1D Kinematics

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The study of motion without consideration for the study of this motion.
Motion is: the change of an object's position over an arbitrary time period.
To study motion, we require a dependable description of position. Such a description must have:

- a constant and mutually-agreed fiducial point (origin)
- a constant and mutually-agreed set of axes

The assembly of the fiducial point and these axes is called a **Frame of Reference** (FoR or simply "frame").
To locate an object in a frame requires a distance and a direction from the fiducial.

1 Vectors

Any variable which requires a magnitude and a direction to make sense is called a vector. Example:

- Force

A quantity that has only magnitude is called a scalar. For example:

- mass
- temperature
- money

We will denote a vector x with an arrow above the letter: $\vec{x}$.

1.1 Stating Vectors

An example: Let us say we have a vector going on a 2D cartesian plane going from the origin to (3, 4). We have two basic coordinate systems to choose from:

1. Polar, in which we have an angle θ and a radius (magnitude) r.
2. Cartesian or rectangular, in which we have an ordered pair (x, y).

For polar form, **all** $\theta's$ are referenced from the $+x$ axis. (Counterclockwise). As long as θ is referenced from the $+x$ axis, your x, y components will always be correct. In polar form, the magnitude is **always** positive.

1.1.1 Other angle issues

Bearings Some of our kinematics work will involve directions. A compass direction is called a bearing (γ) and is measured clockwise from $+y$. However, components of such vectors require a θ reference counter-clockwise from $+x$. Conversion:

$$(\theta = 450 - \gamma) \in [0, 360)$$

Types

- Absolute
  Bearing is an angle clockwise from $+y \in [0, 360)$
  Very typical for professional navigation.

- Relative
  Suppose $\gamma = 128°$

  We can also produce a bearing like this:

  1. Choose the cardinal direction (N, E, S, W) to which this γ most closest points.

  $$S42°E$$

  2. Correct is so many degrees on either side of the chosen cardinal. For N or S, it is E or W, and vice-versa.

  Two basic rules

  1. If correction is $45°$ use NE, NW, etc
  2. If correction is $< 45°$, use trig and common sense.

1.2 $\mathbb{R}^3$ - Spatial Motion

Can use (x, y, z), or (r, θ, ϕ), spherical, or (r, θ, h), cylindrical.
$\theta \in [0, 360)$ or $[-180, 180)$
$\phi \in [-90, 90]$

1.3 $\mathbb{R}^3$

Above $\mathbb{R}^3$ only n-tuples are defined. We cannot visualize them and must trust the theory.

1.4 Lines

For $y = mx$, which passes through the origin, $\theta =$ constant.

For $y = mx + b$, $r = \frac{b}{\sin\theta - m\cos\theta}$ (derivation will be left as an exercise to the reader.)

2 Position Vectors($\vec{r}$)

To analyze motion, it is sessential to locate objects in a frame. A position vector begins at the origin and ends at the object. If the object moves the $\vec{r}$ instantaneously moves to follow it.

3 Displacement($\vec{s}$)

If an object moves, there is a straight vector that points from the start to the finish. This is called **displacement**. It is the net change of position.

In addition or subtraction, vector variable algebra is identical to $\mathbb{R}$.

- Commutivity. $a + b = b + a$
- Associativity. $(a + b) + c = a + (b + c)$

If we move to point A, then to point B, we can simply move to point B.

Unit vectors are vectors with magnitudes of 1. We can represent vectors as a scalar multiplied by a unit vector: $\vec{a} = a\hat{a}$.

There are three special unit vectors: $\hat{\imath}, \hat{\jmath}, \hat{k}$, which represent vectors in x, y, z respectively. Therefore, a rectangular coordinate in three dimensions (x, y, z) can be represented as $x\hat{\imath} + y\hat{\jmath} + z\hat{k}$.

3.1 Distance (d)

The length of the actual path taken is called distance. Since distance is a scalar and is accumulative, the distance is always greater than or equal to zero.

