# Report 2

## Team information (B23-ISE-02):

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## Link to the product:

- `https://github.com/quintet-sdr/optimization-pt2`

## Programming language:

- Rust
- To launch the code: `$ cargo run`

## Linear programming problem:

- Maximization
- Approximation accuracy $\epsilon = 2$
- $\alpha_1 = 0.5$
- $\alpha_2 = 0.9$

## Tests:

In all tests answers for $\alpha_1$ and $\alpha_2$ are the same

(1) **Input:**

- Name: Lab 6 / Problem 1
- Objective function: $F(x_1, x_2) = x_1 + x_2$
- Constraints: $\begin{cases} 2x_1 + 4x_2 \leq 16 \\ x_1 + 3x_2 \geq 9 \\ x_1, x_2 \geq 0 \end{cases}$
- Initial point: [ 0.5 3.5 1 2 ].

**Output:**

- Decision variables: [ 6 1 0 0 ].
- Maximum values: 7.

(2) **Input:**

- Name: Lab 6 / Problem 2
- Objective function: $F(x_1, x_2, x_3) = 9x_1 + 10x_2 + 16x_3$
- Constraints: $\begin{cases} 18x_1 + 15x_2 + 12x_3 \leq 360 \\ 6x_1 + 4x_2 + 8x_3 \leq 192 \\ 5x_1 + 3x_2 + 3x_3 \leq 180 \\ x_1, x_2, x_3 \geq 0 \end{cases}$
- Initial point: $\begin{bmatrix} 1 & 1 & 1 & 315 & 174 & 169 \end{bmatrix}$.

**Output:**

- Decision variables: [ 0 8 20 0 0 96 ].
- Maximum value: 400.

(3) **Input:**

- Name: Lec 6 / Problem 1
- Objective function: $F(x_1, x_2, x_3) = x_1 + 2x_2$
- Constraints: $\begin{cases} x_1 + x_2 + x_3 = 8 \\ x_1, x_2, x_3 \geq 0 \end{cases}$
- Initial point: [ 2 2 4 ].

**Output:**

- Decision variables: [ 0 8 0 ].
- Maximum value: 16.

(4) **Input:**

- Name: Tut 6 / Problem 1
- Objective function: $F(x_1, x_2) = 2x_1 + 5x_2 + 7x_3$

- Constraints: $\begin{cases} 1x_1 + 2x_2 + 3x_3 = 6 \\ x_1, x_2, x_3 \geq 0 \end{cases}$
- Initial point: [ 1 1 1 ].

**Output:**

- Decision variables: [ 0 3 0 ].
- Maximum value: 15.

(5) **Input:**

- Name: No solution 1
- Objective function: $F(x_1, x_2) = 3x_1 + 2x_2$
- Constraints: $\begin{cases} x_1 - x_2 \leq 2 \\ -2x_1 + x_2 \leq -1 \\ x_1, x_2 \geq 0 \end{cases}$
- Initial point: [ 1 1 ].

**Output:**

- No solution.

(6) **Input:**

- Name: No solution 2
- Objective function: $F(x_1, x_2) = 2x_1 + 1x_2$
- Constraints: $\begin{cases} x_1 - x_2 \leq 10 \\ -2x_1 \leq 40 \\ x_1, x_2 \geq 0 \end{cases}$
- Initial point: [ 0 0 ].

**Output:**

- Method is not applicable (unbounded solution).

