Quinlan Lewis
ASEN 3200
9/2/20

HW A-1

4.9

$$V_{uvw} = C_N^0 V_{xyz}$$

$$\hat{u} = 0.26726\hat{\imath} + 0.53452\hat{\jmath} + 0.80178\hat{k}$$
$$\hat{v} = -0.44376\hat{\imath} + 0.80684\hat{\jmath} + 0.38997\hat{k}$$
$$\hat{w} = -0.85536\hat{\imath} - 0.25158\hat{\jmath} + 0.45284\hat{k}$$

$V = -50\hat{\imath} + 100\hat{\jmath} + 75\hat{k}$ ← xyz frame

$$C_N^0 = \begin{bmatrix} 0.26726 & 0.53452 & 0.80178 \\ -0.44376 & 0.80684 & 0.38997 \\ -0.85536 & -0.25158 & 0.45284 \end{bmatrix}$$

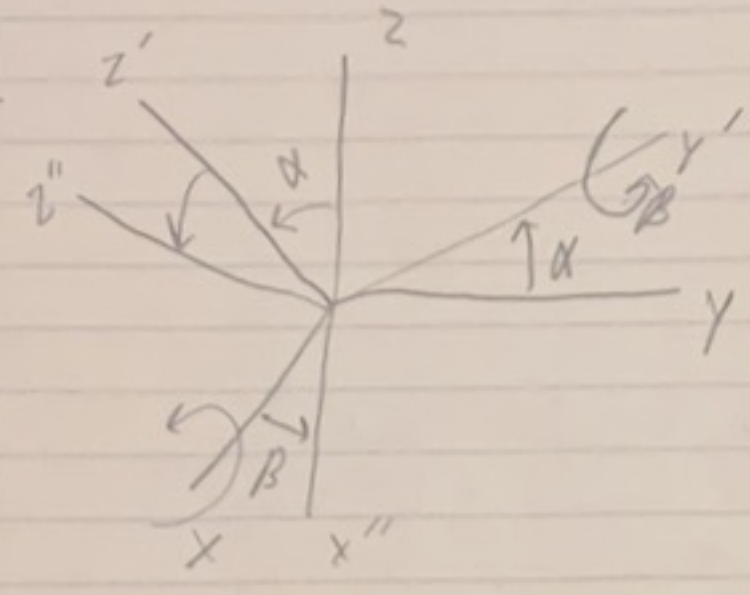
$V_{xyz} = [-50; 100; 75]$

Using Matlab for matrix math...

$V_{uvw} = [100.2; 73.62; 51.57]$

$V = 100.2\hat{u} + 73.62\hat{v} + 51.57\hat{w}$

4.10



$\alpha = 40°$
$\beta = 75°$

rotates about x-axis
then y-axis is
a 1-2 transformation

$$C_1(\alpha) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c\alpha & s\alpha \\ 0 & -s\alpha & c\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(40) & \sin(40) \\ 0 & -\sin(40) & \cos(40) \end{bmatrix}$$

$$C_2(\beta) = \begin{bmatrix} c\beta & 0 & -s\beta \\ 0 & 1 & 0 \\ s\beta & 0 & c\beta \end{bmatrix} = \begin{bmatrix} \cos(75) & 0 & -\sin(75) \\ 0 & 1 & 0 \\ \sin(75) & 0 & \cos(75) \end{bmatrix}$$

$$[Q] = C_2(\beta)\,C_1(\alpha) = \begin{bmatrix} 0.9063 & 0.2717 & -0.3737 \\ 0 & 0.7660 & 0.6428 \\ 0.4226 & -0.5826 & 0.6943 \end{bmatrix}$$

DCM

3 Sequence 3, 2, 1 for 35° each rotation

$Q = C_k(35°)\, C_j(35°)\, C_i(35°)$

(a)
$$= \begin{bmatrix} \cos 35° & \sin 35° & 0 \\ -\sin 35° & \cos 35° & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos 35° & 0 & -\sin 35° \\ 0 & 1 & 0 \\ \sin 35° & 0 & \cos 35° \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos 35° & \sin 35° \\ 0 & -\sin 35° & \cos 35° \end{bmatrix}$$

$$Q = \begin{bmatrix} 0.6710 & 0.4698 & -0.5736 \\ -0.2004 & 0.8547 & 0.4698 \\ 0.7134 & -0.2004 & 0.6710 \end{bmatrix}$$

