🟦 Chapter 12: Heron's Formula

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🔹 A. Main Concepts

Heron's Formula helps us find the area of a triangle when all three sides are known, and height is not given.

📄 **Important Formulas:**

1️⃣ **Semi-perimeter** (s) of triangle with sides a, b, c:
$s = (a + b + c) / 2$

2️⃣ **Area of triangle:**
$A = \sqrt{[s(s - a)(s - b)(s - c)]}$

This formula is known as Heron's Formula. It works for all types of triangles (scalene, isosceles, equilateral).

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🔹 B. Area Formulas of Special Quadrilaterals & Polygons

📐 **Rectangle:**

- Area = length × breadth
- Perimeter = 2 × (length + breadth)
- Diagonal = $\sqrt{(l^2 + b^2)}$

🟦 **Square:**

- Area = side²
- Perimeter = 4 × side
- Diagonal = √2 × side

🔺 **Triangle:**

- Area = ½ × base × height
- With Heron's Formula → when height not given

🔻 **Isosceles triangle (base = a, equal sides = b):**

- Area = ¼ × $\sqrt{(4b^2 - a^2)}$

🔺 **Equilateral triangle (side = a):**

- Area = (√3 / 4) × a^2

🟨 **Parallelogram:**

- Area = base × height

🔷 **Rhombus** (diagonals d_1 and d_2):

- Area = ½ × d_1 × d_2
- Perimeter = 4 × side or 2 × √(d_1^2 + d_2^2)

🧱 **Trapezium** (parallel sides a and b, height h):

- Area = ½ × (a + b) × h

⬡ **Regular Hexagon** (side = a):

- Area = (3√3 / 2) × a^2
  (= 6 × area of equilateral triangle)

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🔷 C. Key Use-Cases of Heron's Formula

- For finding the area of irregular triangles (non-right triangles)
- In word problems involving triangular plots, boards, or tiling
- Also helpful in finding cost (e.g. painting, fencing) based on area

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🔷 D. Concept Explanation

💡 Why use Heron's Formula?

In many problems, the height (altitude) of a triangle is not given, but the sides are. In such cases, we can't use the basic formula:
Area = ½ × base × height

Instead, we calculate semi-perimeter (s), then use Heron's Formula to find the area without knowing the height.

🧠 Tip: Always check if the sum of any two sides is greater than the third side. Otherwise, triangle is not valid.

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📏 E. Useful Notes and Tips

✅ Units: Always write units — cm^2, m^2 etc.
✅ Square roots: If options are in decimals, approximate roots
✅ Use Heron's Formula first, then extend to cost/tiles/perimeter questions
✅ For equilateral triangle: just use direct area formula instead of Heron's

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🧠 Example Walkthrough (without calculations):

Problem: A triangle has sides 7 cm, 8 cm, 9 cm. Find area.
Step 1: Find s = (7+8+9)/2 = 12
Step 2: Apply Heron's Formula:
A = √[12(12 − 7)(12 − 8)(12 − 9)]

A = √[12 × 5 × 4 × 3]
Answer = √720 = approx. 26.83 cm² ✅

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✅ Summary Table:-

Shape	Area Formula
Triangle	√[s(s−a)(s−b)(s−c)] (Heron's Formula)
Rectangle	length × breadth
Square	side²
Parallelogram	base × height
Rhombus	½ × d_1 × d_2
Trapezium	½ × (a + b) × h
Equilateral Triangle	(√3 / 4) × a²
Regular Hexagon	(3√3 / 2) × a²