Bottle Rocket Modeling

By: Quinn Lewis and Patrick Tippens

Introduction

<u>Objectives:</u>

1. Model the trajectory of a bottle rocket using numerical integration of a system of ordinary differential equations
2. Identify the conditions for different phases of the rocket flight
3. Understand how changing the launch conditions effects the flight path of the rocket

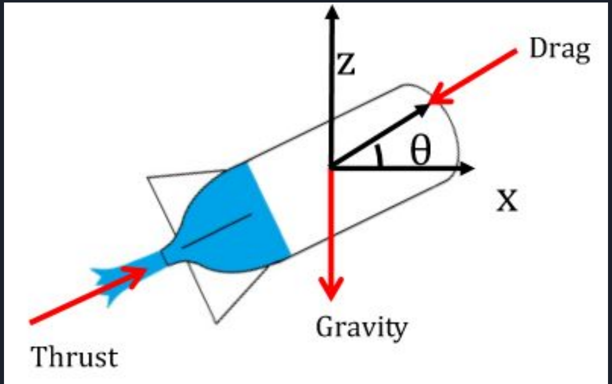
Governing Equations:

- Eq1: $\Sigma Forces = \vec{F} - \vec{D} + m_r \vec{g}$
- Eq8: $F = 2C_d A_t (P-P_a)$
- Eq22: $F = \dot{m}_{air} V_e + (P_a - P_e) A_t$
- Eq24: $\dot{m}_R = -C_d \rho_e A_t V_e$
- Eq:25: $F = 0$
- Eq10: $\dot{m}_R = -C_d A_t (2\rho_w (P-P_a)^{½})$

