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The University of Calgary
Department of Geomatics Engineering
ENGO 634: Principles of Photogrammetry

Absolute Orientation

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ENGO 634: Principles of Photogrammetry
Lab–4: Absolute orientation

Objectives

To perform absolute orientation and check point analysis as part of the overarching aim of labs 2-5 of photogrammetric restitution

Absolute orientation:

Absolutely orient your model using the four control points:

Table 1: Lab model and known object space coordinates

ID	Xm (mm)	Ym (mm)	Zm (mm)	Xg (m)	Yg (m)	Zg (m)
100	-9.5975	96.3215	-153.4711	-399.28	-679.72	1090.96
102	-2.3859	-5.9182	-151.6286	109.7	-642.35	1086.43
104	18.3766	-79.3583	-148.7698	475.55	-538.18	1090.5
105	87.3915	-88.0704	-148.2892	517.62	-194.43	1090.65
200	18.1762	109.7568	-153.585	-466.39	-542.31	1091.55
201	43.8954	7.3577	-150.9432	42.73	-412.19	1090.82
202	-7.5452	-48.379	-151.1187	321.09	-667.45	1083.49
203	50.9073	-90.0254	-148.1656	527.78	-375.72	1092.0

For initial estimates used these points:

$$i = 102, j = 104$$

Table 2: Initial estimated absolution orientation parameters

Parameter	Value
omega (dd)	0.0
phi (dd)	0.0
kappa (dd)	89.8928
lambda	4.981
tX (m)	2785.965
tY (m)	1731.3962
tZ (m)	-4345.4016

Table 3: Estimated absolution orientation parameters

Parameter	Value
omega (dd)	-1.0363
phi (dd)	-1.8319
kappa (dd)	89.8198
lambda	-4.9776

tX (m)	93.9113
tY (m)	-606.6588
tZ (m)	333.3713

Table 4: Residuals for control points

ID	x (m)	y (m)	z (m)
102	0.1279	-0.2696	1.3307
104	-0.2971	0.2484	-7.0479
105	-0.0461	0.0372	2.1773
200	0.0337	0.1974	-1.194
202	0.1817	-0.2134	4.7339
MAE	0.1373	0.1932	3.2967
RMSE	0.1678	0.2099	4.0005

$$Redundancy\ numbers = 1 - diag(A(A^T A)^{-1} A^T)$$

Table 5: Redundancy numbers for control points

ID	x (m)	y (m)	z (m)
102	0.7714	0.7714	0.6898
104	0.6971	0.6969	0.6381
105	0.5326	0.5325	0.0624
200	0.2491	0.2491	0.0714
202	0.7497	0.7497	0.5386

$$cov = (A^T A)^{-1}$$
$$std = sqrt(diag(cov))$$
$$correlation\ matrix = \frac{cov}{np.outer(std, std)}$$

Table 6: Correlation coefficient matrix of the estimated parameters

	omega	phi	kappa	tX	tY	tZ
omega	1.0	-0.3457	0.0366	-0.0	-0.9053	0.3362
phi	-0.3457	1.0	-0.016	0.0	0.3186	-0.9763
kappa	0.0366	-0.016	1.0	-0.0	-0.1242	0.0661
tX	-0.0	0.0	-0.0	1.0	0.0991	0.0464
tY	-0.9053	0.3186	-0.1242	0.0991	1.0	-0.3098
tZ	0.3362	-0.9763	0.0661	0.0464	-0.3098	1.0

Table 7: Transformed control points

ID	x (m)	y (m)	z (m)
102	109.8279	-642.6196	1087.7607
104	475.2529	-537.9316	1083.4521
105	517.5739	-194.3928	1092.8273
200	-466.3563	-542.1126	1090.356
202	321.2717	-667.6634	1088.2239

Comments on Solution Quality:

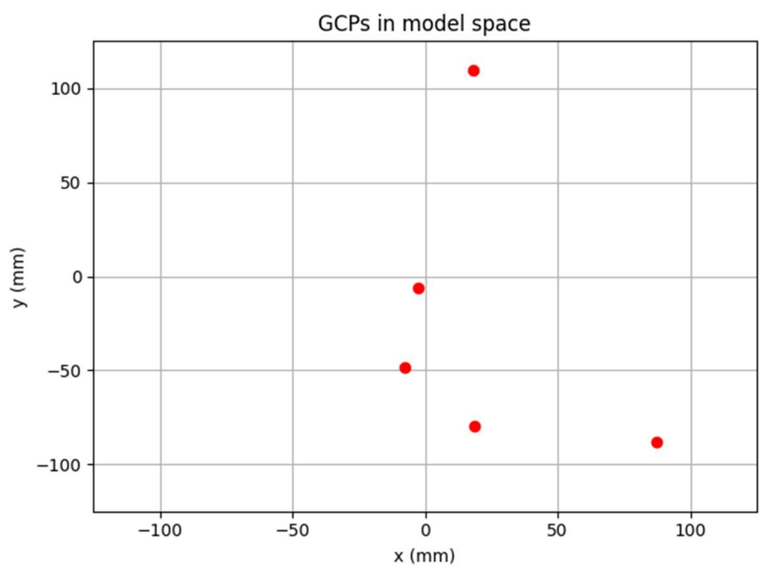
Model fit:
The residuals for the control points are less than 1m for x and y but around 4m for the z coordinate. This is an okay fit for the x and y but not as good in the vertical direction.

