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Volatility spillovers in commodity markets: A large t-vector autoregressive approach

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Contents

1 Introduction

Investments in commodity markets such as energy or agriculture have become more and more important for investors to make money or to hedge risks. It is well-known that in financial investment both price and volatility are crucial criteria for investors and financial analysts to make decision. Unlike price, which is observable, volatility measuring risk of an asset is determined from prices, which makes it hard to forecast volatility. According to Mandelbrot (1963), volatility cluster is one of the stylized facts of financial returns. It turns out that high volatility today might lead to high volatility tomorrow; hence, the historical volatility data plays an essential role in predicting future volatility.

On the other hand, the link between energy and agricultural commodities changed thanks to the biofuel technology. For example, corn and soybean are the source of bioethanol and biodiesel production, which have become technological alternative for traditional fuels (Chang and Su, 2010). It turns out that the volatility of agricultural commodities can affect the volatility of energy commodities - referred as volatility spillovers in this report. Therefore, it may be useful to use volatility of relevant commodities while forecasting volatility of one commodity to achieve more accurate prediction.

In this seminar, we replicate the work of Barbaglia et al. (2019) and summarize the answers for three main research questions:

- Do the volatility spillovers among energy, agricultural and biofuel commodities exist?
- Do these spillovers change over time?
- Is there any improvement in forecast accuracy while considering the fat tails of distribution?

We analyze the volatility spillovers among three huge commodity markets; therefore, we need to include many variables in our model. As a result, we use t-lasso for estimating a large VAR, which allows us to not only estimate model with a large number of time series relative to the time series length, but also take fat-tailed errors into consideration.

Overall, the volatility spillovers among energy, biofuel and agricultural are detected, and these spillovers change from 2014 to 2016. The prediction accuracy of t-lasso model is better than Gaussian lasso for short-term forecast.

The rest of the report is organized as follows. Section 2 provides an introduction to the data set. The t-lasso vector autoregressive model and estimation procedure are presented in section 3. Section 4 is about the empirical result of the volatility spillovers and prediction performance. Conclusion presents some remarkable findings and limitations.

2 Data

The report aims to analyze volatility spillovers and forecast volatility of energy (crude oil, gasoline, natural gas), agricultural (corn, wheat, soybean, sugar, cotton, coffee) and biofuel (ethanol) commodities. The daily price data is collected from Thomson Reuters Eikon database. We use the data from January 3rd, 2012 to October 28th, 2016, which is similar dataset in the original paper. We have 1718 observations for each series in total.

Daily volatility is defined as follows:

$$\upsilon_{t,j} = \frac{1}{4log(2)}\left[log(H_{t,j}) - log(L_{t,j})\right]^2$$

where $H_{t,j}$ and $L_{t,j}$ is the highest price and the lowest price on date t for commodity j. The log transformations $y_t = log(\upsilon_{t,1}), ..., log(\upsilon_{t,J})$ then become the inputs to estimate volatility spillover and forecast volatility.

Barbaglia et al. (2019) finds strong evidence of the stationary of the log volatilities by employing univariate unit root tests. However, we employ Augmented Dickey-Fuller test, one of the unit root test, and fail to reject the null hypothesis that log volatility of crude oil/gasoline has unit root while the stationary holds true for the rest. As can be seen from Figure 1, there is probably break in time series of crude oil and gasoline at the beginning of 2015. We decide to proceed with log volatilities in order to compare the result with the original paper even though the two time series of energy commodity are not stationary.

3 Methods

3.1 Vector Autoregressive (VAR) Model

VAR(p) model consists of a system of regression equations, which is not only estimated by regressing each variable on its lag, but also the lags of other variables in the system.

