# University of Colorado - Boulder

# ASEN 3113 THERMODYNAMICS AND HEAT TRANSFER

# **ASEN 3113: Crib Sheet Exam 3**

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# **Contents**

I	Heat Tra	nnsfer - From Exam 2
	I.A Cor	nduction
	I.B Con	nvection
		liation
		ermal Resistance
II	Steady H	leat Conduction 2
	II.A Con	ntact Resistance
	II.B Hea	at Conduction in Cylinders and Spheres
III	I Transien	at Heat Conduction 4
	III.A Lun	mped System Analysis
		nsient Heat Conduction in Plane Walls, Cylinders and Spheres
		B.1 Dimensionless Parameters
		B.2 Analytical and Graphical Solutions
IV	Radiation	n Heat Transfer
	IV.A Blac	ckbody Radiation
		diative Properties
		chhoff's Law
		in's Displacement Law
		oles

### I. Heat Transfer - From Exam 2

#### A. Conduction

Conduction is the transfer of energy from the more energetic particles of a substance to the adjacent less energetic ones as a result of interactions between the particles.

$$\dot{Q}_{\rm cond} = kA \frac{T_1 - T_2}{\Delta x} = -kA \frac{\Delta T}{\Delta x} \tag{1}$$

where k = thermal conductivity [W/m\*K]

Thermal Diffusivity:

$$\alpha = \frac{\text{Heat conduction}}{\text{Heat storage}} = \frac{k}{\rho c_p} \quad \left( \text{m}^2/\text{s} \right)$$
 (2)

#### **B.** Convection

Convection is the mode of energy transfer between a solid surface and the adjacent liquid or gas that is in motion, and it involves the combined effects of conduction and fluid motion. THIS INCLUDES AIR.

$$\dot{Q}_{\text{conv}} = hA_s \left( T_s - T_{\infty} \right) \tag{3}$$

where h is the convection heat transfer coefficient  $[W/m^2*K]$ 

### C. Radiation

Radiation is the energy emitted by matter in the form of electromagnetic waves (or photons) as a result of the changes in the electronic configurations of the atoms or molecules.

$$\dot{Q}_{\rm rad} = \varepsilon \sigma A_s \left( T_s^4 - T_{\rm surr}^4 r \right) \tag{4}$$

For a blackbody the  $\varepsilon$  is 1.

Combined heat transfer:

$$\dot{Q}_{\text{total}} = \dot{Q}_{\text{conv}} + \dot{Q}_{\text{rad}} = h_{\text{conv}} A_s (T_s - T_{\text{surr}}) + \varepsilon \sigma A_s \left( T_s^4 - T_{\text{surr}}^4 \right) 
\dot{Q}_{\text{total}} = h_{\text{combined}} A_s (T_s - T_{\infty}) \quad (W) 
h_{\text{combined}} = h_{\text{conv}} + h_{\text{rad}} = h_{\text{conv}} + \varepsilon \sigma (T_s + T_{\text{surr}}) \left( T_s^2 + T_{\text{surr}}^2 \right)$$
(5)

### **D.** Thermal Resistance

Conduction:

$$\dot{Q}_{\text{cond, wall}} = \frac{T_1 - T_2}{R_{\text{wall}}} 
R_{\text{wall}} = \frac{L}{kA} \quad (\text{ K/W})$$
(6)

Convection:

$$\dot{Q}_{\rm conv} = \frac{T_s - T_{\infty}}{R_{\rm conv}} \tag{7}$$

$$R_{\rm conv} = \frac{1}{hA_s} \quad (\text{ K/W}) \tag{8}$$

Radiation:

$$\dot{Q}_{\text{rad}} = \varepsilon \sigma A_s \left( T_s^4 - T_{\text{surr}}^4 \right) = h_{\text{rad}} A_s \left( T_s - T_{\text{surr}} \right) = \frac{T_s - T_{\text{surr}}}{R_{\text{rad}}}$$

$$R_{\text{rad}} = \frac{1}{h_{\text{rad}} A_s} \left( \text{ K/W} \right)$$
(9)

