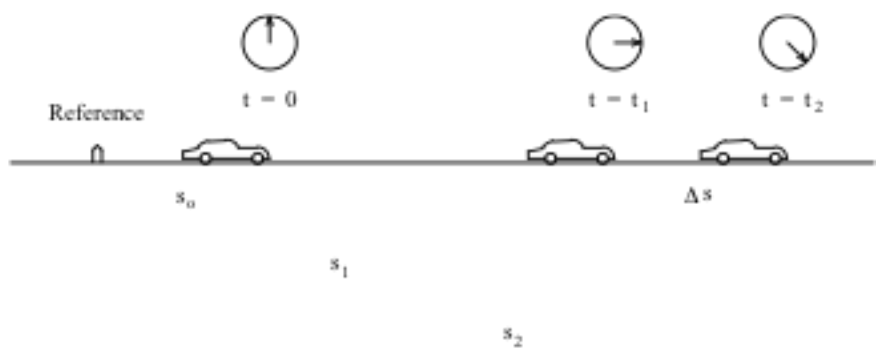
Notes M-6

Introduction to Rectilinear Motion

Rectilinear Motion: Motion in a straight line.

Kinematics: The study of motion without regard to its cause.

Let s equal the distance of an object from a reference position. Let s_o equal the distance at time t = 0, e. g., the time at which the stopwatch is started. Let s_1 = the distance at time t_1 and s_2 = the distance at time t_2. Frequently, the reference position is taken as the position of the body at t = 0. In this case, $s_o = 0$



$$\text{Average Speed:} \quad v_{av} = \frac{s_2 - s_1}{t_2 - t_1} = \frac{\Delta s}{\Delta t}$$

Sample problem: An automobile being tested on a straight road is 700 ft from its starting point when the stopwatch reads 10 seconds and is 800 feet from the starting point when the stopwatch reads 12 seconds. What is the average velocity of the automobile during the interval from t = 10 seconds to t = 12 seconds?

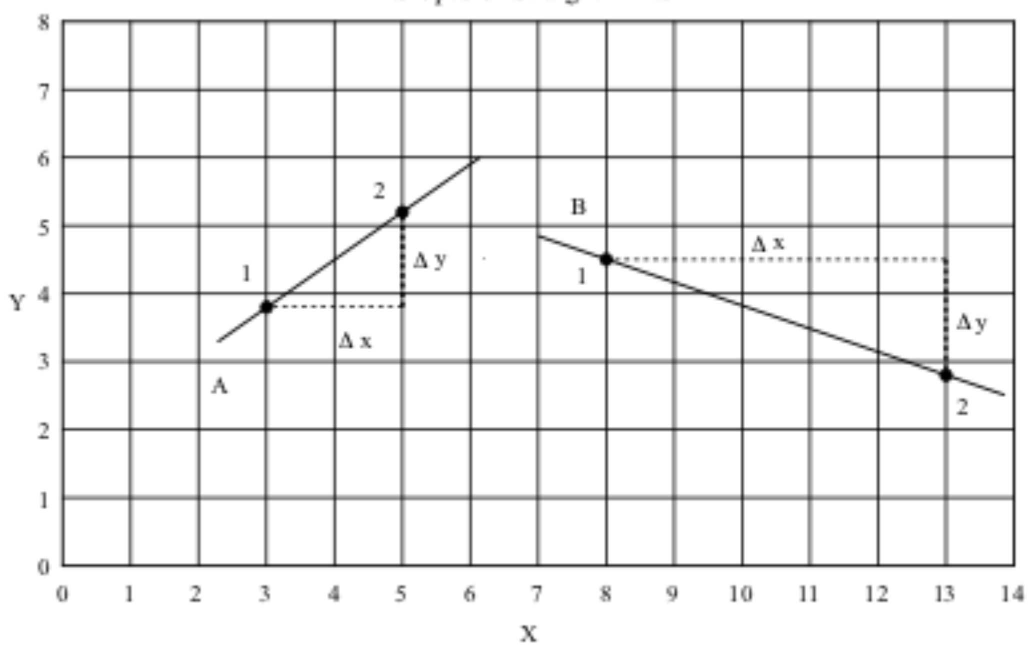
$$v_{av} = \frac{s_2 - s_1}{t_2 - t_1} = \frac{800\text{ ft} - 700\text{ ft}}{12\text{ s} - 10\text{ s}} = 50\frac{\text{ft}}{\text{s}}$$