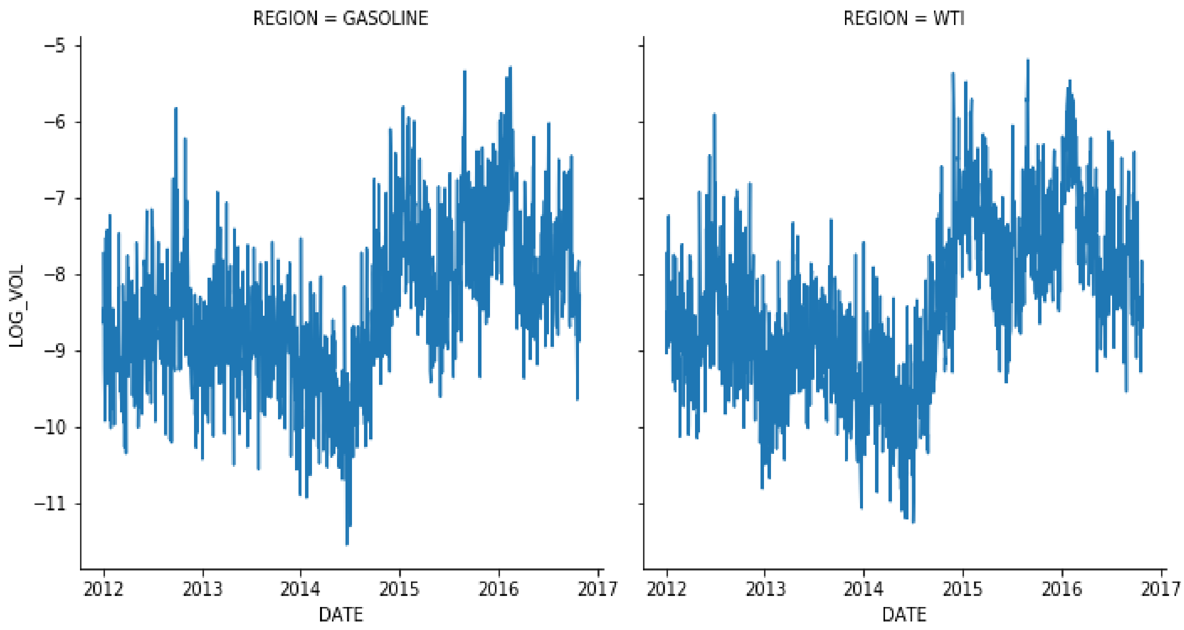

Figure 1: Gasoline and crude oil time series

The system is represented as follows

$$Y_t = c + \sum_{p=1}^{P} B_p y_{t-p} + e_t$$

where $Y_t = log(\upsilon_{t,1}), ..., log(\upsilon_{t,J})$ is J dimensional vector of log volatilities at time t, B_p are JxJ coefficient matrices for p = 1, 2, ..., P and e_t is a J-dimentional white noise process such that $E[e_t] = 0$ and $Var(e_t) = E[e_t e_t^T] = \Sigma_e$ where Σ_u is positive definite.

Basically, the VAR(p) system can be estimated by OLS for equation separately, which results in efficient estimators (Kilian, L., Lütkepohl, H., 2017)). However, we employ the t-lasso VAR(p) so there is no closed form by OLS to solve for the B_p; therefore, we need to have a compact form for VAR(p) system. Without loss of generality, the system for mean centered time series can be rewritten in the more compact form

$$Y = XB + E$$

where $Y = [y'_{P+1}, ..., y'_T]'$ is aNxJ matrix with N = T-P, $X = [X_1, ..., X_P]$ is a NxJP matrix with $X_p = [y'_{P+1-p}, ..., y'_{T-p}]'$ for p = 1,..., P and E is the NxJ white noise with covariance matrix, Σ. The autoregressive coefficients is $B = [B'_1, ..., B'_P]$ with JPxJ dimension.

3.2 Gaussian Lasso Estimation

The application of VAR(p) on estimating macroeconomics or financial model is prone to curse of dimensionality while there are so many relevant variables but limited data available, which increases the chance of ending up with overfitting model if the OLS approach is used. In this case, regularized regression can help to control the coefficient in order to improve the accuracy of out-of-sample forecast. Lasso regression is one of the regularized model and lasso estimation for coefficients matrix of VAR(p) is given by

$$\hat{B}_{lasso} = \underset{B}{\operatorname{argmin}} \frac{1}{2N} tr\left[(Y - XB)(Y - XB)'\right] + \lambda \sum_{i,j=1}^{J} \sum_{p=1}^{P} |B_{p,ij}|$$

where tr(.) is the trace operator, and $\lambda > 0$ is a regularization parameter.

