# Chapter 5: Introduction to Euclid's Geometry

### Geometry in Ancient Civilisations

#### 🔯 Ancient India:

- Geometry (called "Shulba Sutra") came from altar (vedi) construction for Vedic rituals.
- Household rituals used square/circular altars
- Public rituals used complex shapes rectangle, triangle, trapezium 🔺

#### Egypt & Babylonia:

- Used geometry practically for building pyramids, irrigation, etc.
- No proofs only measurements.

#### Greece:

- Geometry became a science of reasoning.
- Thales & Pythagoras introduced logic & deduction 🔍

#### Euclid and the Elements

- Euclid: Greek mathematician, ~300 BCE
- E Famous book: Elements collection of all geometry known then (13 chapters, 465 propositions)

#### He defined:

- 🖊 **Axioms**: Basic truths accepted universally
- **Postulates:** Geometry-specific assumptions
- **Ill Theorems:** Statements proven using axioms/postulates

### Euclid's Axioms (General Truths)

- Things equal to the same thing are equal to one another.
- 2 If equals are added to equals, the wholes are equal.
- 3 If equals are subtracted from equals, the remainders are equal.
- 4 Things which coincide are equal.
- 5 Whole is greater than part.
- Things double of same thing are equal.
- 7 Things halves of same thing are equal.

#### Example:

If Ram = Ravi in weight and both gain 2 kg → they are still equal (Axiom 2)

### 📏 Euclid's Postulates (Geometry-Specific)

- 1 A straight line can be drawn from any point to another.
- 2 A line segment can be extended indefinitely.
- 3 A circle can be drawn with any center and radius.
- 4 All right angles are equal.
- [5] (Most famous) If a line falling on two others makes interior angles less than 180°, the two lines will meet on that side.
- → Rewritten later as Playfair's Axiom.
- Postulate ≠ Theorem → Postulates are accepted without proof.

### Important Terms in Geometry

#### **★** Undefined Terms:

- Point ◆: Location without size or dimension
- Line —: Infinite straight path
- Plane :: Flat surface with no thickness

#### Defined Terms:

- Line segment: Part of line with 2 endpoints
- Ray: Line with one endpoint
- Angle: Two rays with a common start

# Euclid's Geometry Today

#### Modern Geometry:

- Based on undefined terms + axioms + definitions
- Logical system built step by step
- A system is consistent if no contradiction exists

#### Examples:

- "Given 2 points, one and only one line can be drawn"
- "Two lines intersect at most at one point"

## Multiple Choice Practice

- ? Euclid's 2nd axiom: If equals are added to equals → wholes are equal 🗸
- ? 5th Postulate describes line meeting → sum of interior angles < 180° ✓
- ? Postulates are accepted without proof X
- ? Theorems are proven 🐆
- ? Greeks used deductive reasoning 🗸

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### Examples Using Axioms

- 💡 If AB = BC, and BX = BY, show that AX = CY
- → Use: Things equal to same thing are equal
- 💡 If C is midpoint of AB, and D is midpoint of XY
- $\rightarrow$  AB = 2AC, XY = 2XD, and AC = XD
- → So, AB = XY 🗸 (double of equal things)

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### True/False Reasoning

- ✓ True: Pyramid's side faces are triangles
- X False: Not all are equilateral
- ✓ True: In Vedic altars, public ones were complex shaped
- ✓ True: Point, Line, Plane = undefined
- ✓ True: Area of triangle = area of square → equal areas
- ✓ False: Euclid's fourth axiom is not "everything equals itself" it's about right angles

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### Long Answer Explanation

Example: "Square is a polygon with 4 equal sides and all right angles"

#### **→** Define:

- Polygon: Closed figure with 3+ line segments
- Right angle: 90°
- Line, Point → Undefined
- ✓ Use Euclid's First Axiom (sides)
- Use Fourth Postulate (angles)
- → Hence, square has all equal sides and angles

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### Real-World Application

- 🤋 Two salesmen sell same in August
- → In Sept., both double their sales
- → By Euclid's 6th axiom: Double of equals = equal again
- x + y = 10 and x = z
- $\rightarrow$  z + y = 10 (From Axiom 1)

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# **E** Summary Table

Concept	Explanation / Axiom Used
Equal to same ⇒ Equal	Axiom 1
Add equal to equal ⇒ Whole	Axiom 2
Double of same ⇒ Equal	Axiom 6
Right angles equal	Postulate 4
Parallel line unique	Playfair's Axiom (5th Postulate equivalent)
Theorem	Statement that needs proof
Postulate	Geometry-specific truth accepted without proof