**Code:**

crates/pt2-cli/src/main.rs

```rust
use std::{io, panic};

use color_eyre::{eyre::Context, Result};
use crossterm::{
    style::Stylize,
    terminal::{Clear, ClearType, EnterAlternateScreen, LeaveAlternateScreen},
};

mod config;

fn real_main() -> Result<()> {
    const ALPHA_1: f64 = 0.5;
    const ALPHA_2: f64 = 0.9;

    let tests = config::read_tests().wrap_err("tests.json not found")?;

    loop {
        crossterm::execute!(io::stdout(), Clear(ClearType::All))?;

        println!("{}", "[esc to cancel]".cyan());
        let Some(test) = inquire::Select::new("Select a test:", tests.clone())
            .with_vim_mode(true)
            .prompt_skippable()?
        else {
            break;
        };

        for alpha in [ALPHA_1, ALPHA_2] {
            println!("alpha: {alpha:.eps$}", eps = test.eps);

            let iterations = match pt2_core::interior_point(
                test.objective_function.clone(),
                &test.constraints,
                test.initial_point.clone(),
                test.eps,
                alpha,
            ) {
                Ok(it) => it,
                Err(err) => {
                    println!("{err}");
                    continue;
                }
            };

            let last = iterations.last().unwrap();

            let result = match last {
                Ok(it) => it,
                Err(err) => {
                    println!("{err}");
                    continue;
                }
            };

            println!("max: {:.eps$}", result.max, eps = test.eps);
            println!(
                "x:{:.eps$}",
                result.decision_variables.transpose(),
                eps = test.eps
            );
            println!();
        }

        let Some(next) = inquire::Confirm::new("Next test?").prompt_skippable()? else {
            break;
        };
        if !next {
            break;
        };
    }

    Ok(())
}

fn main() -> Result<()> {
    color_eyre::install()?;

    crossterm::execute!(io::stdout(), EnterAlternateScreen)?;
    panic::set_hook(Box::new(|_| {
        crossterm::execute!(io::stdout(), Clear(ClearType::All)).unwrap();
        crossterm::execute!(io::stdout(), LeaveAlternateScreen).unwrap()
    }));

    let result = real_main();
    crossterm::execute!(io::stdout(), LeaveAlternateScreen)?;
    result
}
```

crates/pt2-core/src/lib.rs

```
use na::{DMatrix, DVector};

pub use crate::interfaces::{Constraints, Sign};
use crate::interfaces::{InteriorPoint, NotApplicableError};

mod algorithm;
mod interfaces;

pub fn interior_point(
    objective_function: Vec<f64>,
    constraints: &Constraints,
    initial_point: Vec<f64>,
    eps: usize,
    alpha: f64,
) -> Result<InteriorPoint, NotApplicableError> {
    let (n, m) = get_n_and_m(constraints).ok_or(NotApplicableError)?;

    if constraints
        .iter()
        .any(|row| row.0.len() != objective_function.len())
    {
        return Err(NotApplicableError);
    }

    let initial_point_is_feasible = constraints.iter().all(|(coefficients, sign, rhs)| {
        let constraint_sum: f64 = coefficients
            .iter()
            .zip(&initial_point)
            .map(|(coeff, x)| coeff * x)
            .sum();

        sign.compare(&constraint_sum, rhs)
    });

    if !initial_point_is_feasible {
        return Err(NotApplicableError);
    }

    let no_slack_rows = constraints
        .iter()
        .enumerate()
        .filter_map(|(i, (_, sign, _))| matches!(sign, Sign::Eq).then_some(i));
    let no_slack_cols = no_slack_rows.map(|j| m + j).collect::<Box<[_]>>();
    let slack_cols_count = n - no_slack_cols.len();

    if initial_point.len() != m + slack_cols_count {
        return Err(NotApplicableError);
    }

    let big_a = {
        let left_part_row_elements = constraints
            .iter()
            .flat_map(|(coefficients, _, _)| coefficients)
            .copied();

        let right_part_diagonal_elements = &DVector::from_vec(
            constraints
                .iter()
                .map(|(_, sign, _)| match sign {
                    Sign::Le => 1.,
                    Sign::Eq => 0.,
                    Sign::Ge => -1.,
                })
                .collect(),
        );

        let mut big_a =
            DMatrix::from_row_iterator(n, m, left_part_row_elements).resize_horizontally(m + n, 0.);

        big_a
            .view_mut((0, m), (n, n))
            .set_diagonal(right_part_diagonal_elements);

        big_a.remove_columns_at(&no_slack_cols)
    };

    Ok(InteriorPoint {
        done: false,
        x: DVector::from_vec(initial_point),
        big_a,
        c: DVector::from_vec(objective_function).resize_vertically(m + slack_cols_count, 0.),
        eps: up_to_n_dec_places(i32::try_from(eps).map_err(|_| NotApplicableError)?),
        alpha,
    })
}

fn get_n_and_m(constraints: &Constraints) -> Option<(usize, usize)> {
    Some((constraints.len(), constraints.first()?.0.len()))
}

fn up_to_n_dec_places(n: i32) -> f64 {
    0.1_f64.powi(n) / 2.
}
```