The average acceleration is defined in a similar manner as follows:

$$a_{av} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{\Delta v}{\Delta t}$$

Scalars	Corresponding Vectors
Distance s	Displacement **s**
Speed v	Velocity **v**
Magnitude of acceleration a	Acceleration **a**

Slopes of Straight Lines



$$\text{Slope of line A} = \frac{\text{Rise}}{\text{Run}} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5.2 - 3.8}{5.0 - 3.0} = \frac{1.4}{2.0} = 0.70$$

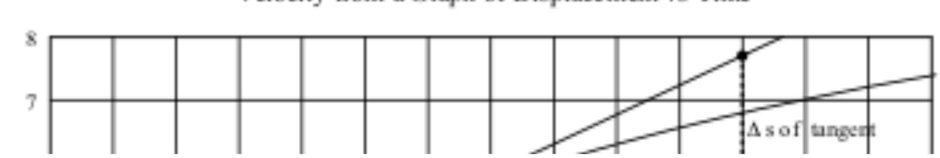
$$\text{Slope of Line B} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2.8 - 4.5}{13.0 - 8.0} = \frac{-1.7}{5.0} = -0.34$$

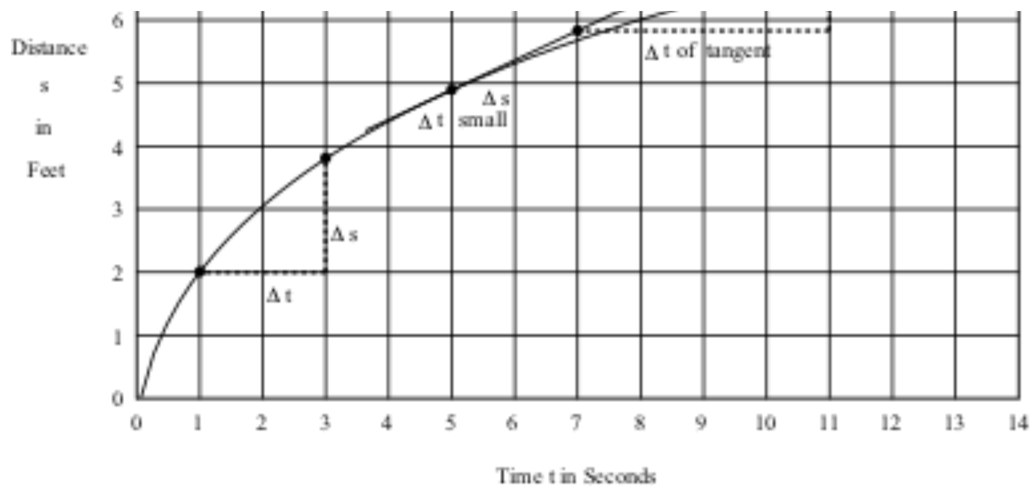
The results should be independent of the choice of points 1 and 2 on the line or of the size or location of the triangles on the line. It could be proven by geometry that two such triangles with sides Δx and Δy on the same line are similar and that their sides are in proportion. However, some students make the mistake of looking for a place where the line comes close to an intersection of grid lines and then pretending that the line passes through that point when it does not. For instance line A does not pass through the point (6, 6). It is best to choose two widely separated points where the line cuts two vertical grid lines. x_1 and x_2 will then be fairly simple numbers, but y_1 and y_2 will have to be read carefully by estimating the position of the points between horizontal grid lines.

Note that the slope of a line can be negative as in the example of line B above. If you mark the two points on the line as 1 and 2, and if you carefully substitute values in the formula, the calculations will give you the correct + or - sign. Some students make the mistake of saying that the slope is Δy over Δx, that Δy is the larger y minus the smaller y, and that Δx is the larger x minus the smaller x. Such students usually miss the minus sign for the slope of a line when a minus sign should be present.

