Time Series Econometrics: Homework assignment 4

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1 Problem 1

Assume the model

$$\Delta \mathbf{y}_t = \boldsymbol{\zeta}_0 \mathbf{y}_{t-1} + \boldsymbol{\epsilon}_t$$

where

$$\mathbf{y}_t = \begin{pmatrix} i_t^{1W} \\ i_t^{10Y} \end{pmatrix}$$

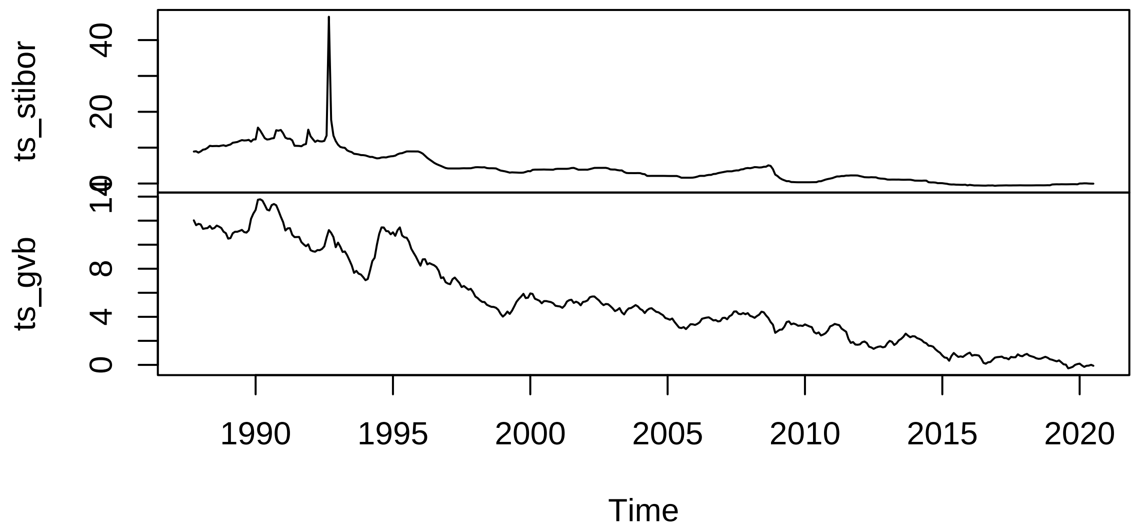
i.e. a 2×1 vector with one-week and ten-year interest rates. The error terms ϵ_t are iid $N(\mathbf{0}, \mathbf{\Omega})$, $t = 1, 2, \ldots, T$.

1.1 Plot the series. Could these potentially be cointegrated?

The first step in the analysis was to plot the complete series from 1987 to 2020. It's important to note that starting in 2020, Riksbank ceased estimating the STIBOR rate, delegating this responsibility to the Swedish Financial Benchmark Facility (SFBF).

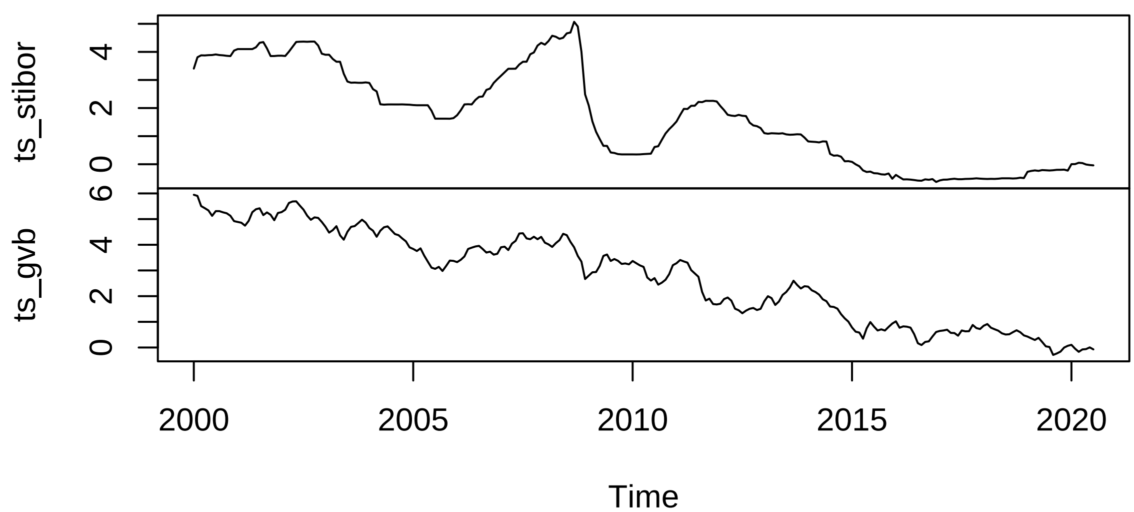
When plotting the complete series, a significant outlier in 1992 is easily noticeable. Further investigation revealed that in the fall of 1992, the Riksbank raised the interest rate to 500 percent in an attempt to defend the krona. However, this defense was unsuccessful, and the krona's exchange rate was allowed to float freely. This dramatic development was rooted in the severe international recession of the early 1990s and the repercussions of German reunification. During the early 1990s, the reunification of Germany and the exchange of Ostmark for D-mark at a 1:1 ratio strained currency arrangements in Europe. For this reason, I decided to filter out the observations from the 1990s, as they represent a crisis period that could bias our analysis.

