Black Scholes Formula

Statistic definitions

n(z) PDF normal Distribution
N(z) CDF normal Distribution

$$n(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2}$$

$$N(z) = \int_{y=-\infty}^{z} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}y^2} dy$$

Black-Scholes theoretical option price

S Spot price
X Strike price
t Time to Maturity
r Riskfree Interest Rate
σ Volatility
C Call-Option Price BS-Model

$$d_1 = \frac{\ln\left(\frac{S}{X}\right) + \left(rt + \frac{1}{2}\sigma^2 t\right)}{\sigma\sqrt{t}} \qquad d_2 = \frac{\ln\left(\frac{S}{X}\right) + \left(rt - \frac{1}{2}\sigma^2 t\right)}{\sigma\sqrt{t}}$$

$$C = S \cdot N(d_1) - Xe^{-rt} \cdot N(d_2)$$

$$P = -S \cdot N(-d_1) + Xe^{-rt} \cdot N(-d_2)$$

Sensitivities

Δ Delta
Γ Gamma
η Vega
ρ Rho
θ Theta

$\Delta:$	$\frac{dC}{dS} = N(d_1)$	$\frac{dP}{dS} = N(d_1) - 1$
$\Gamma:$	$\frac{d^2C}{dS^2} = \frac{n(d_1)}{S\sigma\sqrt{t}}$	$\frac{d^2P}{dS^2} = \frac{n(d_1)}{S\sigma\sqrt{t}}$
$\eta:$	$\frac{dC}{d\sigma} = \sigma\sqrt{t} \cdot n(d_1)$	$\frac{dP}{d\sigma} = \sigma\sqrt{t} \cdot n(d_1)$
$\rho:$	$\frac{dC}{dr} = Xt - e^{-rt} N(d_2)$	$\frac{dP}{dr} = -Xt - e^{-rt} N(-d_2)$
$\theta:$	$\frac{dC}{dt} = \frac{-S\sigma}{2\sqrt{t}} n(d_1) - Xre^{-rt} N(d_2)$	$\frac{dP}{dt} = \frac{S\sigma}{2\sqrt{t}} n(d_1) + Xre^{-rt} N(-d_2)$