Redundancy:
Since we used 5 control points, we had many more observations than unknowns, so the redundancy numbers are generally closer to 1 than 0 meaning that the observations are controlled, and gross error detection is possible.

Correlations:
Omega and tY are highly correlated because a rotation on the x-axis will change the value required from tY. Phi and tZ are highly correlated because a rotation on the y-axis will change the amount of translation needed in z direction. The phi and tZ parameters having such a high correlation suggests that the control points are not distributed well for those parameters as there was not enough information to separate their effects.

Network Geometry Analysis:

Figure 1: Plot of control points in model space



We made sure to pick GCPs that are spread out and form a triangle instead of a straight line. This distribution improves the geometric strength of the network and leads to a more stable and well-conditioned solution.
Reducing the number of GCPs to the three most widely spaced points can lower the error for the control points themselves. However, this reduction significantly decreases the redundancy number for the Z-coordinates, making it nearly zero. As a result, gross errors in elevation cannot be detected, compromising the reliability of the solution. Additionally, with only three GCPs, the check point errors increase compared to a setup with five GCPs, indicating a weaker overall model fit.

If you have bad network geometry small errors in observations can propagate because of high correlations between parameters. The solution could be improved by using more spatially separated control points with the goal of making the highly correlated parameters less correlated. More spatially separated means having less points in a line with each other.

Table 8: Transformed perspective centers

Image	Xm (mm)	Ym (mm)	Zm (mm)	Xo (m)	Yo (m)	Zo (m)
27	0	0	0	93.9113	-606.6588	333.3713
28	92	-1.4649	-1.2609	99.649	-149.1387	354.4145

Table 9: Initial validation angles

Image	omega (dd)	phi (dd)	kappa (dd)
27	0	0	0
28	-0.9724	0.251	-1.7526

Table 10: Rotation matrices for model to image

Image 27	1	0	0
	0	1	0
	0	0	1
Image 28	0.9995	-0.0307	-0.0039
	0.0306	0.9994	-0.0171
	0.0044	0.0170	0.9998

Table 11: Rotation matrices for model to object

M	0.0031	0.9998	-0.0180
	-0.9995	0.0026	-0.0320
	-0.0320	0.0181	0.9993
M^T	0.0031	-0.9995	-0.0320
	0.9998	0.0026	0.0181
	-0.0180	-0.0320	0.9993

Table 12: Rotation matrices for object to image

Image 27	0.0031	-0.9995	-0.0320
	0.9998	0.0026	0.0181
	-0.0180	-0.0320	0.9993
Image 28	-0.0274	-0.9990	-0.0364
	0.9996	-0.0275	0.0
	-0.0010	-0.0363	0.9993

Table 13: Extracted angles from parameters for object to image space rotation

Image	omega (dd)	phi (dd)	kappa (dd)
27	1.8352	-1.0305	-89.8198
28	2.0831	-0.0573	-91.5722

Transforming Tie Points:

Table 14: Initial tie points in model space

ID	x (mm)	y (mm)	z (mm)
T1	-9.9688	14.8164	-156.2243
T2	92.0658	-4.0001	-154.4931
T3	-10.5394	-102.5484	-155.0824
T4	87.0929	-88.3482	-153.1153
T5	-9.4881	96.246	-158.3431
T6	84.9816	102.8444	-157.5453

Table 15: Transformed tie points

ID	x (m)	y (m)	z (m)
T1	6.3439	-681.3422	1107.5485
T2	98.5473	-173.2011	1116.8663
T3	590.552	-682.4992	1112.3379
T4	518.5291	-196.644	1116.8109
T5	-399.1092	-680.3289	1110.8376
T6	-433.355	-210.2969	1121.3077

Transforming Check Points:

Table 16: Transformed check points

ID	x (m)	y (m)	z (m)
100	-399.0471	-680.0976	1086.5789
201	43.0939	-412.4294	1090.5212
203	527.8855	-375.8579	1086.5829

Table 17: Residuals for check points

ID	x (m)	y (m)	z (m)
100	0.2329	-0.3776	-4.3811
201	0.3639	-0.2394	-0.2988
203	0.1055	-0.1379	-5.4171
MAE	0.2341	0.2516	3.3657
RMSE	0.2568	0.2701	4.0261

Calculating Theoretical Accuracy:

Distance between perspective centers in object space:

$$B = \sqrt{(99.65 - 93.91)^2 + (-149.1387 + 606.659)^2 + (354.4 - 333.4)^2} = 458.04m$$