STIBOR 1 week and GVB 10 year

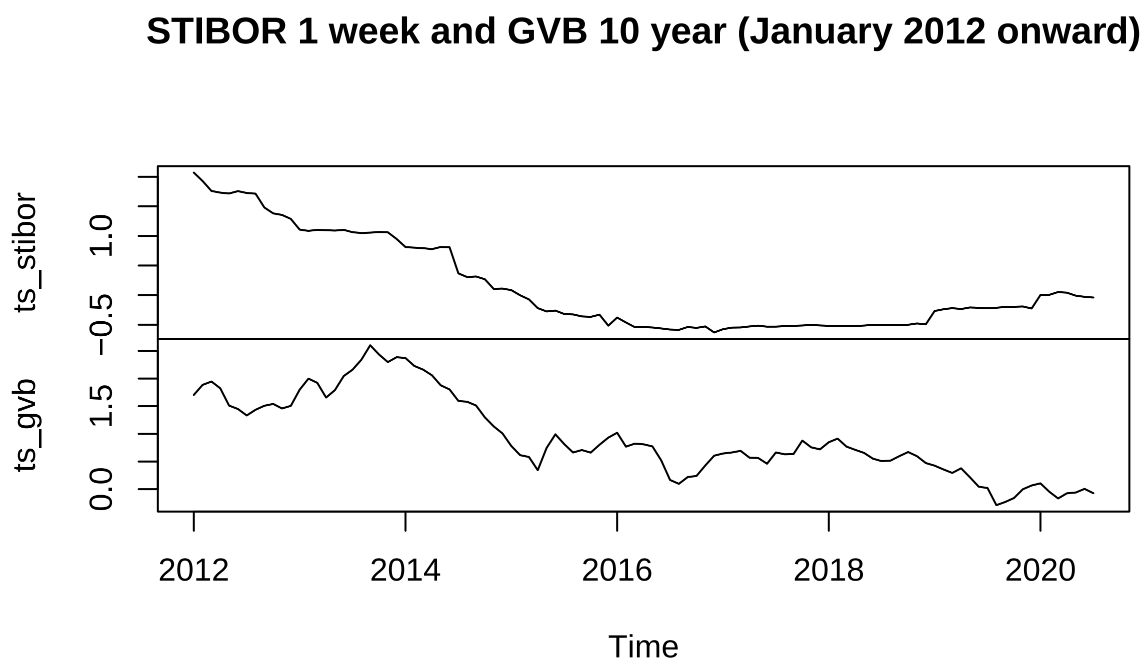


Subsequently, when plotting the series from the 2000s onward, another outlier in 2008 becomes noticeable. These data points are a result of the Global Financial Crisis during those years. Therefore, I made the decision to exclude them from the dataset and work with a final series starting in 2012, when the economy was essentially recovered. With this subset, we can commence our analysis.