### **II. Steady Heat Conduction**

Conduction:

$$\dot{Q}_{\text{cond, wall}} = \frac{T_1 - T_2}{R_{\text{wall}}} 
R_{\text{wall}} = \frac{L}{kA} \quad (\text{ K/W})$$
(10)

Convection:

$$\dot{Q}_{\rm conv} = \frac{T_s - T_{\infty}}{R_{\rm conv}} \tag{11}$$

$$R_{\rm conv} = \frac{1}{hA_s} \quad (K/W) \tag{12}$$

Radiation:

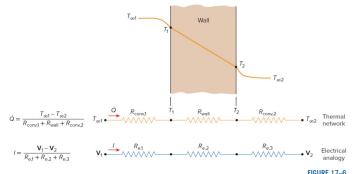
$$\dot{Q}_{\rm rad} = \varepsilon \sigma A_s \left( T_s^4 - T_{\rm surr}^4 \right) = h_{\rm rad} A_s \left( T_s - T_{\rm surr} \right) = \frac{T_s - T_{\rm surr}}{R_{\rm rad}}$$
(13)

$$R_{\rm rad} = \frac{1}{h_{\rm rad} A_{\rm s}} (\text{ K/W}) \tag{14}$$

Radiation Heat transfer coefficient:

$$h_{\text{rad}} = \frac{\dot{Q}_{\text{rad}}}{A_s \left( T_s - T_{\text{surr}} \right)} = \varepsilon \sigma \left( T_s^2 + T_{\text{surr}}^2 \right) \left( T_s + T_{\text{surr}} \right) \quad \left( W/m^2 \cdot K \right)$$
 (15)

Useful example:



The thermal resistance network for heat transfer through a plane wall subjected to convection on both sides, and the electrical analogy.

Fig. 1 Steady State Heat Transfer Through a Wall

More Thermal Resistance Things, in a thermal circuit equivalent the rate of heat conduction, convection, and radiation are all equal.

Rate of heat convection into the wall 
$$\dot{Q} = h_1 A (T_{\infty 1} - T_1) = k A \frac{T_1 - T_2}{L} = h_2 A (T_2 - T_{\infty 2})$$
 Rate of heat convection from the wall 
$$(16)$$

The above equation can also be arranged as:

$$\dot{Q} = \frac{T_{\infty 1} - T_1}{1/h_1 A} = \frac{T_1 - T_2}{L/k A} = \frac{T_2 - T_{\infty 2}}{1/h_2 A} 
= \frac{T_{\infty 1} - T_1}{R_{\text{conv}, 1}} = \frac{T_1 - T_2}{R_{\text{wall}}} = \frac{T_2 - T_{\infty 2}}{R_{\text{conv}, 2}}$$
(17)

$$\dot{Q} = \frac{T_{\text{col}} - T_{\text{co2}}}{R_{\text{total}}}$$

$$R_{\text{total}} = R_{\text{conv},1} + R_{\text{wall}} + R_{\text{conv},2} = \frac{1}{h_1 A} + \frac{L}{k A} + \frac{1}{h_2 A}$$
(18)

### A. Contact Resistance

Thermal Contact Resistance( $R_c$ ) is the resistance to heat transfer offered by an interface.

$$h_c = \frac{\dot{Q}/A}{\Delta T_{\text{interface}}} \left( W/m^2 \cdot K \right)$$
 (19)

$$R_c = \frac{1}{h_c} = \frac{\Delta T_{\text{interface}}}{\dot{Q}/A} \quad \left( \text{m}^2 \cdot \text{K/W} \right)$$
 (20)

Example for copper and insulation:

$$R_{c, \text{ insulation}} = \frac{L}{k} = \frac{0.01 \text{ m}}{0.04 \text{ W/m} \cdot \text{K}} = 0.25 \text{ m}^2 \cdot \text{K/W}$$
 (21)

$$R_{c, \text{ copper}} = \frac{L}{k} = \frac{0.01 \text{ m}}{386 \text{ W/m} \cdot \text{K}} = 0.000026 \text{ m}^2 \cdot \text{K/W}$$
 (22)

