

# Intermediate Value Theorem Project

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## 0.1 Definitions

**Definition 1** (Sequence). A sequence, denoted  $a$  or  $\{a_n\}$ , is a function  $a : \mathbb{N} \rightarrow \mathbb{R}$ .

**Definition 2** (Convergence). A sequence  $\{a_n\}$  converges to  $L \in \mathbb{R}$  if for all  $\varepsilon > 0$  there exists  $N \in \mathbb{N}$  such that for all  $n \geq N$ ,  $|a_n - L| < \varepsilon$ . We say  $\{a_n\}$  converges if there exists  $L \in \mathbb{R}$  such that  $\{a_n\}$  converges to  $L$ .

## 0.2 Theorems

**Theorem 3** (Limit Laws). *Let  $C \in \mathbb{R}$ . Suppose  $\{a_n\}$  converges to  $L$  and  $\{b_n\}$  converges to  $K$ . Then*

- (i)  $\{Ca_n\}$  converges to  $CL$*
- (ii)  $\{a_n + b_n\}$  converges to  $L + K$*

**Theorem 4** (Smale 1958). *There is a homotopy of immersions of  $\mathbb{S}^2$  into  $\mathbb{R}^3$  from the inclusion map to the antipodal map  $a : q \mapsto -q$ .*

*Proof.* This obviously follows from what we did so far. Testing changing my branch □