# An interval type-2 Fuzzy PD controller for ball and beam system

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# An Interval Type-2 Fuzzy PD Controller for Ball and Beam System

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***Abstract*—This paper presents a simple interval type 2 fuzzy proportional derivative controller for ball and beam system. The inner motor position and the outer ball position control loops are operated with the similar controller structures but having different settling times. The proposed controller is validated through simulation runs in MATLAB environment where it is also compared with type 1 fuzzy proportional derivative controller. Results have shown that the proposed controller is robust to beam angle and ball position disturbances, as well as measurement noise and errors.**

***Index Terms*—Interval type 2 fuzzy proportional derivative controller, type 1 fuzzy proportional derivative controller, ball and beam system, MATLAB/Simulink.**

## I. Introduction

Ball and beam system is an under actuated open loop unstable system and is widely used to test the control algorithms. The idea is to control the position of the ball that is free to move on a beam by controlling the angle of the beam using a DC motor. Thus the system can be considered to have two loops namely the inner and outer loops. The inner loop provides the desired beam angle for the outer loop to place the ball at the required distance from the center of the beam. However, this framework requires that the inner loop is much faster than the outer loop and in that case the loops can be designed independently to have different response times. Several control techniques ranging from PID to non-linear control [1-4] have been developed to stabilize this system. However, it is always difficult for these controllers to handle the unmodeled uncertainties and noises present in the system resulting in the performance degradation. On the other hand, soft computing techniques such as fuzzy logic and neural networks can have better performance than classical controllers in the presence of uncertainties and noises as they do not require the complete mathematical description of the system. The use of these techniques has also been reported in literature to control the ball and beam system [5-10]. The vision based fuzzy control system is presented in [5]. The system uses an overhead camera to capture the image of the ball and beam setup. After selecting the region of interest in image, it is converted to grayscale. The corresponding binary image is obtained using a threshold and it is labeled to have the number of objects in the image. The smaller objects representing the noise are removed using a filter and centroid of the ball is determined. The position of the ball is then found using pixel-to-metric conversion. The angle of the beam is found using a potentiometer attached to the motor shaft. The ball position, ball velocity and the beam angle are fed to a fuzzy controller that uses four trapezoidal membership functions for the beam angle and three such membership functions for ball position and ball velocity along with 45 rules to generate the motor commands for regulating the ball position on the beam. A fuzzy PID controller for the ball and beam system is presented in [6] that accepts error and change in error of ball position as inputs, fuzzifies them using seven triangular membership functions and evaluates 49 rules to generate the beam angle commands for the inner PD operated motor control loop to track them. To reduce the rule base, a single input fuzzy controller for ball and beam system is presented in [7] that uses the signed distance method to translate Toeplitz rule matrix to a vector where the entries in the vector represent the distance from the main diagonal line of rule matrix. Operated with seven triangular membership functions, this controller is shown to have better performance in terms of the response time. Although the fuzzy logic controller has better performance, it is not easy to find the optimal scaling factors for membership functions. A neuro-fuzzy algorithm is described in [8] for ball and beam system that determines the scaling factors for the fuzzy controller. A human controls the beam angle using a joystick after obtaining the ball position through a vision system. The real time data thus obtained is used to train a neural network that finds the scaling factors for the fuzzy controller. Other than neural networks, biologically inspired algorithms are also used to fine tune the fuzzy controller. A shuffled frog leaping algorithm (SFLA) is used in [9] to find the optimum parameters for membership functions and scaling gains of fuzzy logic controller with ball and beam system being the plant. With the four states of the plant, each of which is represented by three membership functions, being the inputs and the motor torque, represented by seven singletons, being the output, fuzzy controller evaluates 81 rules to control the system. The SFLA finds the fuzzy controller parameters by minimizing a quadratic function like LQR controller and optimized fuzzy controller is shown to have a slightly better performance than LQR. Other than optimizing the fuzzy controller parameters, the uncertainty can be handled by the use of type-2 fuzzy controller which introduces the concept of footprint of uncertainity (FOU) in the description of membership functions such that the degree of belongingness of certain value lying in the universe of discourse to a fuzzy set is also fuzzy as opposed to a crisp value in case of type-1 fuzzy logic controller or simply the fuzzy controller. However, the type-2 fuzzy controller is computationally complex and therefore a less complex version of this controller is interval type-2 fuzzy controller where the secondary membership grade is set as '1'. The use of interval type-2 fuzzy controller for controlling the ball and beam system is presented in [10]. Similar to [7], signed distance method is used to reduce the number of inputs to fuzzy controller. The single input is fuzzified using seven type-2 fuzzy sets each set being described by two Gaussian membership functions with the same mean but different variances. The output of the controller is also described by the similar type-2 fuzzy sets. The controller evaluates seven rules using Mamdani fuzzy inference mechanism with centroid defuzzification method. The controller is finally compared with single input type-1 fuzzy controller and is shown to have better disturbance and noise rejection properties.