### **B.** Heat Conduction in Cylinders and Spheres

Cylinder:

$$\dot{Q}_{\text{cond, cyl}} = \frac{T_1 - T_2}{R_{\text{cyl}}} \quad (W)$$

$$R_{\text{cyl}} = \frac{\ln(r_2/r_1)}{2\pi Lk} = \frac{\ln(\text{Outer radius/Inner radius})}{2\pi \times \text{Length} \times \text{Thermal conductivity}} \tag{23}$$

$$\dot{Q} = \frac{T_{\text{col}} - T_{\text{co2}}}{R_{\text{total}}} 
R_{\text{total}} = R_{\text{conv}, 1} + R_{\text{cyl}} + R_{\text{conv}, 2} 
= \frac{1}{(2\pi r_1 L)h_1} + \frac{\ln(r_2/r_1)}{2\pi L k} + \frac{1}{(2\pi r_2 L)h_2}$$
(24)

Sphere:

$$\dot{Q}_{\text{cond, sph}} = \frac{T_1 - T_2}{R_{\text{sph}}} \tag{25}$$

$$R_{\rm sph} = \frac{r_2 - r_1}{4\pi r_1 r_2 k} = \frac{\text{Outer radius} - \text{Inner radius}}{4\pi (\text{Outer radius}) \text{(Inner radius)}(\text{Thermal conductivity})}$$
 (26)

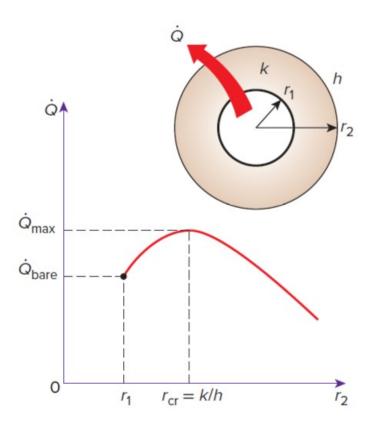
$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{\text{total}}} \tag{27}$$

$$R_{\text{total}} = R_{\text{conv},1} + R_{\text{sph}} + R_{\text{conv},2}$$

$$= \frac{1}{(4\pi r_1^2) h_1} + \frac{r_2 - r_1}{4\pi r_1 r_2 k} + \frac{1}{(4\pi r_2^2) h_2}$$
(28)

Critical Radius of Insulation for a cylindrical body:

$$r_{\text{cr, cylinder}} = \frac{k}{h}$$
 (29)



# FIGURE 17-31

The variation of heat transfer rate with the outer radius of the insulation  $r_2$  when  $r_1 < r_{cr}$ .

Fig. 2 Illustration of Critical Radius

For a sphere:

$$r_{\text{cr, sphere}} = \frac{2k}{h}$$
 (30)

## **III. Transient Heat Conduction**

### A. Lumped System Analysis

Can use lumped system analysis when the Biot Number is LESS THAN or EQUAL TO 0.1:

$$Bi = \frac{hL_c}{k}$$
 (31)

$$L_c = \frac{V}{A_s} \tag{32}$$

Here the Biot number is defied as:

$$Bi = \frac{h}{k/L_c} \frac{\Delta T}{\Delta T} = \frac{\text{Convection at the surface of the body}}{\text{Conduction within the body}}$$

$$Bi = \frac{L_c/k}{1/h} = \frac{\text{Convection resistance within the body}}{\text{Convection resistance at the surface of the body}}$$
(33)

$$\frac{T(t) - T_{\infty}}{T_i - T_{\infty}} = e^{-bt}$$

$$b = \frac{hA_s}{\rho V c_p}$$
(34)

### B. Transient Heat Conduction in Plane Walls, Cylinders and Spheres

#### 1. Dimensionless Parameters

**Dimensionless Temperature:** 

$$\theta(X,\tau) = \frac{T(x,t) - T_{\infty}}{T_i - T_{\infty}}$$
(35)

