CME241 Assignment10

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1 Problem 1

We want to test the performance of the optimal policy we derived in class for the optimal market-making problem. We'll do so by generating 10,000 simulation traces, where each simulation consists of 200 steps, with $\delta t = 0.005$

At each time t, we observe the state, and calculate the optimal action, from which we generate a random probability dependent on this optimal spread that will tell us if we sell or buy the asset. We also let the OB Mid Price do a random walk about the starting value. We then find the average spread generated by this policy, and set this to be a naive deterministic policy. We then compare the performance between these two methods. Note, the hyper-parameters for this experiment are as follows:

$$S_0 = 100, T = 1, \delta t = 0.005, \gamma = 0.1, \sigma = 2, I_0 = 0, k = 1.5, c = 140$$

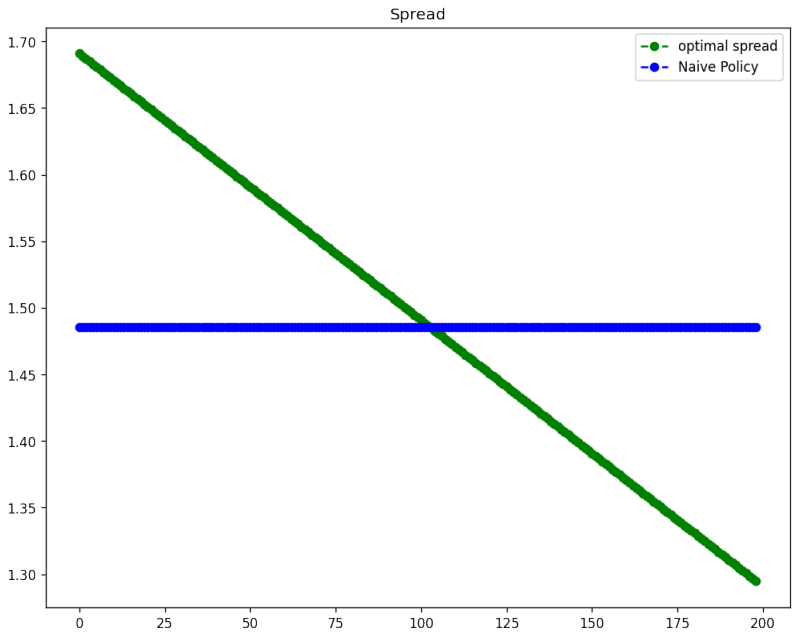
Below, I share some plots from the notebook that capture how the optimal policy fares vs. the naive one:

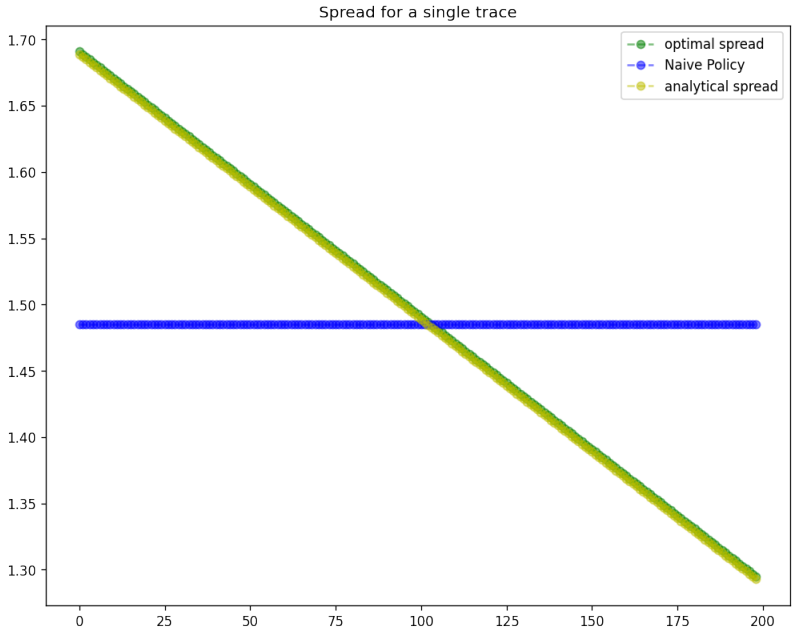
The first two graphs show the spread for (a) the average over all simulation traces at that time step, and (b) for a single trace. I also over-layed the analytical curve, and we see that it matches, which should give us some confidence that our simulation was accurate for the other quantities we computed.