(b) Using matlab functions eig[Q]

$$\text{eigenvalues} = \begin{bmatrix} 0.6009 + 0.7994i \\ 0.6009 - 0.7994i \\ 1.000 + 0.0001 \end{bmatrix} \leftarrow \lambda = 1$$

↓ from MATLAB

eigenvector associated with eigenvalue $\lambda = 1$ is $\begin{bmatrix} 0.4192 \\ 0.8053 \\ 0.4192 \end{bmatrix}$ ← axis of rotation

$e^{i\phi} = \cos\phi \pm i\sin\phi$ using matlab's built-in angle() function the angle of rotation is 53.07°

This angle was checked using Euler's Thm., which returned the original DCM.

Sequence 3,1,3 for 35° each rotation

$$Q = C_x(35°)\, C_i(36°)\, C_x(35°)$$

$$= \begin{bmatrix} \cos 35° & \sin 36° & 0 \\ -\sin 35° & \cos 35° & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos 35° & \sin 35° \\ 0 & -\sin 35° & \cos 35° \end{bmatrix} \begin{bmatrix} \cos 35° & \sin 35° & 0 \\ -\sin 35° & \cos 35° & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(a) $$Q = \begin{bmatrix} 0.4015 & 0.8547 & 0.3290 \\ -0.8547 & 0.2207 & 0.4698 \\ 0.3290 & -0.4698 & 0.8192 \end{bmatrix}$$

(b) eigenvalues: $\begin{bmatrix} 0.2207 + 0.9753i \\ 0.2207 - 0.9753i \\ 1 + 0i \end{bmatrix}$ $\lambda = 1$

eigenvector associated with $\lambda = 1$ = axis of rotation → $\begin{bmatrix} 0.4817 \\ 0 \\ 0.8763 \end{bmatrix}$

again using matlab's angle() function the angle of rotation is 77.75° ← value checked with Euler's thm.

the use of this function comes from the equation

$$e^{i\phi} = \cos\phi \pm i\sin\phi$$

(c) When comparing the sequences of 3,2,1 and 3,1,3 one can see that the resulting eigenvectors, eigenvalues and DCM's are different because of the difference in sequence rotation.

4.26 c 3, 1, 3 Colution $\phi = 50°, \theta = 25°, \psi = 70°$

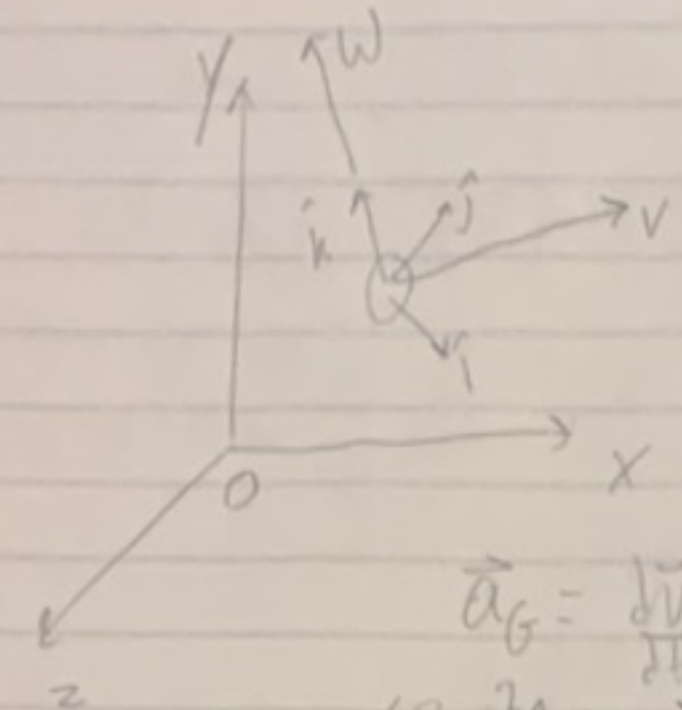
$$C = C_x(\phi)\,C_1(\theta)\,C_x(\psi) = C_x(50°)\,C_1(25°)\,C_x(70°)$$

$$= \begin{bmatrix} \cos 50° & \sin 50° & 0 \\ -\sin 50° & \cos 50° & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos 25° & \sin 25 \\ 0 & -\sin 25° & \cos 25° \end{bmatrix} \begin{bmatrix} \cos 70° & \sin 70° & 0 \\ -\sin 70° & \cos 70° & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -0.4326 & 0.8415 & 0.3237 \\ -0.8094 & -0.5206 & 0.2717 \\ 0.3971 & -0.1445 & 0.9063 \end{bmatrix}$$