3.2 Vector "Locations"

A vector is defined only by its magnitude & direction, never its position. As such, parallel vectors are not only identical and indistinguishable. The direction of a vector can be mathematically described as the slope.

$$y = mx + b$$

So y is a family of all lines of slope m.

4 Rates

A rate is a comparison of a variable and time, typically standardized as **unit time**.

Table 1: SI Base Units

	mks	cgs
Length	m	cm
Time	s	s
Mass	kg	g
Temperature	K	K
Force	N	dynes
Energy	J	ergs

4.1 Velocity ($\vec{v}$)

The rate of change of $\vec{s}$ is defined as the velocity:

$$\vec{v}_{ANG} = \frac{\Delta\vec{s}}{\Delta t}$$

If $\Delta t \to 0$ we obtain a limit

$$\vec{v}_{INST} = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t}$$

$$\vec{v} = \frac{d\vec{s}}{dt}$$

4.2 Acceleration($\vec{a}$)

The rate of change of $\vec{v}$ is called *acceleration*.

$$\vec{a}_{AVG} = \frac{\Delta\vec{v}}{\Delta t}$$

If $\Delta t \to 0$ then another limit,

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta\vec{v}}{\Delta t}$$

$$\vec{a} = \frac{d\vec{v}}{dt}$$

4.3 Higher rates

1. Surge or jerk, in the UK and US respectively
2. Snap
3. Crackle
4. Pop

5 Mathematical Relationships in Kinematics

The interrelations between kinematics variables are firstly derived from definitions and then subject to mathematical rigour. Recall:

$$a = \frac{\mathrm{d}v}{\mathrm{d}t}$$

Hence,

$$\mathrm{d}v = a\mathrm{d}t$$
$$\int \mathrm{d}v = \int a\mathrm{d}t$$
$$v = at + c$$

The c represents $v(0)$. It is practise in physics to write $v(0) = v_0$ seen as an intitial condition. We tend to write polynomials with their terms oriented in order of increasing exponents. This is because it is typical of a Taylor series and they are used extensively to approximate more involved functions. Therefore, our equation becomes

$$v = v_0 + at \tag{1}$$

Our second equation comes from another definition.

$$v = \frac{\mathrm{d}s}{\mathrm{d}t}$$
$$\mathrm{d}s = v\mathrm{d}t$$
$$\mathrm{d}s = (v_0 + at)\mathrm{d}t$$
$$= v_0\mathrm{d}t + at\mathrm{d}t$$

We arrive at:

$$s = s_0 + v_0 t + \frac{1}{2}at^2 \tag{2}$$

Some constraints:

1. a is assumed constant over an interval.
2. Time intervals must be changed if:
    - $\vec{a}$ changes
    - $\vec{v}$ changes sign
    - $\vec{v}$ changes arbitrarily (magic)
3. Question dictates it.

In (1), $t = \frac{v-v_0}{a}$ and substitute this into (2).

$$s = s_0 + v_0\left(\frac{v-v_0}{a}\right) + \frac{1}{2}a\left(\frac{v-v_0}{a}\right)^2$$

$$\vdots$$

$$v^2 = v_0^2 + 2as \tag{3}$$

5.1 Reminder

(3) is only valid iff:

- v has a constant sign.
- Single time interval.

5.2 Caution

$\vec{v}_{AVG}$ is defined as

$$\vec{v}_{AVG} = \frac{\sum \vec{s}}{\sum t}$$

and

$$v_{AVG} = \frac{\sum d}{\sum t}$$

But:

$$v_{AVG} \neq \frac{\sum v_k}{\sum t}$$

6 Graphing kinematics equations

An object starting from rest accelerates at a constant rate of 2ms^-2 until the object has moved 4m. It then stops for 1s. It then goes at -2ms^{-1} for 2s and finally $\vec{a}$ at 1ms^{-2} for 2s. An exercise to the reader: Create a table.