(H – h) = 751.4637599031875m (previous lab)
avg_h = 1089.5500000000002m (previous lab)
f = 153.358mm
S = 751463.76mm / 153.358mm = 4900.06
Pixel Spacing = 0.0119mm/px

$$\sigma = 0.0000119 * 0.5 = 0.00000595\text{m/px}$$
$$\sigma_x = \sigma_y = 4900.06 * 5.95x10^{-6} = 0.0292m/px$$
$$\sigma_z = \frac{\sqrt{2} * 4900.06}{\frac{458.04}{751.464}} * 5.95x10^{-6} = 0.0676m/px$$

Check Points Discussion:
The residuals for the check points are similar to the residuals for the control points, with x and y being less than 1m and z around 4m.
Looking at the individual residuals for the check points it looks like the x values are biased in the positive direction while both the y and z are biased in the negative direction since all the residuals go in the same direction.
The calculated RMSEs are much higher than the theoretical accuracy. The x and y RMSEs are 10 times larger than the theoretical and z is 60 times larger than the theoretical.

Software Validation

Table 18: Validation model and known object space coordinates

ID	Xm (mm)	Ym (mm)	Zm (mm)	Xg (m)	Yg (m)	Zg (m)
30	108.9302	92.5787	-155.7696	7350.27	4382.54	276.42
40	19.5304	96.0258	-156.4878	6717.22	4626.41	280.05
72	71.8751	4.9657	-154.1035	6869.09	3844.56	283.11
127	-0.9473	-7.4078	-154.806	6316.06	3934.63	283.03
112	9.638	-96.5329	-158.0535	6172.84	3269.45	248.1
50	100.4898	-63.9177	-154.9389	6905.26	3279.84	266.47

Table 19: Initial estimated validation absolution orientation parameters

Parameter	Value
omega (dd)	0.0
phi (dd)	0.0
kappa (dd)	18.8601
lambda	7.5826
tX (m)	-52189.0757
tY (m)	-12349.4368
tZ (m)	-1847.087

Table 20: Estimated validation absolution orientation parameters

Parameter	Value
omega (dd)	-0.8241
phi (dd)	-0.7177
kappa (dd)	18.8911
lambda	7.5856

tX (m)	6349.5511
tY (m)	3964.6453
tZ (m)	1458.1142

Table 21: Validation residuals

ID	Xo (m)	Yo (m)	Zo (m)
30	-0.0143	-0.2046	0.0476
40	-0.109	0.3069	-0.1584
72	0.0624	-0.1451	-0.0435
127	0.0439	-0.073	0.2783
112	0.0673	-0.0017	-0.1507
50	-0.0503	0.1175	0.0266
MAE	0.0579	0.1415	0.1175
RMSE	0.0645	0.1714	0.1473

Table 22: Validation transformed points

ID	Xo (m)	Yo (m)	Zo (m)
30	7350.2557	4382.3354	276.4676
40	6717.111	4626.7169	279.8916
72	6869.1524	3844.4149	283.0665
127	6316.1039	3934.557	283.3083
112	6172.9073	3269.4483	247.9493
50	6905.2097	3279.9575	266.4966

Table 23: Transformed validation perspective centers

Image	Xm (mm)	Ym (mm)	Zm (mm)	Xo (m)	Yo (m)	Zo (m)
27	0	0	0	6349.5511	3964.6453	1458.1142
28	92	5.0455	2.1725	7022.3021	3774.6249	1466.3994

Table 24: Initial validation angles

Image	omega (dd)	phi (dd)	kappa (dd)
27	0	0	0
28	0.4392	1.508	3.1575

Table 25: Validation rotation matrices for model to image

Image 27	1	0	0
	0	1	0
	0	0	1
Image 28	0.9981	0.0553	-0.0259
	-0.0551	0.9984	0.0091
	0.0263	-0.0077	0.9996

Table 26: Validation rotation matrices for model to object

M	0.9461	0.3239	0.0072
	-0.3237	0.9460	-0.0177
	-0.0125	0.0144	0.9998

M^T	0.9461	-0.3237	-0.0125
	0.3239	0.9460	0.0144
	0.0072	-0.0177	0.9998

Table 27: Validation rotation matrices for object to image

Image 27	0.9461	-0.3237	-0.0125
	0.3239	0.9460	0.0144
	0.0072	-0.0177	0.9998
Image 28	0.9620	-0.2704	-0.0376
	0.2714	0.9622	0.0242
	0.0296	-0.0334	0.9990

Table 28: Validation extracted angles from parameters for object to image space rotation

Image	omega (dd)	phi (dd)	kappa (dd)
27	1.0121	0.4122	-18.8999
28	1.9164	1.6965	-15.7533

Very similar results to the results in the lecture notes.

Task Summary:

Name	Summary	Amount of Time
Quinn Ledingham	Code development and results compilation.	10 hours
Okoye Akachukwu	Report writing, analysis	6 hours
Fre Ashal	Report editing, initial draft code	6 hours
Innocent Chrisantus	Report review, analysis	6 hours

Appendix A: Code

Code included in submission file called lab4_calc.py.