$\cos^{-1}(-0.4326) = 115.6°$ angle difference between x-axis and X-axis body Inertial

4.4



$V = t^3 \hat{i} + 4\hat{j}$

$\omega = 2t^2 \hat{k}$

@ $t = 2s$.

$$\vec{a}_G = \frac{d\vec{v}}{dt} + \vec{\omega} \times \vec{v}$$

$$\vec{a}_G = (3t^2\hat{i} + 0\hat{j}) + \begin{bmatrix} 0 \\ 0 \\ 2t^2 \end{bmatrix} \times \begin{bmatrix} t^3 \\ 4 \\ 0 \end{bmatrix}$$

$$= (3t^2\hat{i}) + \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & 2t^2 \\ t^3 & 4 & 0 \end{bmatrix}$$

$$= (3t^2)\hat{i} + (-8t^2)\hat{i} + (2t^5)\hat{j} + 0\hat{k}$$

$$= (3t^2 - 8t^2)\hat{i} + (2t^5)\hat{j} \quad @ \; t = 2$$

$$\boxed{\vec{a}_G = -20\hat{i} + 64\hat{j}}$$

9.6 $\omega = \omega_x \hat{\imath} + \omega_y \hat{\jmath} + \omega_z \hat{k}$ B/I

$\omega = \omega_x \hat{\imath} + \omega_y \hat{\jmath}$ F/I

$\vec{\omega}_{B/I} = \vec{\omega}_{B/F} + \vec{\omega}_{F/I}$

$\vec{\omega}_{B/F} = \vec{\omega}_{B/I} - \vec{\omega}_{F/I} = \omega_z \hat{k}$

$\vec{\alpha}_{B/F} = \frac{d}{dt} \omega_{B/I} + \vec{\omega}_{F/I} \times \vec{\omega}_{B/F}$

$\vec{\alpha}_{B/F} = \frac{d}{dt}(\omega_x \hat{\imath} + \omega_y \hat{\jmath} + \omega_z \hat{k}) + (\omega_x \hat{\imath} + \omega_y \hat{\jmath} + \omega_z \hat{k}) \times (\omega_z \hat{k})$

$= \frac{d}{dt}(\omega_x \hat{\imath} + \omega_y \hat{\jmath} + \omega_z \hat{k}) + \begin{pmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \omega_x & \omega_y & \omega_z \\ 0 & 0 & \omega_z \end{pmatrix}$

$\vec{\alpha}_{B/F} = 0 + \omega_y \omega_z \hat{\imath} - \omega_x \omega_z \hat{\jmath}$

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%Quinlan Lewis
%ASEN 3200
%last modified: 8/31/20

clear all; clc;

%declare angles of rotation
theta = 35;
phi = 35;
psi = 35;

C_3 = [cosd(psi) sind(psi) 0; -sind(psi) cosd(psi) 0; 0 0 1];
C_1 = [1 0 0; 0 cosd(theta) sind(theta); 0 -sind(theta) cosd(theta)];
C_2 = [cosd(phi) 0 -sind(phi); 0 1 0; sind(phi) 0 cosd(phi)];

%declare order of operations for Q matrix
Q = C_1*C_2*C_3;

%calculate eigenvalues and vectors for finding axis of rotation and angle
%of rotation
[v,d] = eig(Q);
eigenvalues = diag(d);
eigenvectors = v;

%Assigns u hat and u tilda vectors
u_hat = eigenvectors(:,3);
u_tilda = [0 -u_hat(3) u_hat(2); u_hat(3) 0 -u_hat(1); -u_hat(2) u_hat(1) 0];

%find value of rotation angle
phi = angle(eigenvalues(1))*180/pi;

%plug values into euler's theorem to check to see if angle of rotation is
%correct
E_T = cosd(phi)*eye(3,3) + (1 - cosd(phi))*(u_hat)*(u_hat') - sind(phi)*u_tilda;
```