📘 Chapter 10: Circles

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🔷 Basic Definitions

📌 **Circle:** The set of all points in a plane which are at a fixed distance (radius) from a fixed point (centre).

📌 Key Terms:

- Radius: Distance from centre to any point on the circle
- Diameter: A chord that passes through the centre (2 × radius)
- Chord: A line segment joining two points on the circle
- Arc: A part of the circle's boundary
- Segment: Area between chord and arc
- Sector: Area enclosed between two radii and the arc
- Cyclic Quadrilateral: A quadrilateral whose all vertices lie on a circle

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🧠 Key Results & Theorems

✅ Equal chords of a circle subtend equal angles at the centre
✅ Equal angles at centre imply equal chords
✅ A line from the centre ⟂ to a chord bisects it
✅ A line that bisects a chord passes through the centre ⟂ to it
✅ Only one circle can pass through three non-collinear points
✅ Equal chords are equidistant from the centre
✅ Chords equidistant from the centre are equal
✅ Equal chords have congruent arcs, and vice versa
✅ Congruent arcs subtend equal angles at the centre
✅ ∠ at centre = 2 × ∠ at any other point on the circle (on same arc)
✅ Angles in the same segment are equal
✅ If ∠ subtended by segment is equal at two points, then the 4 points lie on a circle (concyclic)
✅ Opposite angles of a cyclic quadrilateral add up to 180°
✅ If sum of opposite angles = 180°, quadrilateral is cyclic

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📝 Multiple Choice Practice (With Diagrams)

🧠 Sample:

1. If arc AXB = 75° and arc A′YB′ = 25°, then arc ratio = 3:1 ✅
2. In circle, chords AB and CD ⟂ from centre, and ∠POQ = 150°, find ∠APQ → 75° ✅

🧮 Examples:

- Diameter = 34 cm, chord = 30 cm → Distance from centre = 8 cm
- In triangle with AB ⟂ BC, AB = 12, BC = 16 → Radius of circle = 10 cm
- If ∠ABC = 20°, then ∠AOC = 40° (double)

- If AC = BC and AB is diameter → ∠CAB = 90°
- ∠OAB = 40° → ∠ACB = 70° (using arc rule)
- ∠AOB = 90°, ∠ABC = 30° → ∠CAO = 60° ✅

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🧾 True/False Reasoning

1. Chords equal distance from centre → Equal ✅
2. Through 3 collinear points → ❌ circle not possible
3. If AB is diameter → $AC^2 + BC^2 = AB^2$ ✅ (Pythagorean in semicircle)
4. If ∠BAC = ∠BDC = 45° → A, B, C, D are concyclic ✅
5. ∠ADC = 120° with diameter AB → ∠CAB = 30° ✅

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📐 Short Answers With Explanation

1. If arc AXB = ½ arc BYC → ∠BOC = 120°
2. If ∠ABC = 45°, then ∠AOC = 90° → OA ⊥ OC ✅

🧠 **Concept:** ∠ at circumference is half of ∠ at centre for same arc!

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✨ Theorems & Long Conceptual Reasoning

🧠 **Theorem:** Two circles can't intersect in more than 2 points
→ Because only one unique circle can pass through 3 non-collinear points

🧠 **Theorem:** Of all chords passing through a point inside a circle, the shortest one is perpendicular to the diameter passing through that point ✅

🧠 ΔABC inscribed, P any point on minor arc BC → PA is angle bisector of ∠BPC ✅

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🔍 Constructions and Reasoning Based Geometry

📘 Examples:

- Equal chords intersect → segments also equal
- Trapezium with non-parallel sides equal → Cyclic ✅
- A circle with radius 2 cm, chord = 2 cm → ∠ in major segment = 45°
- Opposite angles' bisectors meet on the circle → they form a diameter ✅
- If AB = 2AC in a circle, p and q are distances from centre → $4q^2 = p^2 + 3r^2$ ✅

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📊 Summary Table:-

Concept	Result / Rule
∠ at centre = 2 × ∠ at circle	Always true for same arc
Opposite angles of cyclic quadrilateral	Sum = 180°
Equal chords	Equal distance from centre, same arc
Diameter subtending angle	Always 90° (angle in semicircle)
Only one circle through 3 points	True (must be non-collinear)
Equal chords produce equal arcs	And vice versa
Angle in same segment	Always equal
Chord nearest to centre	Is the longest