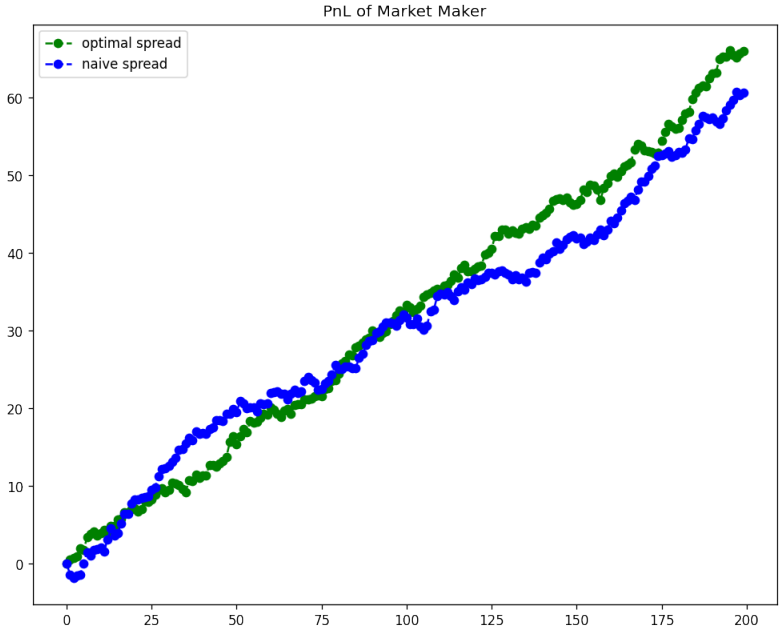
Note, all of the following graphs after the second are averaged over the simulation traces at each specific time step. We can now see the PnL differences between the two strategies. The optimal policy fared slightly better than the naive policy, but gives empirical evidence that our strategy worked.

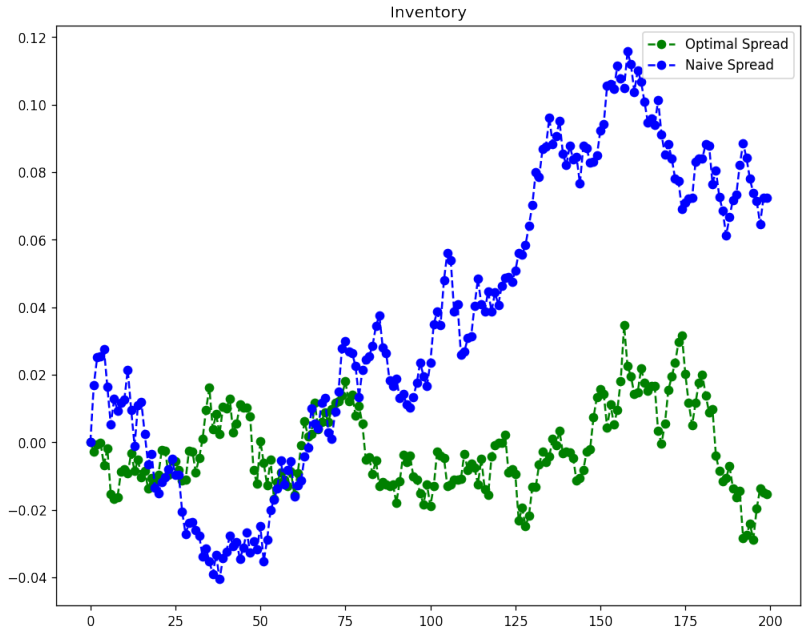
Also, the inventory in the optimal policy tends to stay closer to 0 which is what we would like, since it lowers our risk of being on the wrong side of market movements.

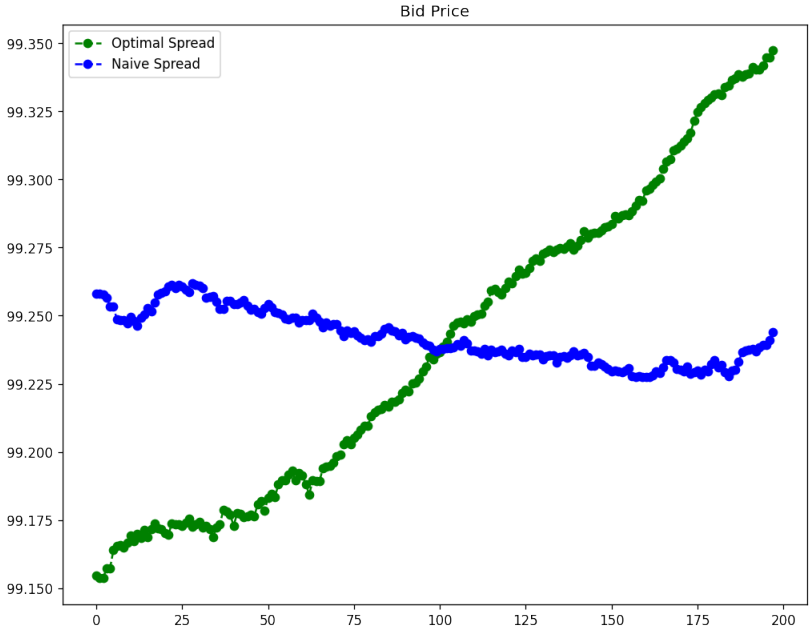
I also include the bid, ask, mid price, lifts, and hit plots for completeness.