Dimensionless distance from the center:

$$X = \frac{x}{L} \tag{36}$$

Dimensionless heat transfer coefficient(Biot Number):

$$Bi = \frac{hL}{k} \tag{37}$$

Dimensionless Time(Fourier Number):

$$\tau = \frac{\alpha t}{L^2} = \text{Fo} \tag{38}$$

$$\theta = \frac{T(t) - T_{\infty}}{T_i - T_{\infty}} = e^{-bt} = e^{-\frac{hAt}{\rho V c_r}} = e^{-\text{BiFo}}$$
(39)

### 2. Analytical and Graphical Solutions

For solving these problems, determine the biot number then use tables and interpolation to determine the needed lambda and A values for the equations.

One term approximations for non-center of wall: Plane wall:

$$\theta_{\text{wall}} = \frac{T(x,t) - T_{\infty}}{T_i - T_{\infty}} = A_1 e^{-\lambda_1^2 \tau} \cos(\lambda_1 x/L), \tau > 0.2$$
 (40)

Cylinder:

$$\theta_{\text{cyl}} = \frac{T(r,t) - T_{\infty}}{T_i - T_{\infty}} = A_1 e^{-\lambda_1^2 \tau} J_0 \left( \lambda_1 r / r_o \right), \tau > 0.2$$
(41)

Sphere:

$$\theta_{\rm sph} = \frac{T(r,t) - T_{\infty}}{T_i - T_{\infty}} = A_1 e^{-\lambda_1^2 \tau} \frac{\sin(\lambda_1 r / r_o)}{\lambda_1 r / r_o}, \tau > 0.2$$
 (42)

Center of wall solutions: Plane Wall:

$$\theta_{0, \text{ wall}} = \frac{T_0 - T_{\infty}}{T_i - T_{\infty}} = A_1 e^{-\lambda_1^2 \tau}$$
 (43)

Cylinder:

$$\theta_{0, \text{ cyl}} = \frac{T_0 - T_{\infty}}{T_i - T_{\infty}} = A_1 e^{-\lambda_1^2 \tau}$$
(44)

Sphere:

$$\theta_{0,\text{sph}} = \frac{T_0 - T_{\infty}}{T_i - T_{\infty}} = A_1 e^{-\lambda_1^2 \tau}$$
(45)

#### IV. Radiation Heat Transfer

Electromagnetic waves are characterized by their frequency  $\nu$  or wavelength  $\lambda$  where c is the speed of light( $c_0 = 2.9979 * 10^8$ ):

$$\lambda = \frac{c}{v} \tag{46}$$

Each photon of frequency  $\nu$  is considered to have an energy of:

$$e = hv = \frac{hc}{\lambda} \tag{47}$$

where  $h = 6.626069 * 10^{-34}$ .

$$\dot{Q}_{\rm emit} = \sigma A_{\rm s} T_{\rm s}^4 \tag{48}$$

$$\dot{Q}_{\text{incident}} = \sigma A_s T_{\text{surr}}^4 \tag{49}$$

### A. Blackbody Radiation

Blackbody is a perfect emitter and absorber of radiation. The blackbody emmissive power:

$$E_b(T) = \sigma T^4 \quad \left( \text{W/m}^2 \right) \tag{50}$$

here  $\sigma = 5.670 * 10^{-8}$  which is the steffan-boltzmann constant.

spectral emissive power:

$$E_{b\lambda}(\lambda, T) = \frac{C_1}{\lambda^5 \left[ \exp\left( C_2 / \lambda T \right) - 1 \right]} \quad \left( W / m^2 \cdot \mu m \right)$$
 (51)

$$C_1 = 2\pi h c_0^2 = 3.74177 \times 10^8 \text{ W} \cdot \mu \text{m}^4/\text{m}^2$$
  
 $C_2 = h c_0/k = 1.43878 \times 10^4 \mu \text{m} \cdot \text{K}$  (52)