Gaussian lasso estimation for the coefficients matrix of VAR(p) is obtained be assuming E follows a multivariate normal distribution $N(0, \Sigma)$ (Barbaglia et al, 2019). $\hat{B}$ is given by

$$(\hat{B}_G, \hat{\Omega}) = \underset{B_G, \Omega}{\operatorname{argmin}} \frac{1}{2N} tr\left[(Y - XB)\Omega(Y - XB)'\right] - \frac{1}{2} log|\Omega| + \lambda \sum_{i,j=1}^{J} \sum_{p=1}^{P} |B_{p,ij}| + \gamma \sum_{i \neq j}^{J} |\omega_{ij}|$$

where $\Omega = \Sigma^{-1}$ is the estimation of the inverse innovation covariance matrix, ω_{ij} is the ijth element of Ω, $|\Omega|$ is the determinant of Ω. $\gamma > 0$ is another regularization parameter to ensure that the estimation of Ω is always feasible. The larger the γ, the more zero elements Ω has.

3.3 t-lasso Estimation

Assume that E follows a multivariate t-distribution $t_v(0, \Psi)$ where $v > 0$ is the degree of freedom, and scale matrix Ψ. According to Barbaglia et al. (2019), there is a way to rewrite Student t-distribution random vector e_t as scale mixture of a multivariate normal $N(0, \Psi)$ and a random variable τ_t following Gamma $\Gamma(v/2, v/2)$ distribution. We have

$$e_t|\tau_t \sim N\left(0, \frac{\Psi}{\tau_t}\right)$$

and

$$\tau_t|e_t \sim \Gamma\left(\frac{v + J}{2}, \frac{v + e_t'\Omega e_t}{2}\right)$$

The expected value of $\hat{\tau}_t = E[\tau_t|e_t] = \left[\frac{v+J}{v+e_t'\Omega e_t}\right]$. Together, t-lasso estimation of the coefficients matrix is given by

$$(\hat{B}_t, \hat{\Omega}) = \underset{B_t,\Omega}{\operatorname{argmin}} \frac{1}{2N} tr\left[(\tau^{1/2}(Y - XB))\Omega(\tau^{1/2}(Y - XB))'\right] - \frac{1}{2} log|\Omega| + \lambda \sum_{i,j=1}^{J} \sum_{p=1}^{P} |B_{p,ij}| + \gamma \sum_{i \neq j}^{J} |\omega_{ij}|$$

where τ is NxN diagonal matrix having $\hat{\tau}_t$ on its diagonal.

3.4 Estimation and Forecast procedure

3.4.1 Estimation

In case we know the degree of freedom v of the multivariate t-distribution of innovations, the expectation maximization algorithm presented in Finegold and Drton (2011) is employed to estimate the t-lasso VAR(p) coefficients matrix. However, we consider a simple case that Ω is constant over time and $\hat{\Omega} = I_J$

Expectation Maximization algorithm. t-Lasso with known v

- In this seminar, we consider only P = 5 or 10 (1 or 2 weeks)
- Y, X, degrees of freedom v.
- $\hat{\Omega} = I_J, \hat{B}_p^{(0)} = 0, e_t^{(0)}$ the tth row of $Y - X\hat{B}^{(0)}$ for t = 1, ...,N.
- Repeat the following steps for m = 0, 1, 2,...:
  - Recompute the weights: $\hat{\tau}_t = \frac{v+J}{v+(e_t^{(m)})'\hat{\Omega}^{(m)}e_t^{(m)}}$.
  - Compute $\hat{B}^{(m+1)}$ by minimizing the Gaussian lasso loss with inputs $Y = (\hat{\tau})^{1/2}Y$ and $X = (\hat{\tau})^{1/2}X$, with $\hat{\tau}$ is the diagonal matrix having $\hat{\tau}_t$ on its diagonal. Set $e_t^{(m+1)}$ be the tth row of $Y - X\hat{B}^{(m+1)}$.
- Iterate until the relative change between the losses of two successive iterations becomes smaller than the lower bound.
- $\hat{B} = \hat{B}^{(m+1)}$.

Among all models with different λ and γ, the optimal one is defined by minimizing the Bayesian Information Criterion (BIC):

- $BIC_\lambda = -2logL_\lambda + df_\lambda log(N)$
- $BIC_\gamma = -2logL_\gamma + df_\gamma log(N)$

where $logL_\lambda$ or $logL_\gamma$ is the estimated likelihood $(\frac{1}{2N}tr\left[(Y - XB)\Omega(Y - XB)'\right])$, df_λ is the number of non-zero elements of $\hat{B}$, and df_γ is the number of non-zero lower diagonal elements of $\hat{\Omega}$

3.4.2 Forecast Performance Measurement

The performance of forecasting model is measured by the mean absolute forecast error (MAE). The smaller MAE is, the higher accuracy level that model has. MAE is given by

$$MAE = \frac{1}{N - h - W + 1} \sum_{t=W}^{N-h} MAE_t$$

with $MAE_t = \frac{1}{J}\sum_{j=1}^{J}\left|\hat{y}_{t+h,j}^{(t)} - y_{t+h,j}\right|$, where N is the sample size, J the number of time series, W is the window width and h is the forecast horizon.