This paper also proposes an interval type-2 fuzzy controller but with lesser membership functions and smaller rule base than [10] to control the ball and beam system. Both the inner and outer fuzzy control loops accepts the error and change in error as inputs, each of which is described by two type-2 triangular fuzzy sets. The outputs of the control loops are described by three singletons. The controllers use four rules for mapping between the inputs and output universes of discourse. The performance of the proposed controller is analyzed in terms of its noise and disturbance rejection capability by simulating it in MATLAB/Simulink environment. The corresponding type-1 fuzzy controller is also simulated to compare the performance of the two controllers.

## II. Ball And Beam Model

A schematic of the ball and beam system is shown in Fig. 1. The ball is free to move on a rectangular beam which is rotated about its center using a DC motor. By balancing the forces acting on the ball using Newton second law of motion and using the small angle approximation, the position of the ball given the beam angle can be described as:

$$\frac{X(s)}{\alpha(s)} = \frac{7.007}{s^2} \tag{1}$$

Where, $\alpha$ is the beam angle and $x$ is the distance of the ball from the beam center.

The required beam angle to balance the ball at a specified distance is provided by a DC geared motor when voltage is applied to it. This transfer function can be given as [11]:

$$\frac{\alpha(s)}{V(s)} = \frac{66.394}{s(s+37.386)} \tag{2}$$

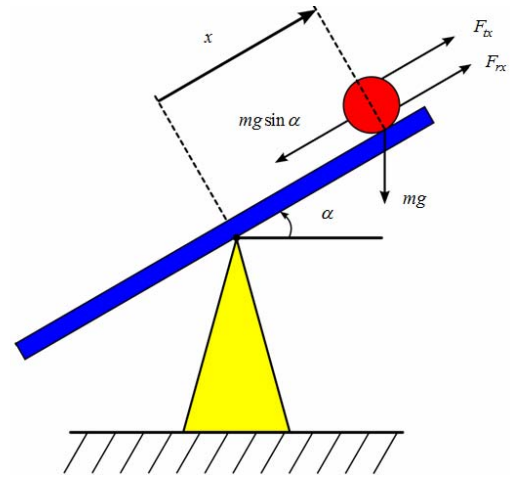


Fig. 1. Ball and beam system

The design of fuzzy PD controllers for the two loops use the gains determined by the classical PD controllers. The structure of the employed PD controller is shown in Fig. 2. The inner loop is designed to have a peak time of 0.1 sec and a damping ratio of 0.707. With $G(s)$ in Fig.2 replaced by (2), the gains for this loop are found to be:

$$K_{pm} = 29.7214, K_{dm} = 0.3830 \tag{3}$$

Where, the subscript 'm' represents the gains for motor loop. The outer loop is designed to have a peak time of 2 sec and a damping ratio of 0.707. With $G(s)$ in Fig.2 replaced by (1), the gains for this loop are found to be:

$$K_{pb} = 0.7048, K_{db} = 0.4487 \tag{4}$$

Where, the subscript 'b' represents the gains for ball-beam loop. The control law for the PD controller is:

$$u(t) = K_p.e - K_d.\dot{y} = K_p.e + K_d.\dot{e} \tag{5}$$

In discrete domain, (5) can be given as:

$$u_n = K_p\left(e_n + \frac{K_d}{K_p}.\frac{e_n - e_{n-1}}{T_s}\right) = K_p\left(e_n + T_d.ce_n\right) \tag{6}$$

