DEPARTAMENTO DE ELETRÓNICA, TELECOMUNICAÇÕES
E INFORMÁTICA

MESTRADO INTEGRADO EM ENG. DE COMPUTADORES E TELEMÁTICA

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DESEMPENHO E DIMENSIONAMENTO DE REDES

ASSIGNMENT GUIDE NO. 1

IMPACT OF TRANSMISSION ERRORS IN THE PERFORMANCE OF A NETWORK LINK

Assignment Guide No. 1

1. Assignment Description

Implement the following 3 tasks using MATLAB to obtain the requested numerical solutions. At the end, submit a report with the answers to all questions and including not only the numerical solutions but also the MATLAB codes used to obtain them.

Task 1

RECALL FROM THEORETICAL CLASSES:

The probability function of a binomial random variable with parameters n and q is:

$$f(i) = \binom{n}{i} q^i (1-q)^{n-i} \quad , i = 0,1, \ldots n$$

where $\binom{n}{i} = \frac{n!}{i!(n-i)!}$.

The probability function of a geometric random variable with parameter p is:

$$f(i) = p(1-p)^i \ , i = 0,1,2, \ldots \qquad \text{or} \qquad f(i) = p(1-p)^{i-1} \ , i = 1,2,3, \ldots$$

On a given wireless link supporting the exchange of data frames, the bit error rate (*i.e.*, the probability of each bit being received with error due to propagation or interference factors) is q. Assume that errors in different bits are statistically independent (in this case, the number of errors in a data frame is a binomial random variable.

Assume that the link capacity is 54 Mbps (= 54×10^6 bits per second) and that data frames must be transmitted with a time gap of 10 μs (= 10×10^{-6} seconds). Each frame is composed by one header of 36 Bytes, a data field of up to 8000 Bytes and a FCS (Frame Check Sequence) of 4 bytes. Assume that the FCS enables the receiver to detect with probability 100% when a frame has transmission errors. Assume also that the receiver discards all frames received with errors.

1.a. Assuming that the bit error rate is $q = 0$, draw a plot of the link data rate as a function of the data field size from 100 Bytes up to 8000 Bytes. What do you conclude?

1.b. Draw a plot of the frame discard probability as a function of bit error rate from $q = 10^{-8}$ up to $q = 10^{-3}$ when the frame data field size is B = 100 bytes. Add to the figure the same plots for B = 200, 1000 and 8000 bytes. What do you conclude?

1.c. Draw a plot of the link data rate as a function of bit error rate from $q = 10^{-8}$ up to $q = 10^{-3}$ when the frame data field size is B = 100 bytes. Add to the figure the same plots for B = 200, 1000 and 8000 bytes. Assuming that a good link performance is to provide a data rate of at least 40 Mbps, what do you conclude?

1.d. Consider that 30% of frames has a data field size B = 100 bytes and 70% of frames has a data field size B = 1500 bytes. Draw a plot of the link data rate as a function of bit error rate from $q = 10^{-8}$ up to $q = 10^{-3}$. What do you conclude?

1.e. Consider now that the data field size B, in number of Bytes, is the sum of a constant value (100 Bytes) and a geometric random value with parameter p = 0.025. Draw a plot of the link data rate as a function of bit error rate from $q=10^{-8}$ up to $q=10^{-3}$. What do you conclude?

Task 2

RECALL FROM THEORETICAL CLASSES:

<u>Bayes' law</u>: consider a set of mutually exclusive events F_1, F_2, …, F_n such that its union is the set of all possible outcomes of a random experiment. Knowing that event E has occurred, the probability of event F_j, with *j* = 1, 2, …, *n*, is given by:

$$P(F_j|E) = \frac{P(EF_j)}{P(E)} = \frac{P(E|F_j)P(F_j)}{\sum_{i=1}^{n} P(E|F_i)P(F_i)}$$

Consider a wireless link used by two stations for data communications. The link can be either in a normal state with a probability of *p* or in an interference state with a probability of 1 – *p*. Consider the following probability values: *p* = 99%, *p* = 99.9%, *p* = 99.99% and *p* = 99.999%.

