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3111 Exam 3 Crib
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Three Assumptions of T.A.T.

1) max thickness is less than 10% of chord length
2) small camber
3) small A.O.A.

Cambered & Asymmetric Airfoils

$$V_\infty\left(\alpha - \frac{dz}{dx}\right) = \frac{1}{2\pi}\int_0^{\pi} \frac{\gamma(\theta)\sin\theta\, d\theta}{\cos\theta - \cos\theta_0}$$

MAP: $\boxed{\frac{x}{c} = \frac{1}{2}(1-\cos\theta)}$

[for $\frac{dz}{dx}(x) \to \frac{dz}{dx}(\theta)$ and bounds of integration]

$$A_0 - \sum_{n=1}^{\infty} A_n \cos(n\theta_0) = \alpha - \frac{dz}{dx}$$

$$\boxed{A_0 = \alpha - \frac{1}{\pi}\int_0^{\pi}\frac{dz}{dx}\, d\theta}$$

$$\boxed{A_n = \frac{2}{\pi}\int_0^{\pi}\frac{dz}{dx}\cos(n\theta)\, d\theta}$$

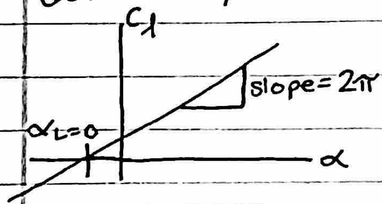
$$\boxed{\Gamma = cV_\infty\left[\pi A_0 + \frac{\pi}{2}A_1\right]} = \frac{c}{2}\int_0^{\pi} 2V_\infty\left[A_0\frac{(1+\cos\theta)}{\sin\theta} + \sum_{n=1}^{\infty} A_n \sin(n\theta)\right]\sin\theta\, d\theta$$

KJ: $L = \rho_\infty V_\infty \Gamma$

$$\boxed{c_l = 2\pi\left[A_0 + \frac{1}{2}A_1\right]} = \frac{L'}{q_\infty c \cdot 1}$$

$$\boxed{\alpha_{L=0} = \alpha - \left[A_0 + \frac{1}{2}A_1\right]} = \frac{1}{\pi}\int_0^{\pi}\frac{dz}{dx}(1-\cos\theta)\, d\theta$$

$\frac{dc_l}{d\alpha} = 2\pi$; $\alpha_{L=0}$ = offset



$$\boxed{C_{M_{LE}} = -\frac{\pi}{2}\left[A_0 + A_1 - \frac{1}{2}A_2\right] = -\frac{1}{4}\left[c_l + \pi(A_1 - A_2)\right] \text{ if } c_l \text{ is known}}$$

$$\boxed{C_{M_{\frac{c}{4}}} = \frac{\pi}{4}(A_2 - A_1)}$$

$$\boxed{x_{cp} = \frac{1}{4} + \frac{(A_1 - A_2)\pi}{4c_l}} = \frac{-C_{M_{LE}}}{c_l}$$

[aerodynamic center] → independent of α, but depends on A_1 and A_2 which depend on $\frac{dz}{dx}$ (camber shape!)

Symmetric Airfoils $\left(\frac{dz}{dx} = 0\right)$

$$V_\infty \alpha = \frac{1}{2\pi}\int_0^{\pi}\frac{\gamma(\theta)\sin\theta\, d\theta}{\cos\theta - \cos\theta_0}$$

$$\gamma(\theta) = 2\alpha V_\infty \frac{(1+\cos\theta)}{\sin\theta}$$

$$\left.\int_0^{\pi}\frac{\cos(n\theta)\, d\theta}{\cos\theta - \cos\theta_0} = \frac{\pi\sin(n\theta_0)}{\sin\theta_0}\right\} \text{ Glauert's Integral (for any } n)$$

$L' = \rho V_\infty^2 \alpha c\pi$

$c_l = 2\pi\alpha$

$L' x_{cp} = -M'_{LE}$

$M'_{LE} = -q_\infty \alpha c^2 \cdot \pi/2$

$C_{M'_{LE}} = -\frac{c_l}{4} = \frac{M'_{LE}}{q_\infty c^2}$

Kutta Condition – tangent flow @ TE

$$\gamma(TE) = \gamma(\pi) = 0$$

Trig Orthogonalities

$$\int_0^{\pi}\cos^2\theta\, d\theta = \frac{\pi}{2} \qquad \int_0^{\pi}\sin^2\theta\, d\theta = \frac{\pi}{2}$$

$$\int_0^{\pi}\cos\theta\sin^2\theta\, d\theta = 0 \qquad \int_0^{\pi}\sin(\theta)\sin(n\theta) = 0 \text{ for } n>1$$

$$\int_0^{\pi}\cos\theta\sin\theta\sin(n\theta) = \begin{cases}\pi/4, & n=2\\ 0, & n>2\end{cases}$$