If suitable drafting equipment is available, it might be easier to measure Δy and Δx by adjusting dividers to the size of the sides of the triangle -- and then setting the dividers on the scale of the graph. Otherwise, it is necessary to find Δy and Δx by subtracting coordinates of points 1 and 2 as was done above. Even in the latter case it is helpful in understanding the problem to sketch in the triangle with sides Δy and Δx.

Velocity from a Graph of Displacement vs Time





Sample problem: A body is moving in a straight line with a velocity described by the graph above. Find the average velocity from t = 1 second to t = 3 seconds.

$$v_{av} = \frac{\Delta s}{\Delta t} = \frac{s_2 - s_1}{t_2 - t_1} = \frac{3.8 - 2.0}{3.0 - 1.0} = \frac{1.8}{2.0} = 0.90\frac{ft}{s}$$

Sample problem: Find the instantaneous velocity at t = 5 s for the same body.

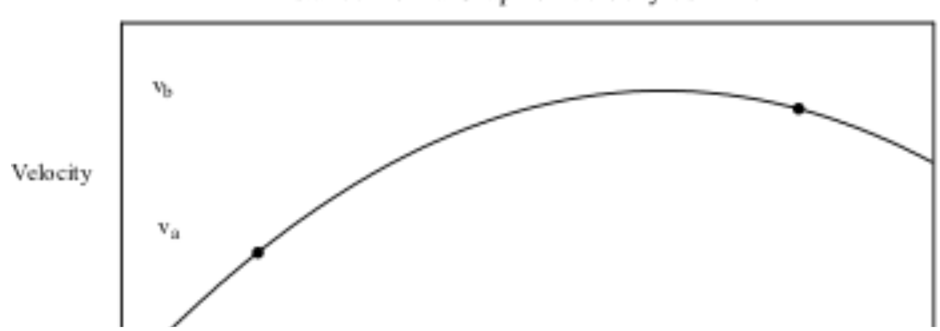
$$v = \frac{\Delta s}{\Delta t}$$ for Δs and Δt very small*

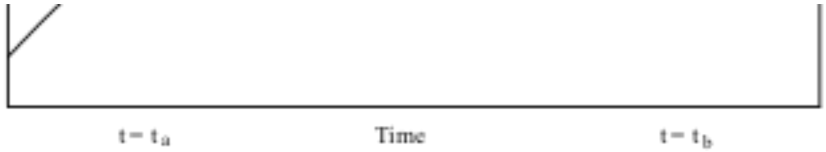
We could select a very small Δt centered about the point on the curve at which t = 5 seconds; and we could find the corresponding small Δs. See the small triangle in the figure above. But we would obtain a poor accuracy. Instead, we draw a tangent line to the curve at the point where t = 5 seconds. To draw the tangent line, we hold a straight edge near the curve, so that it is parallel to the small part of the curve about the point in question. We touch the pencil point to the point in question on the curve. We move the straight edge closer to the curve, always keeping it parallel to the curve, until the straight edge touches the pencil. We then draw the tangent line. We choose two widely spaced points on the tangent line and measure the slope of the tangent line. Compare the two triangles against the tangent line in the figure above. The small triangle actually has two points on the curve, rather than the tangent line, but the small part of the curve nearly coincides with the tangent line. The two triangles are similar and their corresponding sides are in proportion.

$$v = \frac{\text{small } \Delta s}{\text{small } \Delta t} = \frac{\Delta s \text{ of tangent}}{\Delta t \text{ of tangent}} = \frac{s_2 - s_1}{t_2 - t_1} = \frac{7.7 - 5.8}{11.0 - 7.0} = \frac{1.9}{4} = 0.48\frac{ft}{s}$$