Dimensionless Blackbody Radiation Function:

$$f_{\lambda}(T) = \frac{\int_{0}^{\lambda} E_{b\lambda}(\lambda, T) d\lambda}{\sigma T^{4}}$$
 (53)

this value can be obtained from tables once values for lambda and temperature are found

$$f_{\lambda_1 - \lambda_2}(T) = \frac{\int_0^{\lambda_2} E_{b\lambda}(\lambda, T) d\lambda - \int_0^{\lambda_1} E_{b\lambda}(\lambda, T) d\lambda}{\sigma T^4} = f_{\lambda_2}(T) - f_{\lambda_1}(T)$$
(54)

### **B.** Radiative Properties

Emissivity - the ratio of the radiation emitted by the surface at a given temperature to the radiation emitted by a blackbody at the same temperature.

spectral hemispherical emissivity;

$$\varepsilon_{\lambda}(\lambda, T) = \frac{E_{\lambda}(\lambda, T)}{E_{b\lambda}(\lambda, T)} \tag{55}$$

total hemispherical emissivity:

$$\varepsilon(T) = \frac{E(T)}{E_b(T)} \tag{56}$$

OR

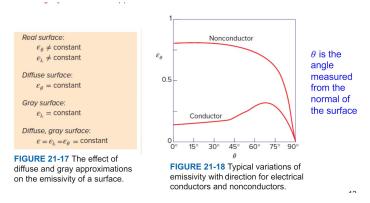
$$\varepsilon(T) = \frac{E(T)}{E_b(T)} = \frac{\int_0^\infty \varepsilon_\lambda(\lambda, T) E_{b\lambda}(\lambda, T) d\lambda}{\sigma T^4}$$
 (57)

The average emissivity can be determined by taking the average of 3 different wavelengths:

$$\varepsilon_{\lambda} = \begin{cases} \varepsilon_{1} = \text{constant}, & 0 \leq \lambda < \lambda_{1} \\ \varepsilon_{2} = \text{constant}, & \lambda_{1} \leq \lambda < \lambda_{2} \\ \varepsilon_{3} = \text{constant}, & \lambda_{2} \leq \lambda < \infty \end{cases}$$
 (58)

$$\varepsilon_{1}(T) = \frac{\varepsilon_{1} \int_{0}^{\lambda_{1}} E_{b\lambda} d\lambda}{E_{b}} + \frac{\varepsilon_{2} \int_{\lambda_{1}}^{\lambda_{2}} E_{b\lambda} d\lambda}{E_{b}} + \frac{\varepsilon_{3} \int_{\lambda_{2}}^{\infty} E_{b\lambda} d\lambda}{E_{b}}$$

$$= \varepsilon_{1} f_{0-\lambda_{1}}(T) + \varepsilon_{2} f_{\lambda_{1}-\lambda_{2}}(T) + \varepsilon_{3} f_{\lambda_{2}-\infty}(T)$$
(59)



emissivity relationships

Absorbtivity:

$$\alpha = \frac{\text{Absorbed radiation}}{\text{Incident radiation}} = \frac{G_{\text{abs}}}{G}, \quad 0 \le \alpha \le 1$$
 (60)

Reflectivity:

$$\rho = \frac{\text{Reflected radiation}}{\text{Incident radiation}} = \frac{G_{\text{ref}}}{G}, \quad 0 \le \rho \le 1$$
 (61)

Transmissivity:

$$\tau = \frac{\text{Transmitted radiation}}{\text{Incident radiation}} = \frac{G_{\text{tr}}}{G}, \quad 0 \le \tau \le 1$$
 (62)

Where G is the irradiation: Radiation Flux incident on a surface

$$G_{abs} + G_{ref} + G_{tr} = G$$

$$\alpha + \rho + \tau = 1$$
(63)

Here if it is an opaque surface then the transmissivty,  $\tau$ , goes to 0.