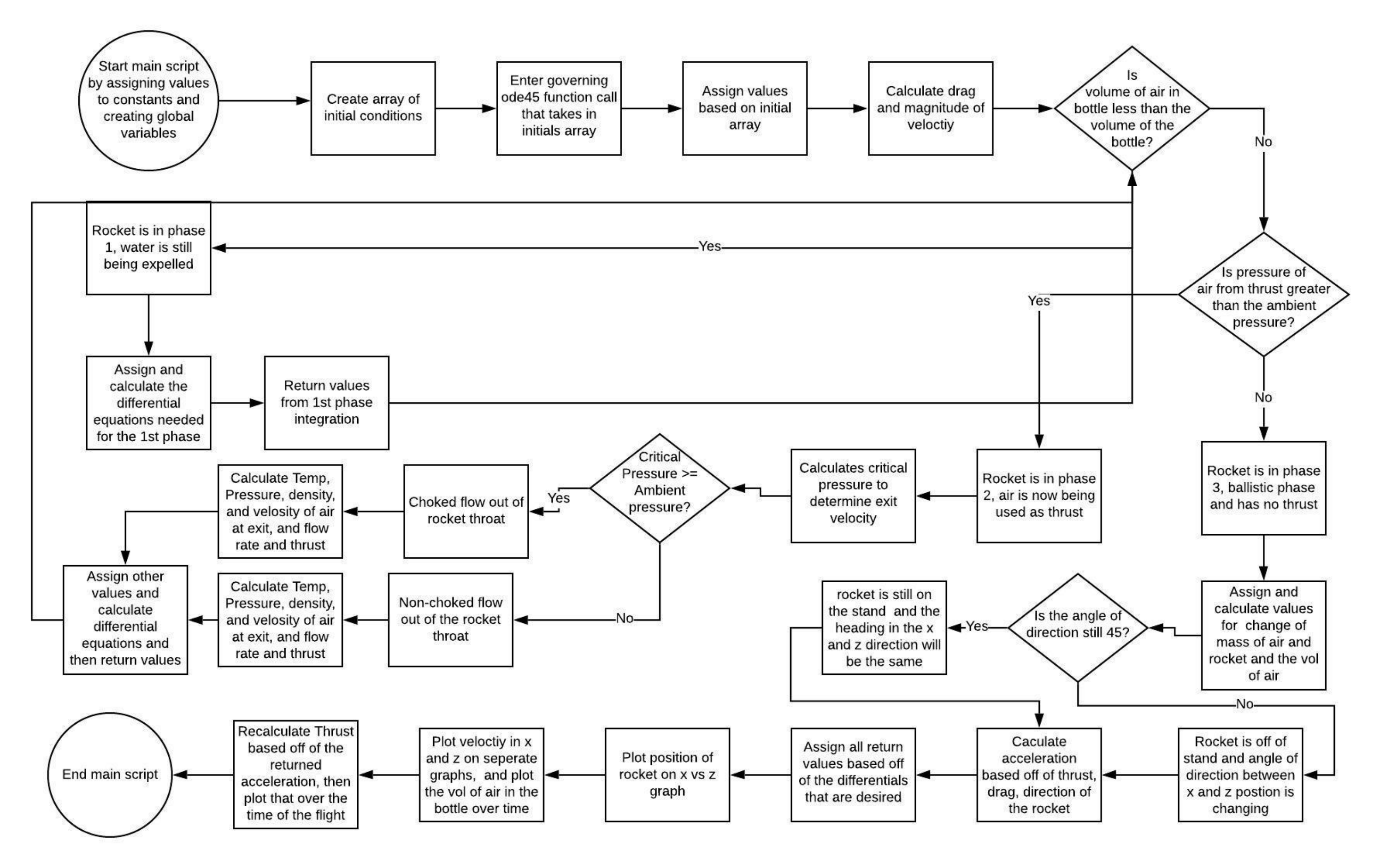
Methodology

10 Step Method:

1. Purpose
   a. Develop model for a bottle rocket
2. Knowns
   a. Given values for initial conditions and verification case
3. What do we need to find?
   a. Need to find functions for position and thrust
4. Make assumptions
   a. Assuming the gas is ideal, incompressible flow, thermodynamic relations apply
5. Sketch out problem
6. Fundamental Principles?
   a. Numerical integration through matlab(ode45)
   b. Ideal gas law
   c. Newton's 1st law for force balancing
   d. 3 phases of a bottle rocket
7. Alternate approaches to consider
8. Develop step-by-step process
9. Check program for bugs
10. Reality Check

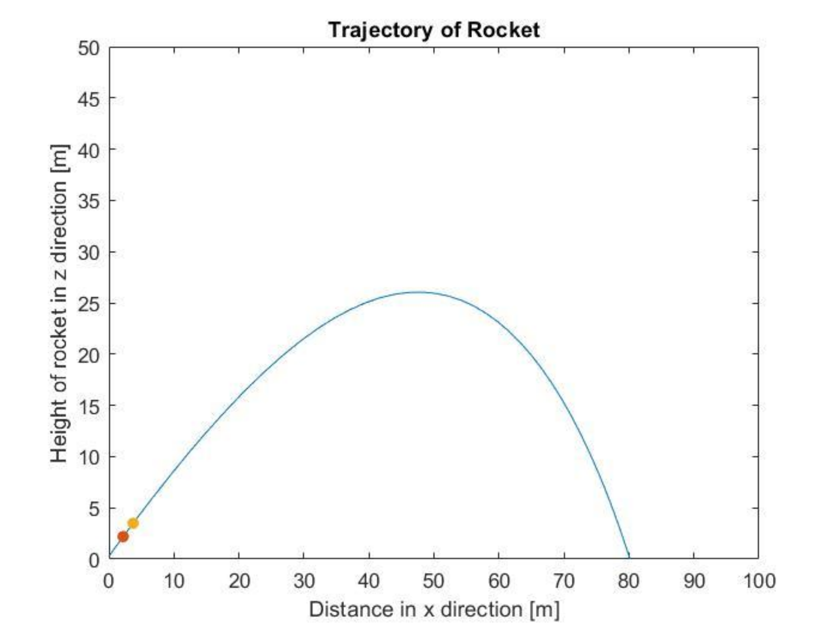


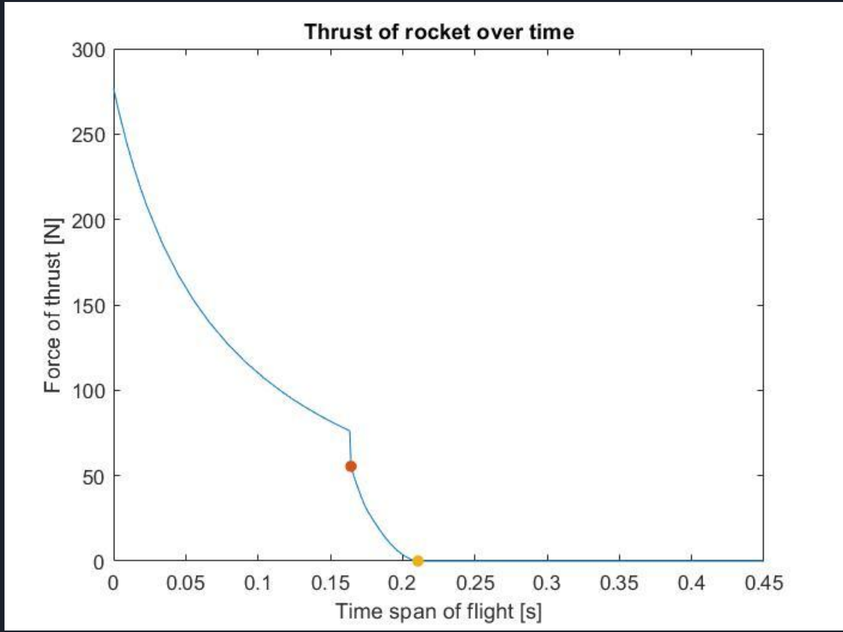
Methodology: flow chart of code



Results

- Max height reached: 26.04 m
- Distance in x traveled: 80.57 m
- Red Dot: transition from 1st to 2nd phase
- Yellow Dot: Transition from 2nd to 3rd phase