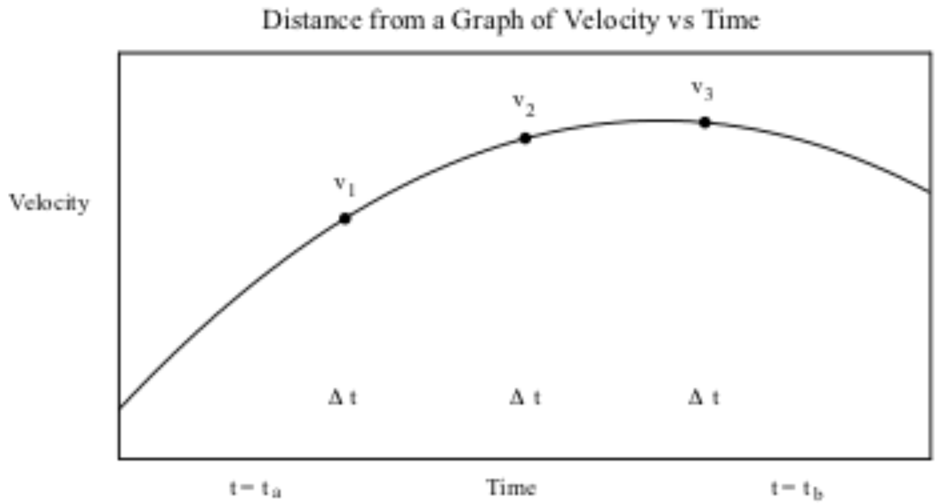
*Or in calculus notation: $v = \frac{ds}{dt}$

Distance from a Graph of Velocity vs Time





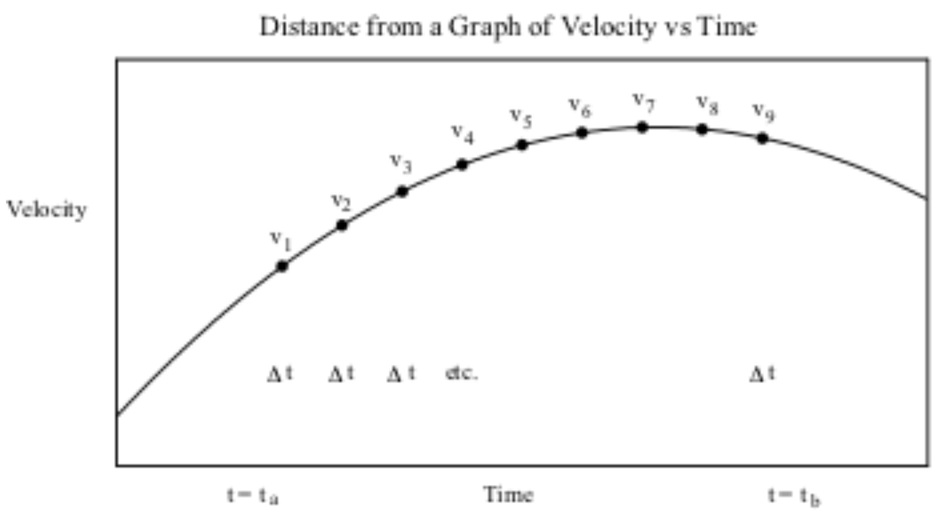
Distance traveled from time t_a to time t_b: $s_{ab} = v_{av}\Delta t = v_{av}(t_b - t_a)$ But v_{av} not usually $= \frac{v_a + v_b}{2}$ Divide up the time interval from t_a to t_b into three equal intervals. v_1 is an estimate of the average velocity during the first interval; and s_1 is the distance traveled during the first interval.



Distance traveled from t_a to t_b:

$$s_{ab} = s_1 + s_2 + s_3 \approx v_1\Delta t + v_2\Delta t + v_3\Delta t$$
$$\approx \text{area under staircase}$$

Now divide the interval from t_a to t_b into a larger number of smaller intervals Δt. v_1 is still an estimate of the average velocity in interval number 1; but now it is a better estimate.



Distance traveled from time t_a to time t_b: s_{ab} still $\approx$ area under staircase.

Area under staircase $\approx$ area under curve.

Divide the interval from t_a to t_b into a very large number n of very small intervals Δt Distance traveled from time t_a

Divide the interval from t_a to t_b into a very large number n of very small intervals Δt. Distance traveled from time t_a to time t_b:

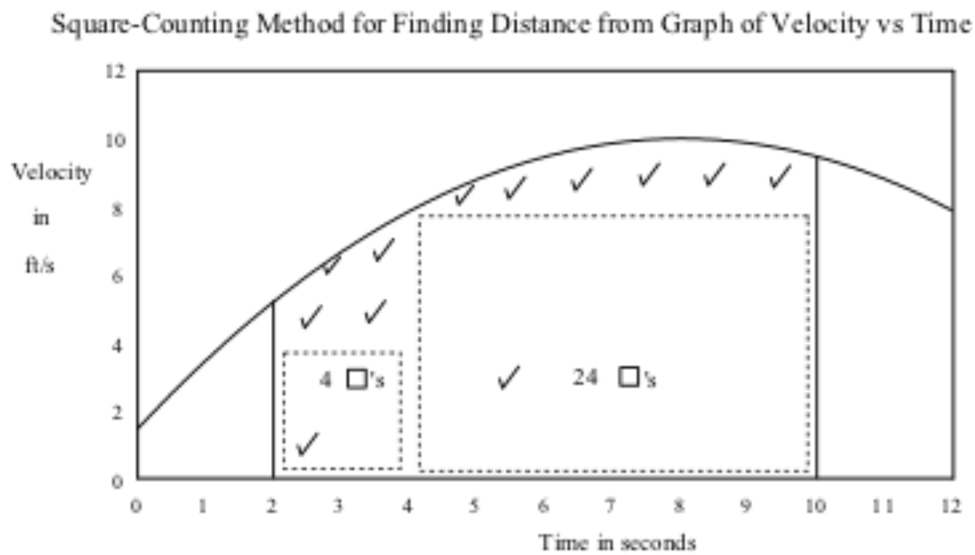
$$s_{ab} = v_1 \Delta t + v_2 \Delta t + \boxed{?} + v_n \Delta t = \sum_{i=1}^{n} v_i \Delta t$$

$$= \text{ area under staircase } = \text{ area under curve}$$

When n is extremely large and Δt is extremely small, the following symbols are used for the above summation:

$$s_{ab} = \int_{t_a}^{t_b} v \, dt$$

This notation is from integral calculus. The statement is read as: "S sub ab is the integral from t sub a to t sub b of v dt." But the distance can be calculated from a graph of velocity vs time without calculus by measuring the area under the curve. **Sample Problem:** A body is moving in a straight line with a speed given by the graph below. Find the distance traveled between t = 2 s and t = 10 s.



Distance: s = Area under curve from t = 2 s to t = 10 s.

Number of squares:

24
4
3
.9
.1
.6
.2
.5
.8
.8
———
34.9

Area of each square:

2 ft/s x 1 s = 2 ft

$$s = 34.9 \text{ squares} \times 2 \frac{\text{ft}}{\text{square}} = 69.8 \text{ feet}$$

36

Notes M-6

Name: Quinn Dougherty

An automobile being tested on a straight road is 400 feet from its starting point when the stopwatch reads 8.0 seconds and is 550 feet from the starting point when the stopwatch reads 10.0 seconds.

A. What was the average velocity of the automobile during the interval from t = 8.0 seconds to t = 10.0 seconds?

(550 - 400) / (10 - 8) ft/s = 75 ft/s

B. What was the average velocity of the automobile during the interval from t = 0 s to t = 10.0 s? (Assume that the stopwatch read t = 0 and started at the same time as the auto.)