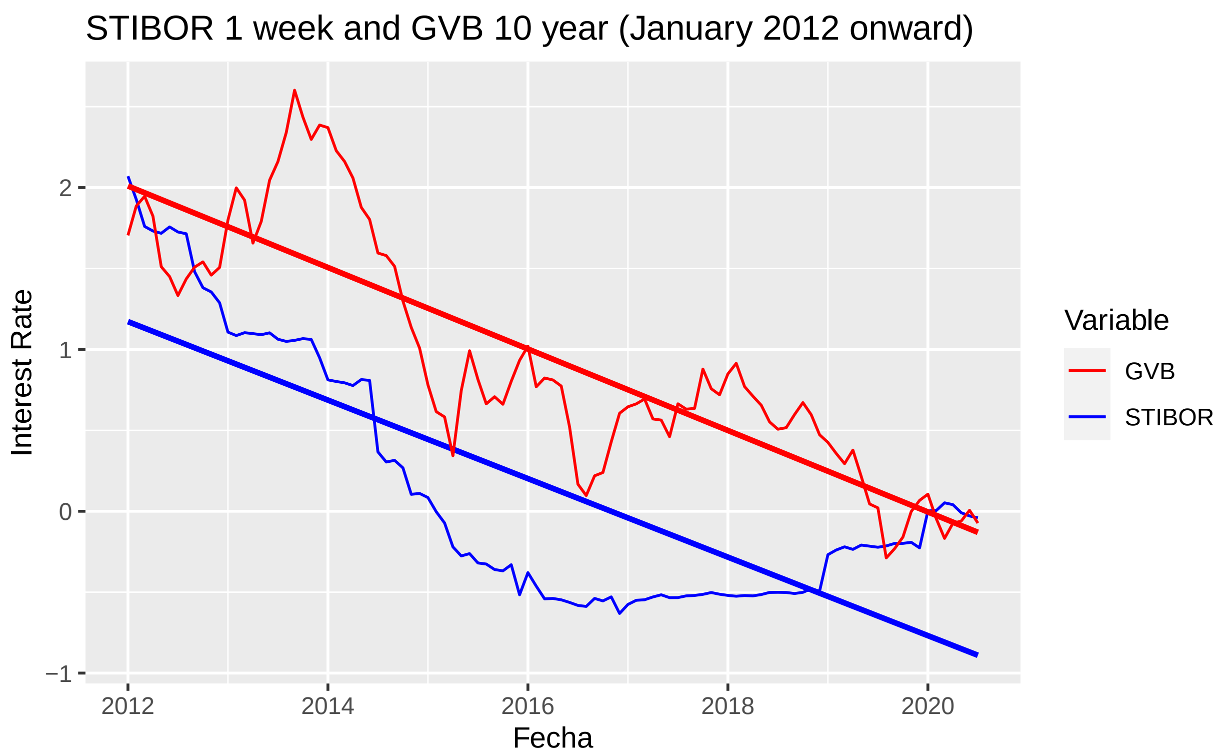
STIBOR 1 week and GVB 10 year (January 1995 onward)