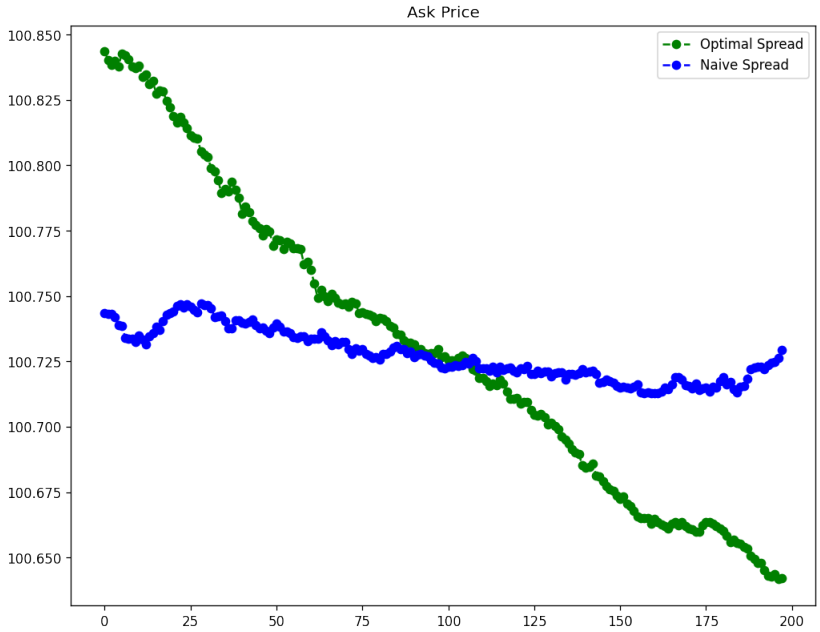

Spread
optimal spread
Naive Policy
1.70
1.65
1.60
1.55
1.50
1.45
1.40
1.35
1.30
0
25
50
75
100
125
150
175
200

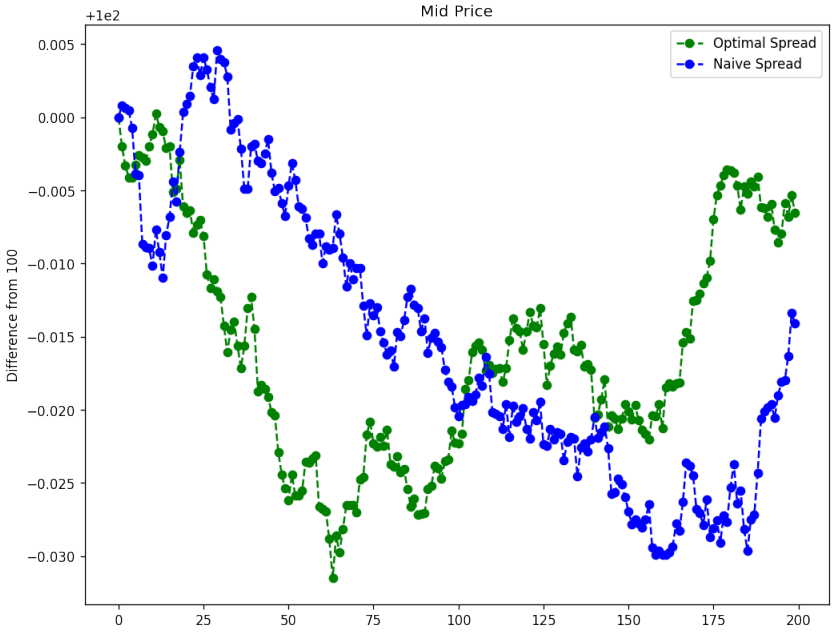

Spread for a single trace
optimal spread
Naive Policy
analytical spread
1.70
1.65
1.60
1.55
1.50
1.45
1.40
1.35
1.30
0
25
50
75
100
125
150
175
200