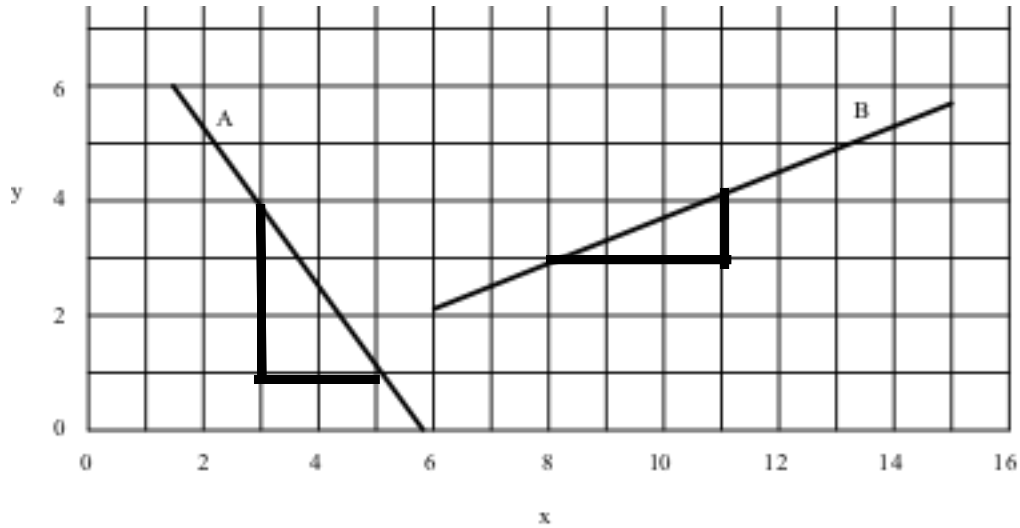
(550 - 0) / (10 - 0) ft/s = 55 ft/s

C. If the automobile averages 100 ft/s from t = 10.0 s to t = 20.0 s, what distance does it travel during this interval?

d = vt, v <- 100 ft/s, t <- 20-10 s => d = 100 ft/s * (20 - 10) s = 1000 ft

D. The automobile has a special speedometer calibrated in feet/s instead of in miles/hour. At t = 8 s the speedometer reads 65 ft/s; and at t = 10 s it reads 80 ft/s. What is the average acceleration during this interval?

Name: ______________________________



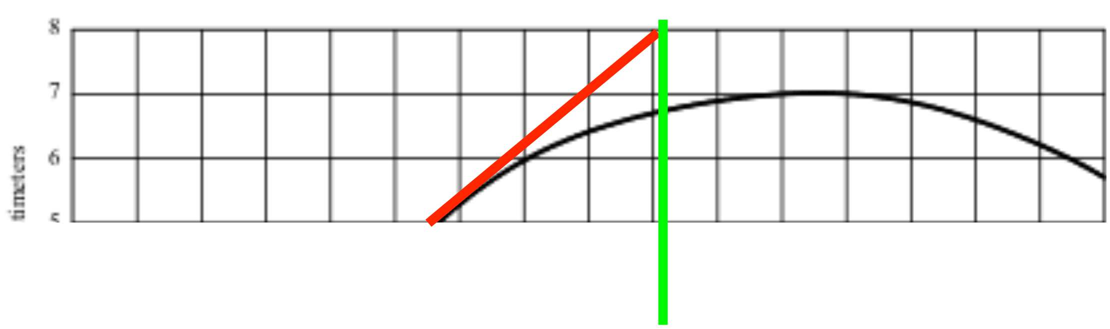
Determine the slopes of lines A and B. Draw in triangles and indicate where you are measuring Δx and Δy. (Note that neither line as drawn passes through intersections of grid lines.) Try to read coordinates of points to the nearest tenth of a division by estimating the position of the points between grid lines.

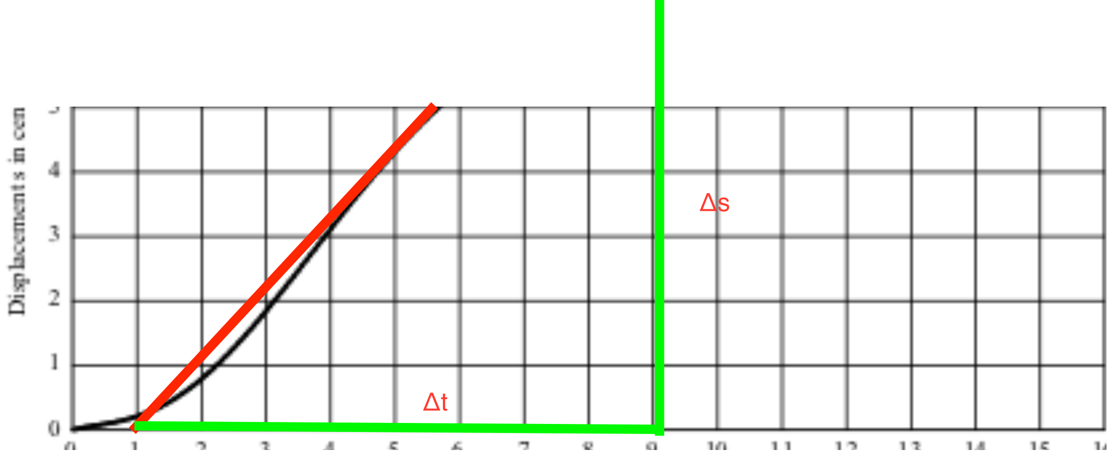
Slope of Line A = (4-1) / (5-3) = 3/2

Slope of Line B = (4 - 3) / (11 - 8) = 1/3

Name: ____

Finding velocity from a curve of displacement vs time for a body moving in a straight Line





