# Hipercuádricas.

Def $\Rightarrow H = \{ p \in A \ / \ (1, p_R^t) \cdot \hat{C} (1, p_R)^t = 0 \}$ con $\hat{C} = \left( \begin{array}{c|c} a & z^t \\ \hline z & C \end{array} \right)$

$C \equiv$ núcleo cuadrático asociado a $\hat{C}$, $C \in S_n(\mathbb{R}) \setminus \{0\}$

Números que definen hipercuádricas:

$R_H := \text{rang}(\hat{C})$, $r_H := \text{rang}(C)$, $S_H := |\hat{t} - \hat{s}|$ Número de 1 menos número de -1 en la forma de Sylvester de la $\hat{C}$, $s_H := |t - s|$ lo mismo que antes pero de $C$

Clasificación de cónicas (hipercuádricas de $\mathbb{R}^2$):

- Recta doble (Tipo I, $t = 1$, $s = 0$): $R_H = r_H = 1$, $S_H = s_H = 1$ $\left( \begin{array}{c|cc} 0 & 0 & 0 \\ \hline 0 & 1 & 0 \\ 0 & 0 & 0 \end{array} \right)$ $x^2 = 0$
- Punto (Tipo I, $t = 2$, $s = 0$): $R_H = r_H = 2$, $S_H = s_H = 2$ $\left( \begin{array}{c|cc} 0 & 0 & 0 \\ \hline 0 & 1 & 0 \\ 0 & 0 & 1 \end{array} \right)$ $x^2 + y^2 = 0$
- Rectas secantes (Tipo I, $t = 1$, $s = 1$): $R_H = r_H = 2$, $S_H = s_H = 0$ $\left( \begin{array}{c|cc} 0 & 0 & 0 \\ \hline 0 & 1 & 0 \\ 0 & 0 & -1 \end{array} \right)$ $x^2 - y^2 = 0$
- Vacío (Tipo II, $t = 1$, $s = 0$): $R_H = r_H + 1 = 2$, $S_H = s_H + 1 = 1$ $\left( \begin{array}{c|cc} 1 & 0 & 0 \\ \hline 0 & 1 & 0 \\ 0 & 0 & 0 \end{array} \right)$ $x^2 + 1 = 0$

•) Rectas paralelas (Tipo II, $t=0, s=1$): $R_H = r_H+1 = 2$, $S_H = s_H - 1 = 0$ $\left(\begin{array}{c|cc} 1 & 0 & 0 \\ \hline 0 & 0 & 0 \\ 0 & 0 & -1 \end{array}\right)$ $1 - y^2 = 0$

•) Vacío (Tipo II, $t=2, s=0$): $R_H = r_H + 1 = 3$, $S_H = s_H + 1 = 3$ $\left(\begin{array}{c|cc} 1 & 0 & 0 \\ \hline 0 & 1 & 0 \\ 0 & 0 & 1 \end{array}\right)$ $1 + x^2 + y^2 = 0$

•) Hipérbola (Tipo II, $t=1, s=1$): $R_H = r_H + 1 = 3$, $S_H = s_H + 1 = 1$ $\left(\begin{array}{c|cc} 1 & 0 & 0 \\ \hline 0 & 1 & 0 \\ 0 & 0 & -1 \end{array}\right)$ $1 + x^2 - y^2 = 0$

•) Elipse (Tipo II, $t=0, s=2$): $R_H = r_H + 1 = 3$, $S_H = s_H - 1 = 1$ $\left(\begin{array}{c|cc} 1 & 0 & 0 \\ \hline 0 & -1 & 0 \\ 0 & 0 & -1 \end{array}\right)$ $1 - x^2 - y^2 = 0$

•) Parábola (Tipo III, $t=1, s=0$): $R_H = r_H + 2 = 3$, $S_H = s_H = 1$ $\left(\begin{array}{c|cc} 0 & 0 & 1 \\ \hline 0 & 1 & 0 \\ 1 & 0 & 0 \end{array}\right)$ $x^2 + 2y = 0$

Clasificación de Cuádricas (hipercuádricas $\mathbb{R}^3$):

•) Plano (Tipo I, $t=1, s=0$): $R_H = r_H = 1$, $S_H = s_H = 1$ $\left(\begin{array}{c|ccc} 0 & 0 & 0 & 0 \\ \hline 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$ $x^2 = 0$

•) Recta (Tipo I, $t=2, s=0$): $R_H = r_H = 2$, $S_H = s_H = 2$ $\left(\begin{array}{c|ccc} 0 & 0 & 0 & 0 \\ \hline 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$ $x^2 + y^2 = 0$

