

Kinematics

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The study of motion without consideration for the study of this motion.

Motion is: the change of an object's position over an arbitrary time period.

To study motion, we require a dependable description of position. Such a description must have:

- a constant and mutually-agreed fiducial point (origin)
- a constant and mutually-agreed set of axes

The assembly of the fiducial point and these axes is called a **Frame of Reference** (FoR or simply “frame”).

To locate an object in a frame requires a distance and a direction from the fiducial.

0.1 Vectors

Any variable which requires a magnitude and a direction to make sense is called a vector. Example:

- Force

A quantity that has only magnitude is called a scalar. For example:

- mass
- temperature
- money

We will denote a vector x with an arrow above the letter: \vec{x} .

0.1.1 Stating Vectors

An example: Let us say we have a vector going on a 2D cartesian plane going from the origin to (3, 4). We have two basic coordinate systems to choose from:

1. Polar, in which we have an angle θ and a radius (magnitude) r .
2. Cartesian or rectangular, in which we have an ordered pair (x, y) .

For polar form, **all** θ 's are referenced from the $+x$ axis. (Counterclockwise). As long as θ is referenced from the $+x$ axis, your x, y components will always be correct. In polar form, the magnitude is **always** positive.

0.1.2 Other angle issues

Bearings Some of our kinematics work will involve directions. A compass direction is called a bearing (γ) and is measured clockwise from $+y$. However, components of such vectors require a θ reference counterclockwise from $+x$. Conversion:

$$(\theta = 450 - \gamma) \in [0, 360)$$

Types

- Absolute
Bearing is an angle clockwise from $+y \in [0, 360)$
Very typical for professional navigation.

- Relative
Suppose $\gamma = 128^\circ$

We can also produce a bearing like this:

1. Choose the cardinal direction (N, E, S, W) to which this γ most closest points.

$S42^\circ E$

2. Correct is so many degrees on either side of the chosen cardinal. For N or S, it is E or W, and vice-versa.

Two basic rules

1. If correction is 45° use NE, NW, etc
2. If correction is $< 45^\circ$, use trig and common sense.

1 \mathbb{R}^3 - Spatial Motion

Can use (x, y, z) , or (r, θ, ϕ) , spherical, or (r, θ, h) , cylindrical.

$\theta \in [0, 360)$ or $[-180, 180)$

$\phi \in [-90, 90]$

1.1 \mathbb{R}^3

Above \mathbb{R}^3 only n-tuples are defined. We cannot visualize them and must trust the theory.

1.2 Lines

For $y = mx$, which passes through the origin, $\theta = \text{constant}$.

For $y = mx + b$, $r = \frac{b}{\sin \theta - m \cos \theta}$ (derivation will be left as an exercise to the reader.)

2 Position Vectors(\vec{r})

To analyze motion, it is essential to locate objects in a frame. A position vector begins at the origin and ends at the object. If the object moves the \vec{r} instantaneously moves to follow it.

3 Displacement(\vec{s})

If an object moves, there is a straight vector that points from the start to the finish. This is called **displacement**. It is the net change of position.

In addition or subtraction, vector variable algebra is identical to \mathbb{R} .

- Commutativity. $a + b = b + a$
- Associativity. $(a + b) + c = a + (b + c)$

If we move to point A, then to point B, we can simply move to point B.

Unit vectors are vectors with magnitudes of 1. We can represent vectors as a scalar multiplied by a unit vector: $\vec{a} = a\hat{a}$.

There are three special unit vectors: $\hat{i}, \hat{j}, \hat{k}$, which represent vectors in x, y, z respectively. Therefore, a rectangular coordinate in three dimensions (x, y, z) can be represented as $x\hat{i} + y\hat{j} + z\hat{k}$.

3.1 Distance (d)

The length of the actual path taken is called distance. Since distance is a scalar and is accumulative, the distance is always greater than or equal to zero.

3.2 Vector “Locations”

A vector is defined only by its magnitude & direction, never its position. As such, parallel vectors are not only identical and indistinguishable. The direction of a vector can be mathematically described as the slope.

$$y = mx + b$$

So y is a family of all lines of slope m .

4 Rates

A rate is a comparison of a variable and time, typically standardized as **unit time**.

Table 1: SI Base Units

	mks	cgs
Length	m	cm
Time	s	s
Mass	kg	g
Temperature	K	K
Force	N	dynes
Energy	J	ergs

4.1 Velocity (\vec{v})

The rate of change of \vec{s} is defined as the velocity:

$$\vec{v}_{ANG} = \frac{\Delta \vec{s}}{\Delta t}$$

If $\Delta t \rightarrow 0$ we obtain a limit

$$\vec{v}_{INST} = \lim_{\Delta t \rightarrow 0} \frac{\Delta \vec{s}}{\Delta t}$$

$$\vec{v} = \frac{d\vec{s}}{dt}$$

4.2 Acceleration(\vec{a})

The rate of change of \vec{v} is called *acceleration*.

$$\vec{a}_{AVG} = \frac{\Delta \vec{v}}{\Delta t}$$

If $\Delta t \rightarrow 0$ then another limit,

$$\vec{a} = \lim_{\Delta t \rightarrow 0} \frac{\Delta \vec{v}}{\Delta t}$$

$$\vec{a} = \frac{d\vec{v}}{dt}$$

4.3 Higher rates

1. Surge or jerk, in the UK and US respectively
2. Snap
3. Crackle
4. Pop