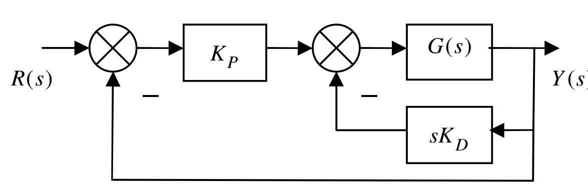


Fig. 2. PD controller structure

## III. Fuzzy Logic Controller

The interval type-2 fuzzy logic controller (IT2FLC) is a two input, single output controller, as shown in Fig. 3. The error ($e$) between the reference input and the plant output forms one input while the change in the plant output ($c.e$) is used as second input to the controller. Both the inputs are described by two interval type-2 fuzzy sets (IT2FS) namely Negative ($\tilde{N}$) and Positive ($\tilde{P}$) as shown in Fig. 4. Such a fuzzy set is bounded from above and below by a type-1 fuzzy set and the area between these two fuzzy sets is defined as footprint of uncertainty (FOU) which provides a mean to model the process uncertainties. Thus the degree of membership of the input variable to a particular IT2FS is a range instead of a crisp value e.g., the degree of belongingness of $e$ to $\tilde{N}$ is the interval $\left[\mu_{\underline{N}}(e), \mu_{\bar{N}}(e)\right]$. The definition of IT2FS, $\tilde{N}$ with $e$ being the universe of discourse follows from [12]:

$$\tilde{N} = \int_{e\in E}\int_{v\in J_e} 1\Big/(e,v), \quad J_e \subseteq [0,1] \tag{7}$$

The single output from IT2FLC is described by three singletons namely Negative ($\tilde{N}_o$), Zero ($\tilde{Z}_o$) and Positive ($\tilde{P}_o$), as shown in Fig. 5. The singletons are also blurred version of the ones used in T1FLC. Thus the output singleton in IT2FLC is also described by an interval e.g., $\tilde{N}_o$ is defined to form the interval $\left[u_{\underline{N}_o}, u_{\bar{N}_o}\right]$. The rule base of IT2FLC contains four rules described as:

R1: If $e$ is $\tilde{N}$ and $c.e$ is $\tilde{N}$ Then $u$ is $\tilde{N}_o$

R2: If $e$ is $\tilde{N}$ and $c.e$ is $\tilde{P}$ Then $u$ is $\tilde{Z}_o$

R3: If $e$ is $\tilde{P}$ and $c.e$ is $\tilde{N}$ Then $u$ is $\tilde{Z}_o$

R4: If $e$ is $\tilde{P}$ and $c.e$ is $\tilde{P}$ Then $u$ is $\tilde{P}_o$

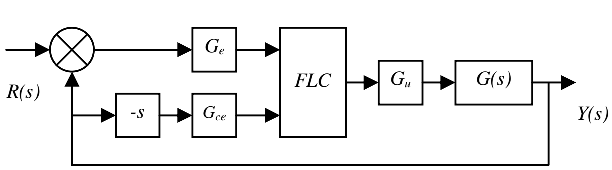


Fig. 3. Fuzzy controller structure

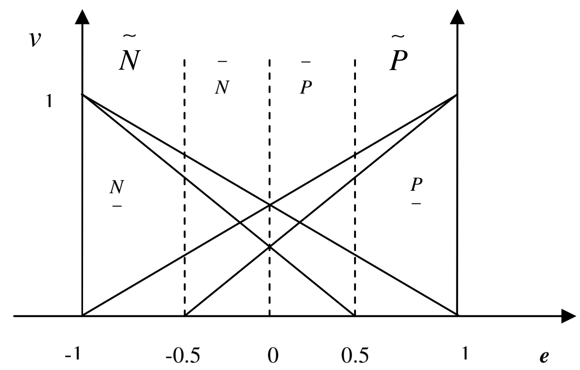


Fig. 4. Type-2 fuzzy sets for error/change in error input

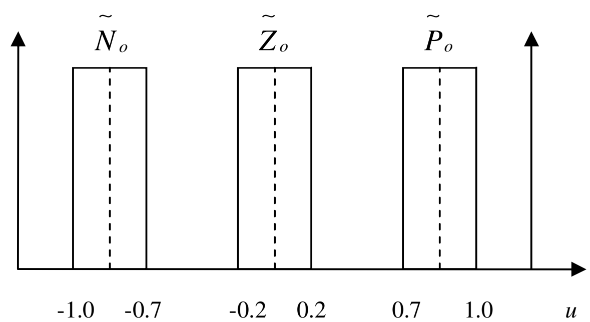


Fig. 5. Type-2 singletons for output

After defining the rule base, following steps are performed to compute the output of IT2FLC:

1) For the input vector, x ($e$, $c.e$), the degree of membership of its elements to each IT2FS, X ($\tilde{N}$, $\tilde{P}$) is determined to lie in the interval $\left[\mu_{\underline{X}}(\text{x}), \mu_{\bar{X}}(\text{x})\right]$. Thus there will be four such membership intervals.

2) The firing interval of all the rules is determined. Let $X' = \left[\left(\tilde{N},\tilde{N}\right),\left(\tilde{N},\tilde{P}\right),\left(\tilde{P},\tilde{N}\right),\left(\tilde{P},\tilde{P}\right)\right]$ be the rule vector for IT2FLC. For $n^{th}$ rule in the vector, firing interval can be described as $W_n = \left[\underline{w}_n, \bar{w}_n\right]$, where the subscript '$n$' is the rule index. The mapping between the $n^{th}$ pair of $X'$ and $W$ can be given as:

$$W_n = \left[\min\left(\mu_{\underline{X}'_n}(e), \mu_{\underline{X}'_n}(ce)\right), \min\left(\mu_{\bar{X}'_n}(e), \mu_{\bar{X}'_n}(ce)\right)\right] \tag{7}$$

3) Type reduction is performed by employing center-of-sets-type-reducer method in order to combine $\left[\underline{w}_n, \bar{w}_n\right]$ computed in previous step and the corresponding rule consequent $\left[\underline{u}_n, \bar{u}_n\right]$. Let $U^{'} = \left[\tilde{N}_o, \tilde{Z}_o, \tilde{Z}_o, \tilde{P}_o\right]$ be the vector describing the consequent part of the rules in the rule vector $X^{'}$.

$$U_{\cos}(\mathrm{x}) = \bigcup_{\substack{w_n \in W_n \\ u_n \in U_n}} \frac{\sum_{n=1}^{4} w_n u_n}{\sum_{n=1}^{4} w_n} = \left[u_l, u_r\right] \tag{8}$$

Where, the points $u_l$ and $u_r$ are computed using the efficient extended iterative algorithm with stop condition (EIASC) as described in [13]. The defuzzified output is then found as:

$$u = \frac{u_l + u_r}{2} \tag{9}$$

The scaling gains for IT2FLC are the same as that of T1FLC. The FLC gains are found in terms of PD controller gains using the linear approximation of the fuzzy control law and comparing it with the PD control law [14]. The fuzzy control law, using the structure in Fig. 3, can be given as:

$$\begin{aligned} u_n &= G_u.f(G_e.e_n, G_{ce}.ce_n) \\ &= G_u.(G_e.e_n + G_{ce}.ce_n) \\ &= G_u.G_e(e_n + \frac{G_{ce}}{G_e}.ce_n) \end{aligned} \tag{10}$$

Comparing (10) with (6), the FLC gains can be given as:

$$\begin{aligned} G_u &= \frac{K_p}{G_e} \\ G_{ce} &= \frac{K_d.G_e}{K_p} \end{aligned} \tag{11}$$

Where, the choice of $G_e$ depends upon the maximum error signal. $G_e$ is selected as $\frac{5.\pi}{9}$ for the motor control loop which corresponds to the maximum permissible beam angle deviation, $\left[-50^{\mathrm{O}}, +50^{\mathrm{O}}\right]$ while $G_e$ is set as $20$ for ball and beam loop which corresponds to a maximum possible change of 20cm in the ball position. The scaling gains found in (11) are further tuned for ball and beam loop to have better response time in such a way that these changes do not drive the fuzzy controller inputs to saturation. The modified scaling factors are given as:

$$\begin{aligned} G_u &= \frac{K_p}{G_e - k.G_e}, 0 \le k < 1 \\ G_{ce} &= \frac{K_d.(G_e - l.G_e)}{K_p}, 0 \le l < 1 \end{aligned} \tag{12}$$

## IV. Simulation And Results

The IT2FLC for ball and beam system is simulated in MATLAB environment. Both the inner motor control and outer ball and beam loop are driven with the same controller structure but with different response times. The inner loop is made twenty times faster than outer loop so that the motor provides required beam angle quickly as demanded by the outer loop for the ball to stabilize and track the reference inputs. The T1FLC and PD controllers are also simulated in order to draw a comparison between the transient performance and disturbance rejection capabilities of the controllers. The Simulink block diagram of the control scheme is shown in Fig. 6.