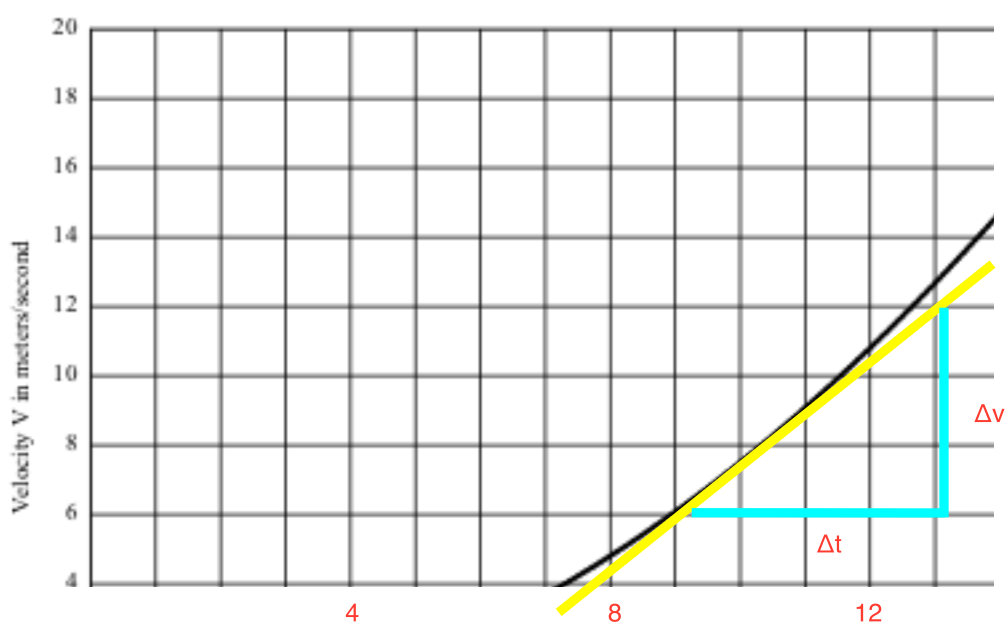
A. What is the average velocity between t = 2 seconds and t = 4 seconds?

(3 - 1) / (4 - 2) = 1

B. What is the instantaneous velocity at t = 6 seconds? Draw in the tangent line required. Show the two points on the tangent line used for measuring the slope. Draw in the right triangle whose hypotenuse is the part of the tangent line between these two points. Label the sides of the triangle □s and □t. (For good accuracy draw a fairly long tangent line and choose the two points fairly far apart on the tangent line.)

Δs/Δt = (8-0) / (9 - 1) = 1

Name: ____



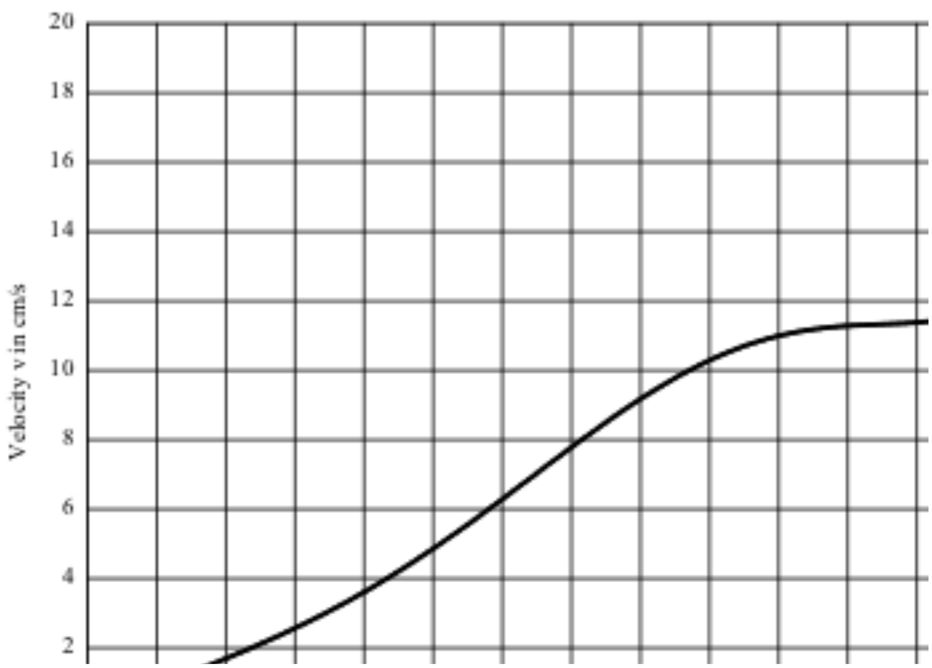
Finding acceleration from a curve of velocity vs time for a body moving in a straight line