Spectral hemispherical equations:

$$\alpha_{\lambda}(\lambda) = \frac{G_{\lambda, \text{abs}}(\lambda)}{G_{\lambda}(\lambda)}$$

$$\rho_{\lambda}(\lambda) = \frac{G_{\lambda, \text{ref}}(\lambda)}{G_{\lambda}(\lambda)}$$

$$\tau_{\lambda}(\lambda) = \frac{G_{\lambda, \text{ref}}(\lambda)}{G_{\lambda}(\lambda)}$$
(64)

OR

$$\alpha = \frac{\int_0^\infty \alpha_\lambda G_\lambda d\lambda}{\int_0^\infty G_\lambda d\lambda}, \quad \rho = \frac{\int_0^\infty \rho_\lambda G_\lambda d\lambda}{\int_0^\infty G_\lambda d\lambda}, \quad \tau = \frac{\int_0^\infty \tau_\lambda G_\lambda d\lambda}{\int_0^\infty G_\lambda d\lambda}$$
 (65)

Diffuse: Radiation is reflected equally in all directions.

### C. Kirchhoff's Law

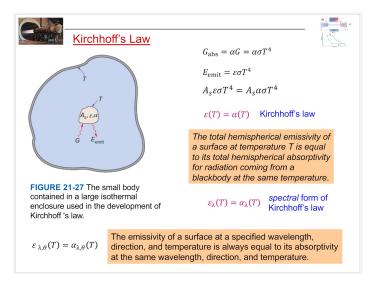


Fig. 4 Kirchhoff's Law info

### D. Wein's Displacement Law

$$(\lambda T)_{\text{max power}} = 2897.8 \mu \text{m} \cdot \text{K}$$
 (66)

### E. Tables

TABLE 21-1					
The wavelength ranges of different colors					
Color	Wavelength band				
Violet Blue Green Yellow Orange Red	0.40-0.44 μm 0.44-0.49 μm 0.49-0.54 μm 0.54-0.60 μm 0.60-0.67 μm 0.63-0.76 μm				