The gains $k$ and $l$ given by (12) are adjusted to bound the variations in linguistic parameters within the defined universe of discourse. The step response of the ball and beam system employing the three controllers is shown in Fig. 7. It can be seen that the transient performance of IT2FLC is better than T1FLC and PD controller. The beam angle tracking by the inner motor loop for the case of IT2FLC is shown in Fig. 8. The square wave reference tracking of the ball position by the three controllers is shown in Fig. 9 while the sine wave reference tracking is shown in Fig. 10. An impulse disturbance on the beam angle measuring 5 degrees is introduced at T=6sec that lasts for one second while the ball position was already stabilized at 10cm. The response of the three controllers in rejecting the impulse disturbance is shown in Fig. 11. It can be seen that IT2FLC has rejected this disturbance in less time as compared to other two controllers. A constant sinusoidal disturbance (0.09V, 50Hz) is now introduced in the ball position which can be considered as an error in its measurement. The response of the two fuzzy controllers is compared in Fig. 12. It can be seen that the settling time for IT2FLC is unaffected and also no overshoot is observed. However, there is an overshoot of 15.7% in the response of T1FLC and its settling time is increased to 1.70sec. A further increase in the amplitude of this disturbance (0.1V) reveals that the percentage overshoot is increased to 38.3% for T1FLC with settling time of 2.08sec while no overshoot is observed for IT2FLC and its settling time remains the same. This response is shown in Fig. 13. By changing the membership functions of T1FLC to Gaussian, the overshoot is eliminated as depicted in Fig. 14 which also helps in reducing the settling time to 1.55sec. However, the performance of IT2FLC with the same triangular membership functions is still better. The comparison of the controllers with respect to settling time, rise time, delay time and disturbance rejection ability is shown in Table 1.