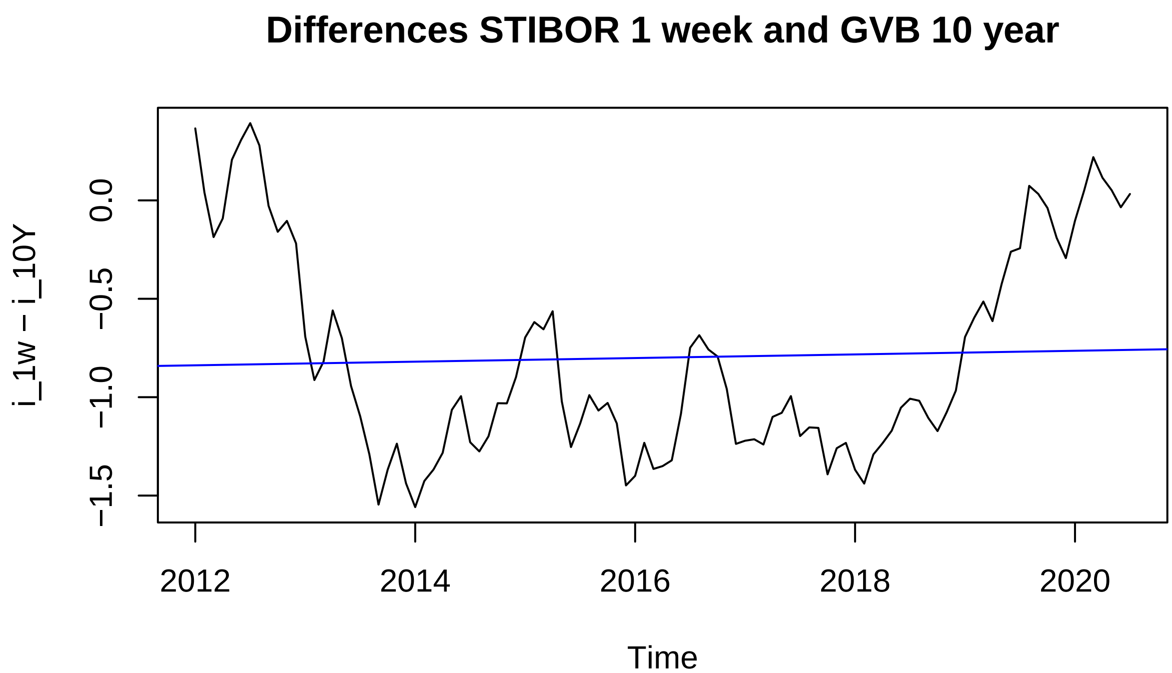
By plotting the series from 2012 onward we can see that both of them seem non-stationary.



We can also observe that both series move together over a long period, suggesting that they share common trends. The series appear to move in a parallel fashion, indicating some form of co-movement over time, which can make us think that they could be cointegrated.



We can further look at the spread between the two interest rates and see if it seems stationary.



Determining cointegration usually requires more than just a visual inspection. Therefore, We can test stationarity of the interest rates and the difference using Augmented Dickey-Fuller tests:

Even though there is evidence that suggest that the interest rates are non-stationary, the spread between the two interest rates also appears to be non stationary, this might make us think that the series are not cointegrated, as cointegration requires that there exists $c \in \mathbb{R}$ where $i_t^{1W} - c \cdot i_t^{10Y} \sim I(0)$

1.2 Estimate the cointegrating rank of the system by a sequence of tests.

Using the AIC, we can determine the lag order of the VAR process which in this case is $K = 2$. We can now estimate the cointegration rank of the system using the results of the Johansen's test with $K = 2$.

```
## AIC(n)
##      2
```

```
##
## ######################
## # Johansen-Procedure #
## ######################
##
## Test type: trace statistic , with linear trend
##
## Eigenvalues (lambda):
## [1] 0.10901182 0.03862126
##
## Values of teststatistic and critical values of test:
##
##           test 10pct  5pct  1pct
## r <= 1 |  3.98  6.50  8.18 11.65
```

```
## r = 0  | 15.64 15.66 17.95 23.52
##
## Eigenvectors, normalised to first column:
## (These are the cointegration relations)
##
##              ts_stibor.l2 ts_gvb.l2
## ts_stibor.l2    1.0000000  1.000000
## ts_gvb.l2       0.9638809 -1.068752
##
## Weights W:
## (This is the loading matrix)
##
##             ts_stibor.l2    ts_gvb.l2
## ts_stibor.d  -0.01836296 -0.009792696
## ts_gvb.d     -0.01370722  0.046937571
```

In this case, there are two eigenvalues: 0.10901182 and 0.0386212. The first hypothesis, $r = 0$, tests for the presence of cointegration, since it is not statistically significant at any level we cannot reject the null hypothesis. The second hypothesis, $r \leq 1$, tests against the alternative hypothesis of $r > 1$, since it is not statistically significant at any level we cannot reject the null hypothesis.Therefore, we can conclude that the rank of the matrix is either 1 or 0. Therefore, we can conclude that the series are probably not cointegrated because we cannot reject r=0.