crates/pt2-core/src/algorithm.rs

```
use na::{DMatrix, DVector};

use crate::interfaces::{Auxiliary, InteriorPoint, Iteration, NoSolutionError};

impl Iterator for InteriorPoint {
    type Item = Result<Iteration, NoSolutionError>;

    fn next(&mut self) -> Option<Self::Item> {
        if self.done {
            return None;
        }

        let size = self.x.len();

        let big_d = DMatrix::from_diagonal(&self.x);

        let big_a_tilde = &self.big_a * &big_d;
        let c_tilde = &big_d * &self.c;

        let big_p = {
            let big_i = DMatrix::identity(size, size);
            let big_a_tilde_tr = big_a_tilde.transpose();
            let Some(inverse) = (&big_a_tilde * &big_a_tilde_tr).try_inverse() else {
                self.done = true;
                return Some(Err(NoSolutionError));
            };
            big_i - big_a_tilde_tr * inverse * &big_a_tilde
        };
        let c_p = &big_p * &c_tilde;

        let Some(nu) = c_p
            .iter()
            .filter_map(|it| (it < &0.).then_some(it.abs()))
            .max_by(|a, b| a.partial_cmp(b).unwrap())
        else {
            self.done = true;
            return Some(Err(NoSolutionError));
        };
        let x_tilde = DVector::from_element(size, 1.) + (self.alpha / nu) * &c_p;

        let new_x = &big_d * &x_tilde;
        if (&new_x - &self.x).norm() < self.eps {
            self.done = true;
        }

        self.x = new_x;

        Some(Ok(Iteration {
            auxiliary: Auxiliary {
                big_d,
                big_a_tilde,
                c_tilde,
                big_p,
                c_p,
                nu,
                x_tilde,
            },
            decision_variables: self.x.clone_owned(),
            max: self.x.dot(&self.c),
        }))
    }
}
```

crates/pt2-core/src/interfaces.rs

```
use na::{DMatrix, DVector};
use serde::Deserialize;
use thiserror::Error;

#[derive(Error, Debug)]
#[error("method is not applicable")]
pub struct NotApplicableError;

#[derive(Error, Debug)]
#[error("problem has no solution")]
pub struct NoSolutionError;

pub struct Auxiliary {
    pub big_d: DMatrix<f64>,
    pub big_a_tilde: DMatrix<f64>,
    pub c_tilde: DVector<f64>,
    pub big_p: DMatrix<f64>,
    pub c_p: DVector<f64>,
    pub nu: f64,
    pub x_tilde: DVector<f64>,
}

pub struct Iteration {
    pub auxiliary: Auxiliary,
    pub decision_variables: DVector<f64>,
    pub max: f64,
}

pub struct InteriorPoint {
    pub(crate) done: bool,
    pub(crate) x: DVector<f64>,
    pub(crate) big_a: DMatrix<f64>,
    pub(crate) c: DVector<f64>,
    pub(crate) eps: f64,
    pub(crate) alpha: f64,
}

pub type Constraints = Box<[(Box<[f64]>, Sign, f64)]>;

#[derive(Clone, Deserialize)]
pub enum Sign {
    #[serde(rename = "<=")]
    Le,
    #[serde(rename = "==", alias = "=")]
    Eq,
    #[serde(rename = ">=")]
    Ge,
}

impl Sign {
    pub fn compare<Lhs, Rhs>(&self, a: &Lhs, b: &Rhs) -> bool
    where
        Lhs: PartialOrd<Rhs>,
    {
        let cmp_function = match self {
            Self::Le => PartialOrd::le,
            Self::Eq => PartialEq::eq,
            Self::Ge => PartialOrd::ge,
        };
        cmp_function(a, b)
    }
}
```