Discussion: Phases of Flight

Phase 1: Water Expulsion:

- Thrust Source: Water Expulsion (Eq8)
- Forces: Thrust, Drag, Gravity
- Rocket mass changes according Eq10

Phase 2: Air Expulsion:

- Thrust Source: Air Expulsion (Eq22)
- Forces: Thrust, Drag, Gravity
- Rocket mass changes according to Eq24

Phase 3: Ballistic :

- Thrust Source: None (Eq25)
- Forces: Drag, Gravity
- Rocket Mass is constant

Discussion: Varying Parameters

<u>Gage Pressure:</u>

- Direct Relationship with launch distance and Thrust

<u>Water Volume:</u>

- Half of volume of bottle is most efficient
- Decrease Volume water → Increase Drag effects
- Increase Volume of Water → Increases Weight

<u>Drag Coefficient:</u>

- Inverse relationship with Distance
- No effect on Thrust

<u>Launch angle:</u>

- 45° is most efficient launch angle.

Conclusion

Final Flight Parameters:

- Launch angle: 45°
- Initial Gage pressure : 72.5psi
- Initial Volume of Water: $0.001m^3$
- Coefficient of Drag: 0.5

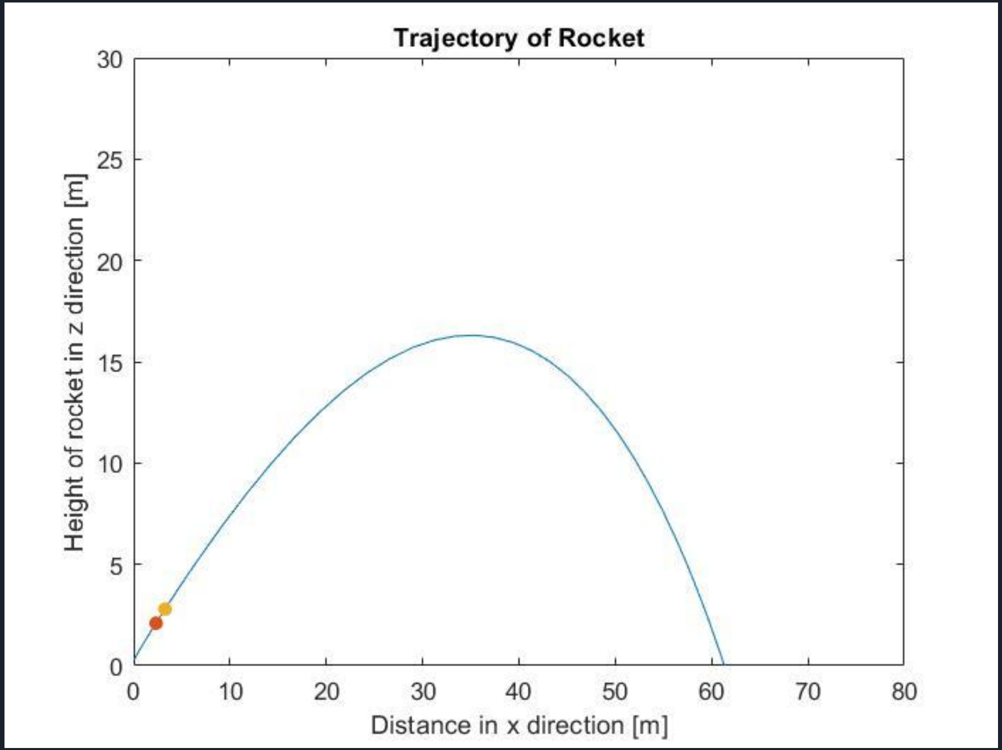
These initial parameters allow our rocket to land within the 80 meters +/- 1 meter mark, which is the goal of our launch.

References

Anderson, J. D., Jr., Introduction to Flight, 7th Ed., McGraw-Hill (2009)

Sutton, G. and Biblarz, O., Rocket Propulsion Elements, 8th Ed., Wiley (2010)

Backup slides: verification trajectory



Backup slides: verification thrust