Fig. 5 wavelength ranges

TABLE 18	-2						TABLE	18-3	
Coefficients used in the one-term approximate solution of transient one- dimensional heat conduction in plane walls, cylinders, and spheres (Bi = $hL/k$ for a plane wall of thickness $2L$ , and $Bi = hL/k$ for a cylinder or sphere of						= hL/k	The zeroth- and first-order Bessel functions of the first kind		
for a plane radius $r_o$ )	wall of thick	kness ZL, an	id BI = nr <sub>o</sub> /F	ctor a cylind	er or spnere	OT	η	$J_0(\eta)$	$J_1(\eta)$
	Plane Wall Cylinder Sphere		nere	0.0	1.0000	0.0000			
Bi	$\lambda_1$	A <sub>1</sub>	λ <sub>1</sub>	A <sub>1</sub>	λ <sub>1</sub>	A <sub>1</sub>	0.1	0.9975 0.9900	0.0499
0.01	0.0998	1.0017	0.1412	1.0025	0.1730	1.0030	0.2	0.9900	0.0993
0.01	0.0998	1.0017	0.1412	1.0025	0.1730	1.0060	0.3	0.9604	0.1463
0.02	0.1410	1.0055	0.1993	1.0030	0.2445	1.0120	0.4	0.9604	0.1960
0.04	0.1987	1.0098	0.3438	1.0148	0.4217	1.0120	0.5	0.9385	0.2423
0.08	0.2791	1.0130	0.3960	1.0197	0.4860	1.0239	0.6	0.9120	0.2867
0.1	0.3111	1.0161	0.4417	1.0246	0.5423	1.0298	0.7	0.8812	0.3290
0.1	0.4328	1.0311	0.6170	1.0483	0.7593	1.0592	0.8	0.8463	0.3688
0.3	0.5218	1.0450	0.7465	1.0712	0.9208	1.0880	0.9	0.8075	0.4059
0.4	0.5932	1.0580	0.8516	1.0931	1.0528	1.1164	0.5	0.0070	
0.5	0.6533	1.0701	0.9408	1.1143	1.1656	1.1441	1.0	0.7652	0.4400
0.6	0.7051	1.0814	1.0184	1.1345	1.2644	1.1713	1.1	0.7196	0.4709
0.7	0.7506	1.0918	1.0873	1.1539	1.3525	1.1978	1.2	0.6711	0.4983
0.8	0.7910	1.1016	1.1490	1.1724	1.4320	1.2236	1.3	0.6201	0.5220
0.9	0.8274	1.1107	1.2048	1.1902	1.5044	1.2488	1.4	0.5669	0.5419
1.0	0.8603	1.1191	1.2558	1.2071	1.5708	1.2732			
2.0	1.0769	1.1785	1.5995	1.3384	2.0288	1.4793	1.5	0.5118	0.5579
3.0	1.1925	1.2102	1.7887	1.4191	2.2889	1.6227	1.6	0.4554	0.5699
4.0	1.2646	1.2287	1.9081	1.4698	2.4556	1.7202	1.7	0.3980	0.5778
5.0	1.3138	1.2403	1.9898	1.5029	2.5704	1.7870	1.8	0.3400	0.5815
6.0	1.3496	1.2479	2.0490	1.5253	2.6537	1.8338	1.9	0.2818	0.5812
7.0	1.3766	1.2532	2.0937	1.5411	2.7165	1.8673	0.0	0.0000	0.5767
8.0	1.3978	1.2570	2.1286	1.5526	2.7654	1.8920	2.0	0.2239	0.5767
9.0	1.4149	1.2598	2.1566	1.5611	2.8044	1.9106	2.1	0.1666	0.5683
							2.2	0.1104	0.5560
									0.5399
							2.4	0.0025	0.5202
40.0	1.5325			1.5993		1.9942	26	-0.0968	0.4708
								-0.0968	0.4708
								-0.1850	0.4097
∞	1.5708	1.2732	2.4048	1.6021	3.1416	2.0000		-0.3202	0.3391
50.0 100.0	1.5400 1.5552	1.2620 1.2699 1.2717 1.2723 1.2727 1.2731 1.2732	2.1795 2.2880 2.3261 2.3455 2.3572 2.3809 2.4048	1.6002 1.6015	2.8363 2.9857 3.0372 3.0632 3.0788 3.1102 3.1416	1.9962 1.9990	2.3 2.4 2.6 2.8 3.0 3.2	0. 0. -0. -0.	0555 0025 0968 1850 2601

Fig. 6 Table for  $\lambda$  and A based off of Bi

TABLE 21-2							
Blackbody radiation functions $f_{\lambda}$							
λ <i>T</i> , μm·K	$f_{\lambda}$	λΤ, μm·K	$f_{\lambda}$				
200	0.000000	6200	0.754140				
400	0.000000	6400	0.769234				
600	0.000000	6600	0.783199				
800	0.000016	6800	0.796129				
1000	0.000321	7000	0.808109				
1200	0.002134	7200	0.819217				
1400	0.007790	7400	0.829527				
1600	0.019718	7600	0.839102				
1800	0.039341	7800	0.848005				
2000	0.066728	8000	0.856288				
2200	0.100888	8500	0.874608				
2400	0.140256	9000	0.890029				
2600	0.183120	9500	0.903085				
2800	0.227897	10,000	0.914199				
3000	0.273232	10,500	0.923710				
3200	0.318102	11,000	0.931890				
3400	0.361735	11,500	0.939959				
3600	0.403607	12,000	0.945098				
3800	0.443382	13,000	0.955139				
4000	0.480877	14,000	0.962898				
4200	0.516014	15,000	0.969981				
4400	0.548796	16,000	0.973814				
4600	0.579280	18,000	0.980860				
4800	0.607559	20,000	0.985602				
5000	0.633747	25,000	0.992215				
5200	0.658970	30,000	0.995340				
5400	0.680360	40,000	0.997967				
5600	0.701046	50,000	0.998953				
5800	0.720158	75,000	0.999713				
6000	0.737818	100,000	0.999905				

Fig. 7  $\lambda$ \*f table