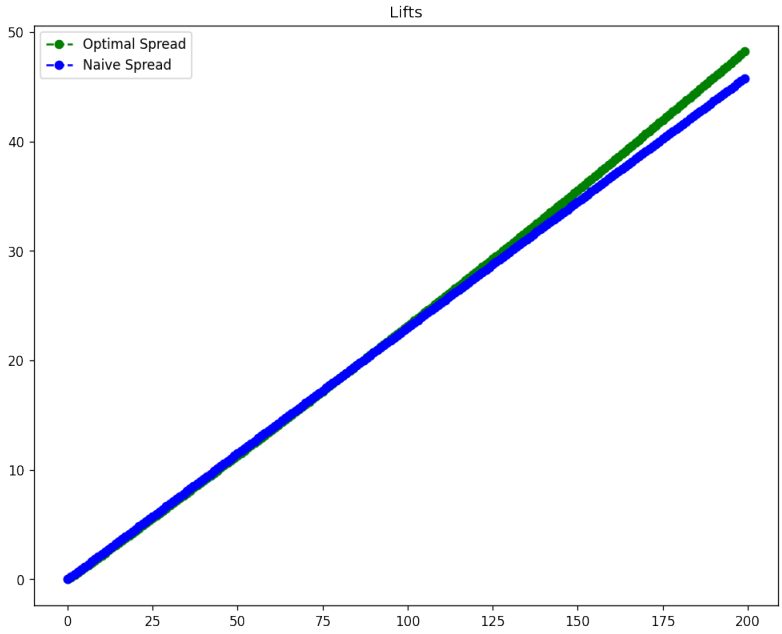

PnL of Market Maker
optimal spread
naive spread
60
50
40
30
20
10
0
0
25
50
75
100
125
150
175
200

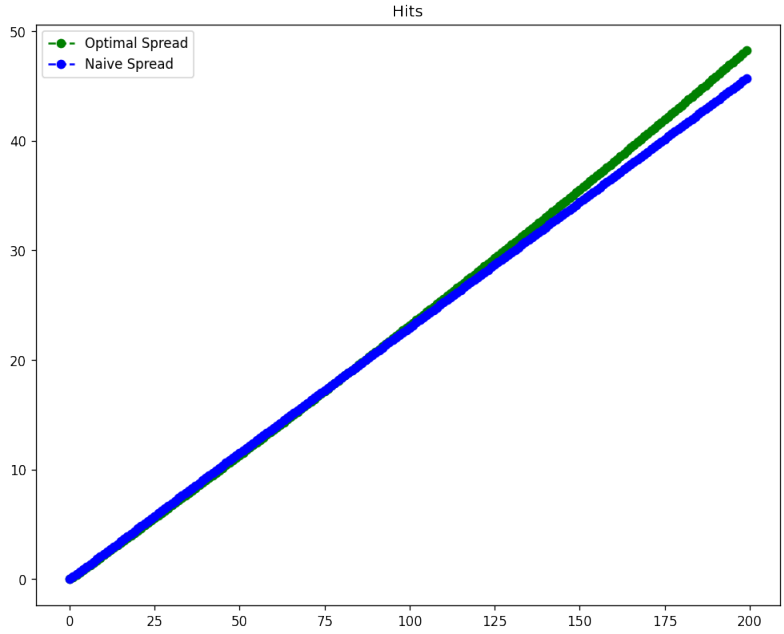

Inventory
Optimal Spread
Naive Spread
0.12
0.10
0.08
0.06
0.04
0.02
0.00
−0.02
−0.04
0
25
50
75
100
125
150
175
200


Bid Price
Optimal Spread
Naive Spread
99.350
99.325
99.300
99.275
99.250
99.225
99.200
99.175
99.150
0
25
50
75
100
125
150
175
200


Ask Price
Optimal Spread
Naive Spread
100.850
100.825
100.800
100.775
100.750
100.725
100.700
100.675
100.650
0
25
50
75
100
125
150
175
200


Mid Price
+1e2
Optimal Spread
Naive Spread
Difference from 100
0.005
0.000
−0.005
−0.010
−0.015
−0.020
−0.025
−0.030
0
25
50
75
100
125
150
175
200


Lifts
Optimal Spread
Naive Spread
50
40
30
20
10
0
0
25
50
75
100
125
150
175
200


Hits
Optimal Spread
Naive Spread
50
40
30
20
10
0
0
25
50
75
100
125
150
175
200