1.3 Assume that the cointegrating rank is 1 and, using Johansen's approach, test that $i_t^{1W} - i_t^{10Y}$ is a cointegrating relation, i.e. that the spread between the two interest rates is stationary

```
##
## ######################
## # Johansen-Procedure #
## ######################
##
## Estimation and testing under linear restrictions on beta
##
## The VECM has been estimated subject to:
## beta=H*phi and/or alpha=A*psi
##
##      [,1]
## [1,]    1
## [2,]   -1
##
## Eigenvalues of restricted VAR (lambda):
## [1] 0.0391
##
## The value of the likelihood ratio test statistic:
## 7.63 distributed as chi square with 1 df.
## The p-value of the test statistic is: 0.01
##
## Eigenvectors, normalised to first column
## of the restricted VAR:
##
##      [,1]
## [1,]    1
## [2,]   -1
```

```
##
## Weights W of the restricted VAR:
##
##                  [,1]
## ts_stibor.d -0.0140
## ts_gvb.d     0.0453
```

We can summarize the test results as:

- The test results suggest that there is no cointegrating relation between the spread between the one-week interest rate and the ten-year interest rate.
- The likelihood ratio test with a p-value of 0.01 rejects the null hypothesis that the spread between the one-week interest rate and the ten-year interest rate is a cointegrating relation.

1.4 Test the same null hypothesis using a simple unit root test.

The ADF test is used to test for unit roots in the data, which would indicate non-stationarity. By observing the p-value of the test we can conclude that there is no evidence to reject the null, meaning $i_t^{1W} - i_t^{10Y}$ may not be stationary.

```
# Perform ADF test with a constant (intercept)
adf_test_const <- adfTest((ts_rates[,1]-ts_rates[,2]), lags = 2, type = "c")

# Perform ADF test with a constant and trend
adf_test_const_trend <-  adfTest((ts_rates[,1]-ts_rates[,2]), lags = 2, type = "ct")

adf_test_const_trend
```

```
##
## Title:
##  Augmented Dickey-Fuller Test
##
## Test Results:
##   PARAMETER:
##     Lag Order: 2
##   STATISTIC:
##     Dickey-Fuller: -1.5022
##   P VALUE:
##     0.783
##
## Description:
##  Sun Oct 29 18:15:22 2023 by user:
```

We can also do an Engle-Granger test which involves regressing one time series on the other and then testing the stationarity of the residuals from the regression:

```
# Regress one series on the other
regression_result <- lm(ts_stibor ~ ts_gvb)

# Obtain the residuals from the regression
residuals <- residuals(regression_result)

# Perform ADF test with a constant
adf_test_residuals <-  adfTest(residuals, lags = 2, type = "c")

# Perform ADF test with a constant and trend
```

```r
adf_test_residuals <-  adfTest(residuals, lags = 2, type = "ct")

adf_test_residuals
```

```
##
## Title:
##  Augmented Dickey-Fuller Test
##
## Test Results:
##   PARAMETER:
##     Lag Order: 2
##   STATISTIC:
##     Dickey-Fuller: -1.2678
##   P VALUE:
##     0.8802
##
## Description:
##  Sun Oct 29 18:15:22 2023 by user:
```

The null hypothesis in the ADF test is that the data series is non-stationary, meaning it has a "unit root" and is not suitable for cointegration analysis. For two time series to be cointegrated, it is expected that the residual series resulting from the regression (as in the Engle-Granger test) should be stationary.

Given the high p-value (0.8802), there is not enough statistical evidence to conclude that the residuals are stationary. This may indicate that the two time series of the interest rates are not cointegrated, and they do not exhibit a significant long-term relationship in their movements.

