2/5	⭐ Always