What is the instantaneous acceleration at t = 10 seconds? Draw the required tangent line. Show the points on the line used to calculate the slope. Draw in the right triangle whose hypotenuse is the part of the tangent line between those two points. Label the sides of the triangle Δv and Δt. (For good accuracy draw a fairly long tangent line and choose the two points fairly far apart on the tangent line.)

(12 - 6) / (13 - 9) = 3/2

Name: ____

4 8 16



An object is moving in a straight line with a velocity as given by the graph below. What distance does it travel between t = 4 s and t = 26 s? (See the example on page 6 of the M-6 notes in the laboratory manual.)

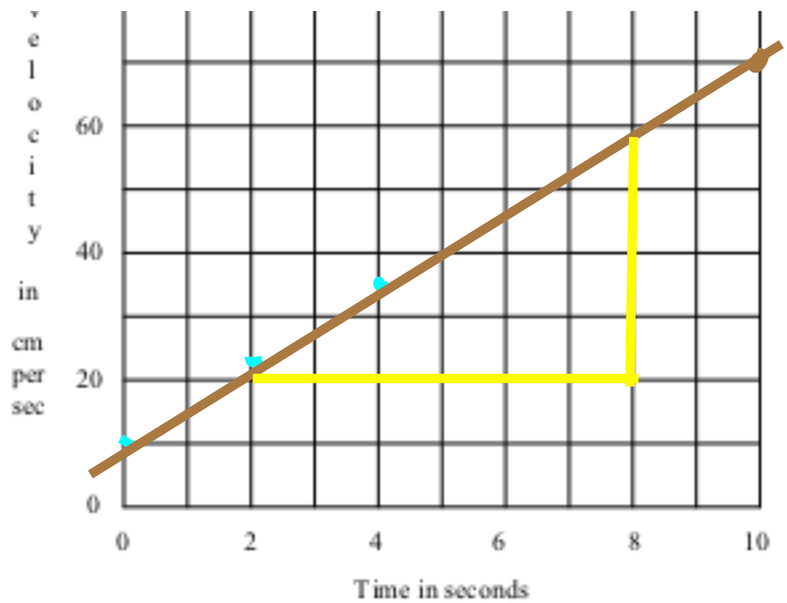
11 + 9.5 + 8 + 6.5 + 5.5 + 2.5 = 43

Name: ____

A. The velocity of a certain body moving in a straight line is given by the formula: v = 10 + 6t where t is in seconds and v is in cm/sec. Complete the table by calculating v at t = 0, 2, 4, ... and 10 s. Plot the curve of velocity versus time on the graph below.

Velocity vs Time

t	v
s	cm/s
0	
2	
4	
6	
8	
10	



B. The shape of the curve is ___linear___

C. Measure the slope of the curve. Indicate where you do so with a triangle. Label its sides.

$$\text{Acceleration} = \frac{(60 - 20)}{(8 - 2)} = \underline{6.666666666666667}\ \text{cm/s}^2$$

D. Find the total distance traveled from t = 0 to t = 10 seconds by calculating the area under the curve.

s = = ___80___ cm

Note: Area of trapezoid: $A = \frac{b}{2}(h_1 + h_2)$

(10 - 0) / 2 * (v(0) + v(10))
= 5 * (10 + 10 + 6(10))
= 20 + 60
= 80