A Appendix: Code

```r
# This chunk sets echo = TRUE as default, that is, print all code.
# knitr::opts_chunk$set can be used to set other notebook generation options, too.
# include=FALSE inside curly brackets makes this block not be included in the pdf.
knitr::opts_chunk$set(echo = TRUE)
knitr::opts_chunk$set(fig.pos = "!H", out.extra = "")
library(ggplot2)
library(readxl)
library(vars)
library(irr)
library(tidyverse)
library(readr)
library(forecast)
library(kableExtra)
library(urca)
library(fUnitRoots)
library(tseries)
# Read data
df_stibor <- read_excel("ADVANCED RESULT_2023-10-24_16_23.xlsx")
df_gvb <- read_excel("ADVANCED RESULT_2023-10-24_16_22.xlsx")

# Create time series for both variables
ts_stibor <- ts(df_stibor$STIBOR %>% as.numeric(),
                start = c(1987, 10), end = c(2020, 7), frequency = 12)
ts_gvb <- ts(df_gvb$GVB %>% as.numeric(),
             start = c(1987, 10), end = c(2020, 7), frequency = 12)

ts_rates <- ts.union(ts_stibor, ts_gvb)

# Plot the series
plot.ts(ts_rates, plot.type = "multiple",
        main = 'STIBOR 1 week and GVB 10 year')
# Filter data
ts_stibor <- window(ts_stibor, start = c(2000, 1))
ts_gvb <- window(ts_gvb, start = c(2000, 1))

ts_rates <- ts.union(ts_stibor, ts_gvb)

# Plot
plot.ts(ts_rates, plot.type = "multiple",
        main = 'STIBOR 1 week and GVB 10 year (January 1995 onward)')
# Filter data
ts_stibor <- window(ts_stibor, start = c(2012, 1))
ts_gvb <- window(ts_gvb, start = c(2012, 1))

ts_rates <- ts.union(ts_stibor, ts_gvb)

# Plot
plot.ts(ts_rates, plot.type = "multiple",
        main = 'STIBOR 1 week and GVB 10 year (January 2012 onward)')
# Filtrar las series temporales desde enero de 2012
ts_stibor <- window(ts_stibor, start = c(2012, 1))
ts_gvb <- window(ts_gvb, start = c(2012, 1))
```

```r
# Crear un marco de datos con las series de tiempo
data <- data.frame(
  Date = time(ts_stibor),
  STIBOR = ts_stibor,
  GVB = ts_gvb
)

# Crear un gráfico utilizando ggplot2 y agregar líneas de tendencia
ggplot(data, aes(x = Date)) +
  geom_line(aes(y = STIBOR, color = "STIBOR")) +
  geom_smooth(aes(y = STIBOR), method = "lm", color = "blue", se = FALSE) +
  geom_line(aes(y = GVB, color = "GVB")) +
  geom_smooth(aes(y = GVB), method = "lm", color = "red", se = FALSE) +
  labs(x = "Fecha", y = "Interest Rate", color = "Variable") +
  scale_color_manual(values = c("STIBOR" = "blue", "GVB" = "red")) +
  ggtitle("STIBOR 1 week and GVB 10 year (January 2012 onward)")

plot.ts(ts_rates[,1]-ts_rates[,2],
        main = 'Differences STIBOR 1 week and GVB 10 year', ylab = 'i_1w - i_10Y')

trend <- lm((ts_rates[,1]-ts_rates[,2]) ~ time(ts_rates[,1]-ts_rates[,2]))
abline(trend, col = "blue")
adfTest(ts_stibor, lags = 12, type = "ct")
adfTest(ts_stibor, lags = 12, type = "c")
adfTest(ts_gvb, lags = 12, type = "ct")
adfTest(ts_gvb, lags = 12, type = "c")
adfTest((ts_rates[,1]-ts_rates[,2]), lags = 12, type = "ct")
adfTest((ts_rates[,1]-ts_rates[,2]), lags = 12, type = "c")

var <- VAR(ts_rates, type='const', lag.max=12)
var$p


cointegration_test <- ca.jo(ts_rates, type = "trace", ecdet = "none", K = 2)
summary(cointegration_test)
M <- matrix(c(1, -1), nrow = 2)
test_I1 <- blrtest(z = cointegration_test, H=M, r=1)
summary(test_I1)
# Perform ADF test with a constant (intercept)
adf_test_const <- adfTest((ts_rates[,1]-ts_rates[,2]), lags = 2, type = "c")

# Perform ADF test with a constant and trend
adf_test_const_trend <-  adfTest((ts_rates[,1]-ts_rates[,2]), lags = 2, type = "ct")

adf_test_const_trend
# Regress one series on the other
regression_result <- lm(ts_stibor ~ ts_gvb)

# Obtain the residuals from the regression
residuals <- residuals(regression_result)

# Perform ADF test with a constant
adf_test_residuals <-  adfTest(residuals, lags = 2, type = "c")
```

```r
# Perform ADF test with a constant and trend
adf_test_residuals <-  adfTest(residuals, lags = 2, type = "ct")

adf_test_residuals
```