3.5 Volatility Spillovers

Volatility spillover is built from pieces of variance decompositions of forecast error. We employed the method from Diebold and Yilmaz (2015) to measure spillovers. According to Barbaglia (2019), volatility spillover is the proportion of volatility forecast error variance of one commodity that can be caused by a shock in the volatility of another commodity. First, the h-step ahead forecast error is given by

$$\hat{y}_{t+h} - y_{t+h} = e_{t+h}$$

and

$$e_{t+h} = \sum_{p=0}^{h-1} \theta_p e_{t+h-p}$$

Then, according to Barbaglia et al. (2019), the proportion of the variance of the h-step-ahead forecast error contributed by the error in component k is as follow

$$w_{h,jk} = \frac{\sigma_{kk}^{-1} \sum_{p=0}^{h-1} (\delta_j' \theta_p \Sigma \delta_k')^2}{\sum_{p=0}^{h-1} \delta_j' \theta_p \Sigma \theta_p' \delta_j}$$

where σ_{kk} is the kkth entry of Σ, δ_j is the selection vector with length J having unity entry as its jth element and zeros elsewhere, and similarly for δ_k.

Finally, the volatility spillover from the commodity k to commodity j at horizon h is given by

$$s_{h,k \to j} = 100 * \frac{w_{h,jk}}{\sum_{k=1}^{J} w_{h,jk}}$$

4 Results

We analyze the data using rolling window with window width W = 220 days. For every $t > W$, we estimate t-lasso VAR(P) model for log volatilities. We run gird search to seek for optimal model, and use BIC as selection criteria.

4.1 Volatility Spillover

We concentrate on the volatility spillovers for three ending times of the 220-day rolling window. In order to make the graphical illustration clearer, we visualize only the 15% highest volatility spillovers on heatmaps. Figure 2, Figure 3 and Figure 4 show the spillovers for the time window ending on June 17, 2014, February 5, 2015 and October 28, 2016 respectively. Overall, the volatility spillovers among commodities within each industry are obvious, and there are notable spillover among commodities from different industries.

- Volatility spillovers among energy and biofuel commodities

Volatility spillovers among energy and biofuel commodities are not detected in all three heatmaps. This finding is different from the result in Barbaglia et al. (2019) where the link energy-biofuel is determined for all three time periods.

- Volatility spillovers among energy and agricultural commodities

In the three heatmaps, energy commodities have spillovers from and towards agricultural commodities. However, the detailed links are quite different. In 2014, spillovers between natural gas, sugar and corn are determined, while the spillovers of natural gas to soybean are detected in 2015. Moreover, the spillover from sugar to crude oil can be seen from heatmap in 2016.

- Volatility spillovers among agricultural and biofuel commodities

The spillover is only detected in 2016. According to Barbaglia et al (2019), energy prices dramatically declined during 2015, which make biofuel less competitive in the market, resulting in weaker volatility spillovers among agricultural and biofuel commodities.

- Volatility spillovers among commodities within industries

Crude oil has strong volatility spillover from and towards gasoline in all heatmaps. For agricultural commodities, volatility spillovers of corn, wheat, sugar and soybean are significant.

The findings support answers for the first two research questions. First, there are evidences for the volatility spillovers among the three sections. Second, although the general spillovers persist over the three periods, the specific links change over time according to the set of commodities in each section.