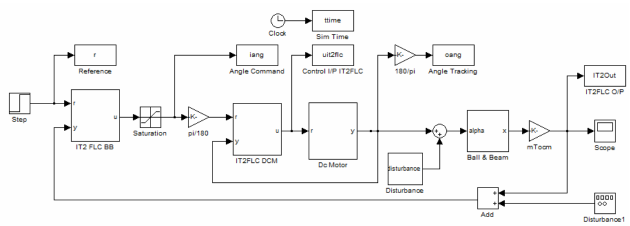


Fig. 6. Simulink block diagram for IT2FLC/T1FLC/PD

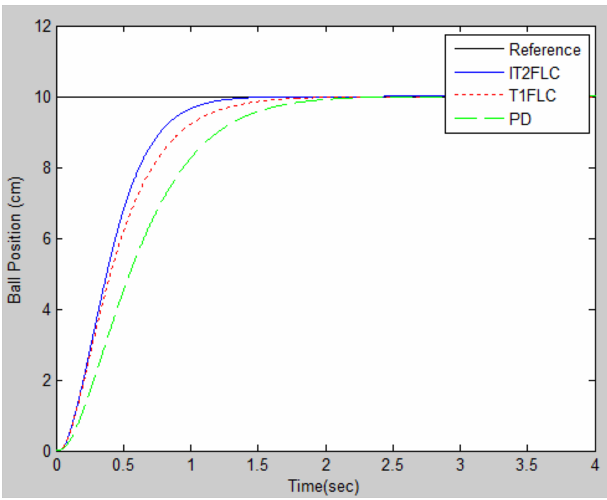


Fig. 7. Step response of the controllers

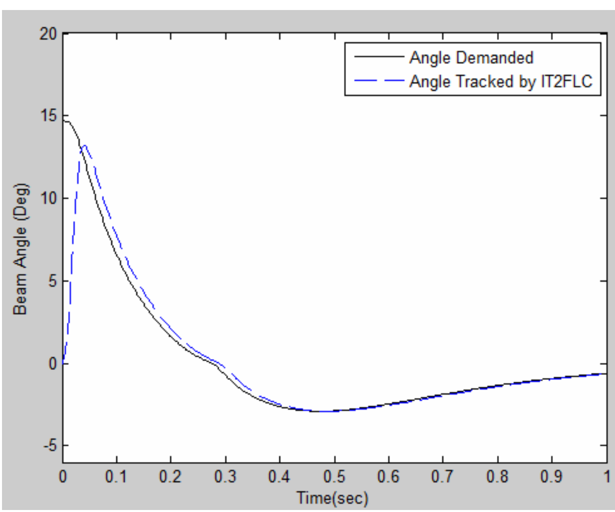


Fig. 8. Angle tracking by IT2FLC motor loop

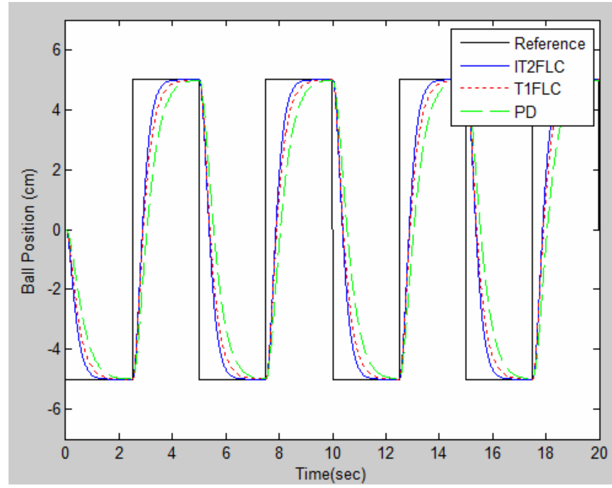


Fig. 9. Square wave reference tracking by the controllers

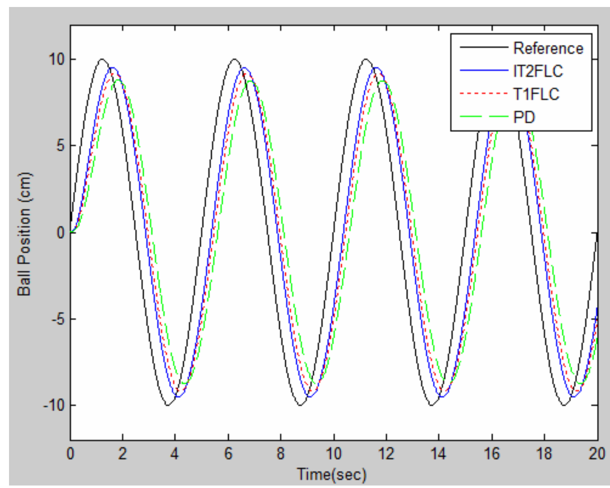


Fig. 10. Sine wave reference tracking by the controllers

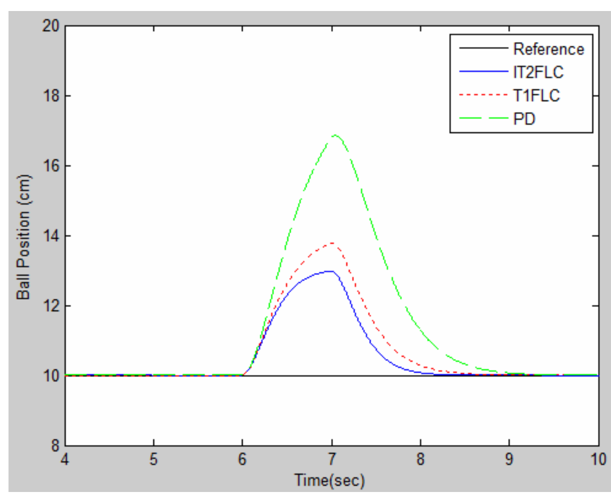


Fig. 11. Rejection of impulse disturbance in beam angle

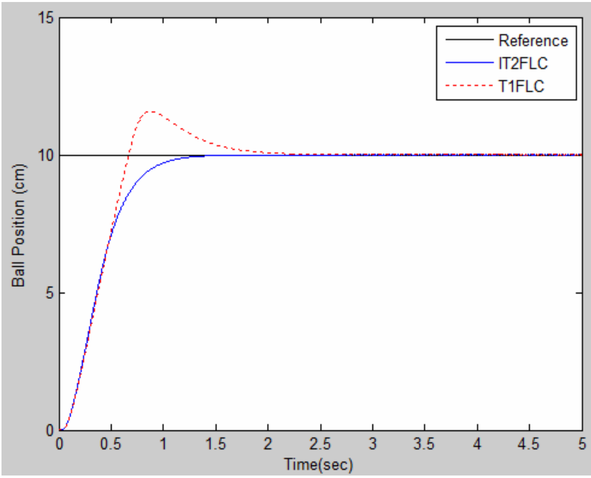


Fig. 12. Step response with constant sinusoidal disturbance in ball position (0.09V, 50Hz)

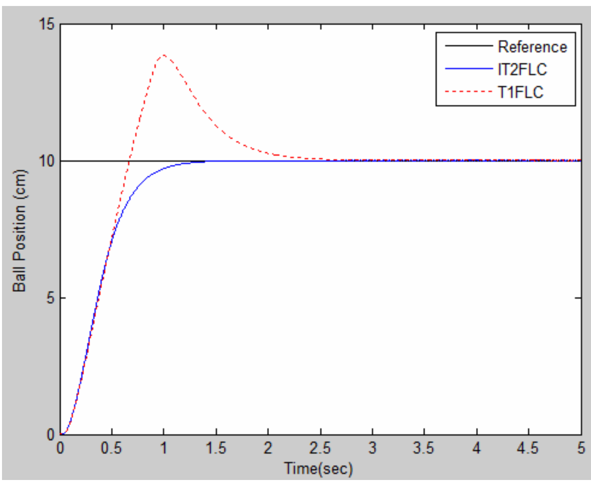


Fig. 13. Step response with constant sinusoidal disturbance in ball position (0.1V, 50Hz)

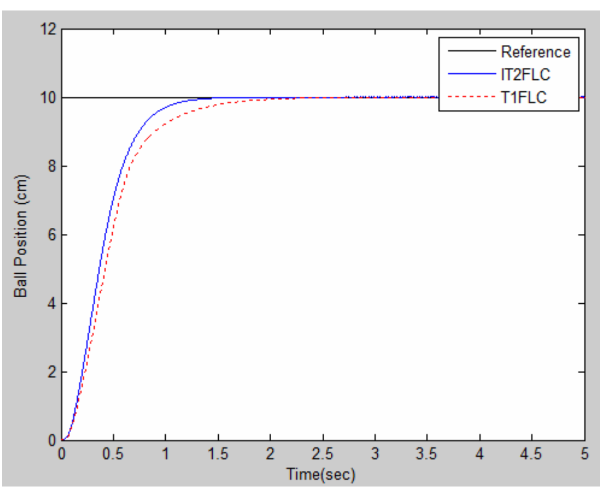


Fig. 14. Step response with constant sinusoidal disturbance in ball position and Gaussian membership functions for T1FLC (0.1V, 50Hz)

TABLE I. COMPARISON OF CONTROLLERS

| | | | |
|---|---|---|---|
| Settling Time (sec) | | IT2FLC | 1.11 |
| | | T1FLC | 1.41 |
| | | PD | 1.75 |
| Rise Time (sec) | | IT2FLC | 0.64 |
| | | T1FLC | 0.79 |
| | | PD | 1.02 |
| Delay Time (sec) | | IT2FLC | 0.36 |
| | | T1FLC | 0.41 |
| | | PD | 0.55 |
| Disturbance Rejection | Peak Attained (cm) | IT2FLC | 3.00 |
| | | T1FLC | 3.80 |
| | | PD | 6.90 |
| | Rejection Time (sec) | IT2FLC | 1.80 |
| | | T1FLC | 2.11 |
| | | PD | 2.67 |

## V. CONCLUSIONS

An interval type-2 fuzzy logic controller is presented to stabilize ball and beam system. The controller uses lesser membership functions with smaller rule base and yet shows good performance in reference tracking and disturbance rejection. It is compared with a type-1 fuzzy logic controller that has the same membership functions and rule base. With the same criterion to set the scaling gains for the fuzzy controllers, IT2FLC shows better transient performance and is found to handle the disturbances better than T1FLC.

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