The two stations exchange from time to time a set of *n* consecutive control frames to decide if the link is in interference state. The probability of a control frame being received with one or more errors is at most 0.01% in normal state and is 50% in interference state.

Both stations determine with a 100% probability if the control frames have been received with errors. The stations decide that the link is in interference state when the *n* consecutive control frames are received with errors.

A false positive is when a station decides wrongly that the link is in interference state and a false negative is when a station decides wrongly that the link is in normal state.

2.a. For each value of *p*, determine the probability of the link being in the interference state and in the normal state when one control frame is received with errors (fulfil the following table). What do you conclude?

	p(normal)	*p*(interference)
p = 99%		
p = 99.9%		
p = 99.99%		
p = 99.999%		

2.b. For each value of *p* and for *n* = 2, 3, 4 and 5, determine the probability of false positives. Fulfil the follow table:

	Probability of false positives			
	n = 2	*n* = 3	*n* = 4	*n* = 5
p = 99%				
p = 99.9%				
p = 99.99%				
p = 99.999%				

2.c. For each value of p and for n = 2, 3, 4 and 5, determine the probability of false negatives. Fulfil the follow table:

	Probability of false negatives			
	$n = 2$	$n = 3$	$n = 4$	$n = 5$
$p = 99\%$				
$p = 99.9\%$				
$p = 99.99\%$				
$p = 99.999\%$				

2.d. Describe and justify the influence of the values of p and n observed in the results obtained in **2.b** and **2.c**.

2.e. Assume that we aim a system where both false positive and false negative probabilities are not higher than 0.1%. From the results obtained in **2.b** and **2.c**, what is the best value of n to be used if the probability of the normal state is $p = 99.99\%$.

2.f. Repeat the previous exercise **2.e** but now considering that the probability of the normal state is $p = 99.999\%$.

Task 3

RECALL FROM THEORETICAL CLASSES:

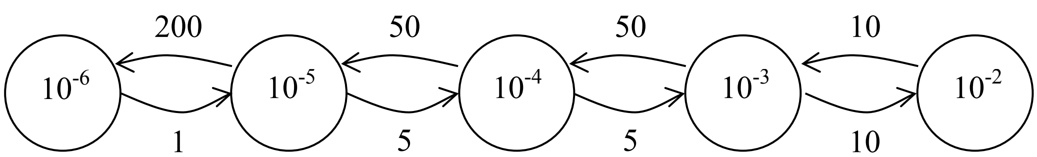
Birth-dead Markov chain: if λ_i is the birth rate of state *i* and μ_i is the dead rate of state *i*, than, the steady-state probability of state 0 is:

$$\pi_0 = \frac{1}{1 + \sum_{i=1}^{\infty} \frac{\lambda_0 \lambda_1 \cdots \lambda_{i-1}}{\mu_1 \mu_2 \cdots \mu_i}}$$

and the steady-state probability of state *n* > 0 is:

$$\pi_n = \frac{\lambda_0 \lambda_1 \cdots \lambda_{n-1}}{\mu_1 \mu_2 \cdots \mu_n} \cdot \pi_0$$

Consider a wireless link between multiple stations for data communications. The bit error rate (*ber*) introduced by the wireless link due to the variation along with time of the propagation and interference factors is approximately given by the following Markov chain:



where the state transition rates are in number of transitions per hour.

3.a. What is the average percentage of time the link is on each of the five possible states?

3.b. What is the average bit error rate of the link?

3.c. What is the average time duration (in minutes) that the link is on each of the five possible states?

3.d. Consider that the link is in interference state when its bit error rate is 10^{-3} or higher. What is the probability of the link being in interference state? And what is the average time duration (in minutes) of the interference state?