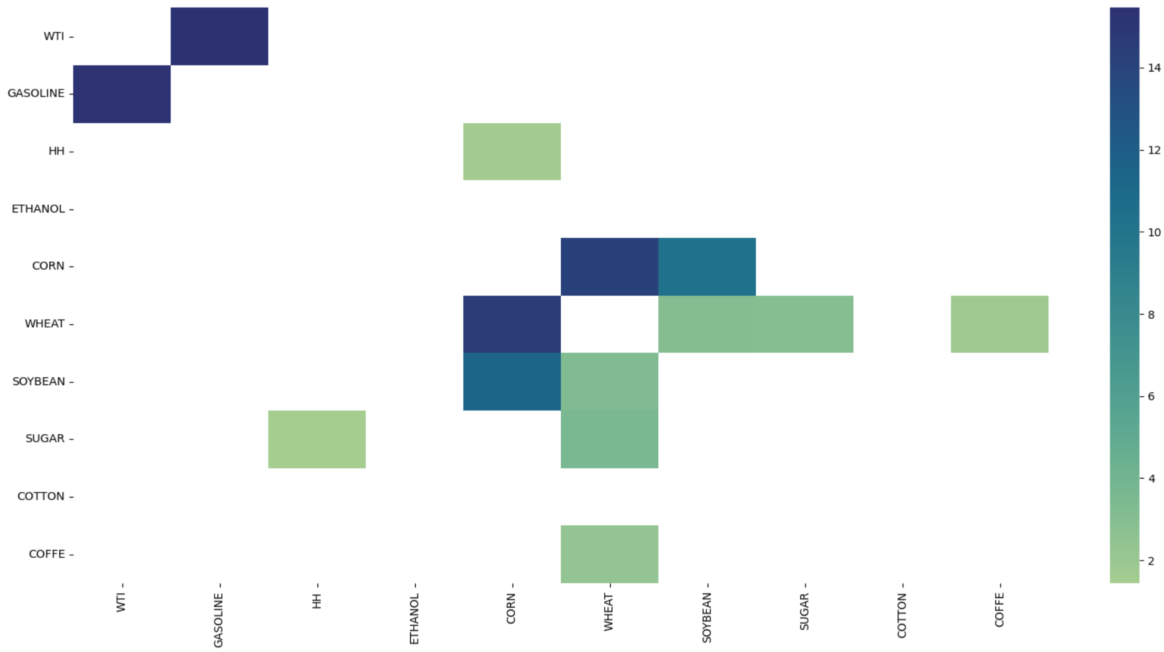

Figure 2: Heatmap of volatility spillovers: June 17th, 2014

4.2 Forecast Accuracy

We employ Gaussian lasso and t-lasso model to forecast the vector of log volatilities for forecast horizon $h = 1, 5, 20, 60$, that is one day, one week, four weeks and three months respectively, and compare the performance according to the mean absolute error (MAE). We re-estimate model every Tuesday and use the estimated parameters for predictions in

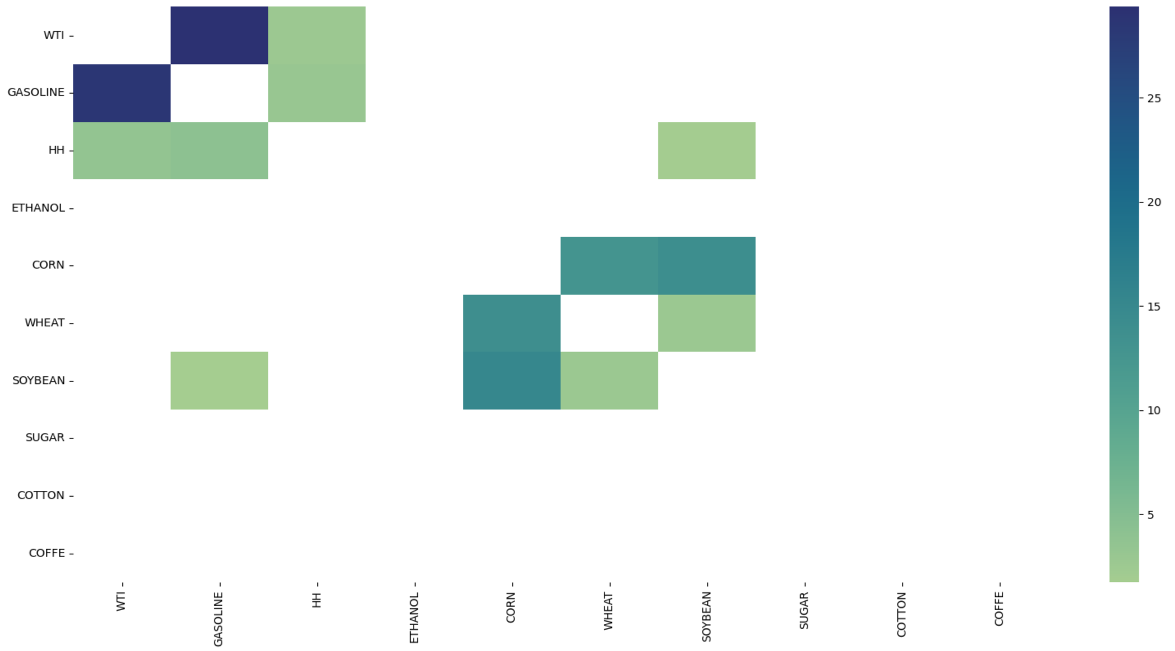

Figure 3: Heatmap of volatility spillovers: February 5th, 2015

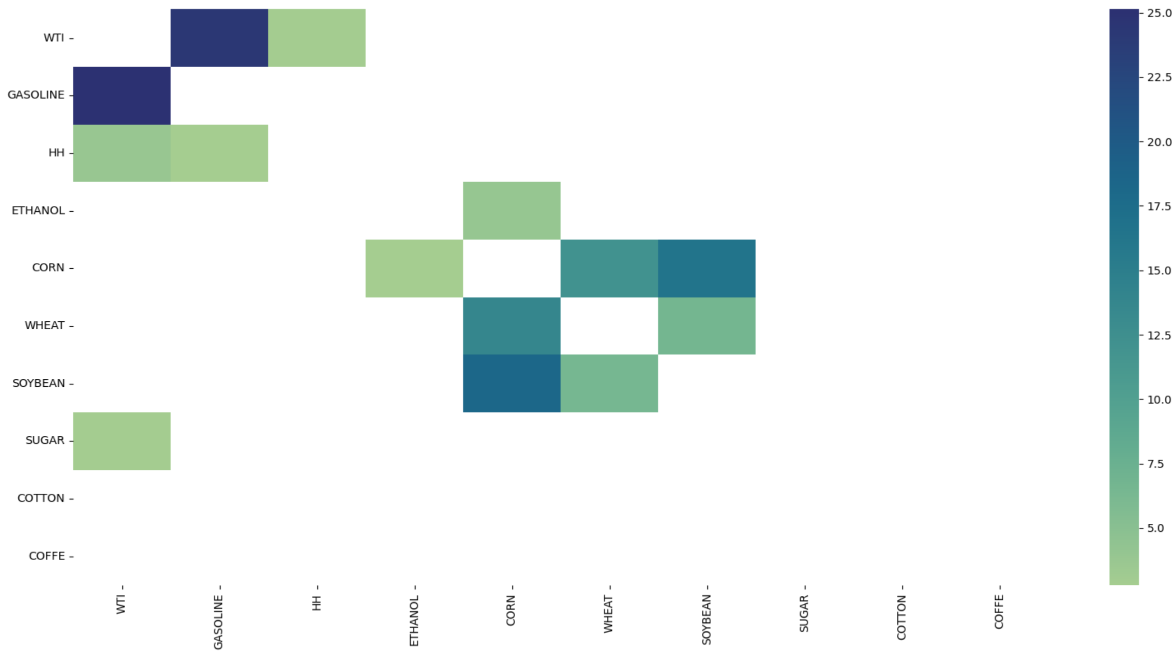

Figure 4: Heatmap of volatility spillovers: October 28th, 2016

the next 5 working days.

The MAE of the t-lasso and Gaussian lasso is presented in Table 1. The MAE of t-lasso is smaller in case of $h = 1, 5$ (short-term), while the performance of Gaussian lasso is better in terms of MAE for mid-term and long-term forecast ($h = 20, 60$). The result is partly different from the findings in Barbaglia (2019) for mid-term and long-term prediction as a result of constant Ω assumption.

Table 1: Mean absolute error of t-lasso and Gaussian lasso for h-step ahead=1, 5, 20, 60

h-step ahead	t-lasso	Gaussian lasso
1	0.864	0.88
5	0.957	0.975
20	1.309	1.231
60	2.632	2.216

5 Conclusion

To summarize, this study focuses on the application of t-lasso and Gaussian lasso VAR model on computing volatility spillover and forecasting daily log volatilites of energy, agricultural and biofuel commodities . I highlight the three main findings. First, we find the volatility spillovers among energy, agricultural and biofuel commodities. Second, the spillovers among commodities change over time. Third, the improvement of considering fat tails of of innovation distribution is not obvious; however, it is probably due to our assumption of constant Ω.

There are several possible improvements. We can only allow higher number of lags to capture the information in the past. In this seminar, we allows no more than lag 10 due to the computational expense. Besides, we can consider the dynamic Ω while forecasting log of volatility to improve the prediction accuracy.

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