•) Planos secantes (Tipo I, $t=1, s=1$): $R_H = r_H = 2$, $S_H = s_H = 0$ $\left(\begin{array}{c|ccc} 0 & 0 & 0 & 0 \\ \hline 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$ $x^2 - y^2 = 0$

•) Punto (Tipo I, $t=3, s=0$): $R_H = r_H = 3$, $S_H = s_H = 3$ $\left(\begin{array}{c|ccc} 0 & 0 & 0 & 0 \\ \hline 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right)$ $x^2 + y^2 + z^2 = 0$

•) Cono (Tipo I, $t=2, s=1$): $R_H = r_H = 3$, $S_H = s_H = 1$ $\left(\begin{array}{c|ccc} 0 & 0 & 0 & 0 \\ \hline 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{array}\right)$ $x^2 + y^2 - z^2 = 0$

- Vacío (Tipo II, $t=1, s=0$): $R_H = r_H + 1 = 2$, $S_H = s_H + 1 = 2$ $\left(\begin{array}{c|ccc} 1 & 0 & 0 & 0 \\ \hline 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$ $1 + x^2 = 0$
- Par de planos paralelos (Tipo II, $t=0, s=1$) $= R_H = r_H + 1 = 2$, $S_H = s_H - 1 = 0$ $\left(\begin{array}{c|ccc} 1 & 0 & 0 & 0 \\ \hline 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$ $1 - x^2 = 0$
- Vacío (Tipo II, $t=2, s=0$): $R_H = r_H + 1 = 3$, $S_H = s_H = 3$ $\left(\begin{array}{c|ccc} 1 & 0 & 0 & 0 \\ \hline 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$ $1 + x^2 + y^2 = 0$
- Cilindro hiperbólico (Tipo II, $t=1, s=1$): $R_H = r_H + 1 = 3$, $S_H = s_H + 1 = 1$ $\left(\begin{array}{c|ccc} 1 & 0 & 0 & 0 \\ \hline 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$ $1 + x^2 - y^2 = 0$
- Cilindro elíptico (Tipo II, $t=0, s=2$): $R_H = r_H + 1 = 3$, $S_H = s_H - 1 = 1$ $\left(\begin{array}{c|ccc} 1 & 0 & 0 & 0 \\ \hline 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$ $1 - x^2 - y^2 = 0$
- Vacío (Tipo II, $t=3, s=0$): $R_H = r_H + 1 = 4$, $S_H = s_H + 1 = 4$ $\left(\begin{array}{c|ccc} 1 & 0 & 0 & 0 \\ \hline 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right)$ $x^2 + y^2 + z^2 + 1 = 0$
- Hiperboloide de 2 hojas (Tipo II, $t=2, s=1$): $R_H = r_H + 1 = 4$, $S_H = s_H + 1 = 2$ $\left(\begin{array}{c|ccc} 1 & 0 & 0 & 0 \\ \hline 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{array}\right)$ $1 + x^2 + y^2 - z^2 = 0$
- Hiperboloide de 1 hoja (Tipo II, $t=1, s=2$): $R_H = r_H + 1 = 4$, $S_H = s_H - 1 = 0$ $\left(\begin{array}{c|ccc} 1 & 0 & 0 & 0 \\ \hline 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{array}\right)$ $1 + x^2 - y^2 - z^2 = 0$
- Elipsoide (Tipo II, $t=0, s=3$): $R_H = r_H + 1 = 4$, $S_H = s_H - 1 = 2$ $\left(\begin{array}{c|ccc} 1 & 0 & 0 & 0 \\ \hline 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{array}\right)$ $1 - x^2 - y^2 - z^2 = 0$
- Cilindro parabólico (Tipo III, $t=1, s=0$): $R_H = r_H + 2 = 3$, $S_H = s_H = 1$ $\left(\begin{array}{c|ccc} 0 & 0 & 0 & 1 \\ \hline 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{array}\right)$ $x^2 + 2z = 0$
- Paraboloide elíptico (Tipo III, $t=2, s=0$): $R_H = r_H + 2 = 4$, $S_H = s_H = 2$ $\dashrightarrow$ $\left(\begin{array}{c|ccc} 0 & 0 & 0 & 1 \\ \hline 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{array}\right)$ $x^2 + y^2 + 2z = 0$
- Paraboloide hiperbólico (Tipo III, $t=1, s=1$): $R_H = r_H + 2 = 4$, $S_H = s_H = 0$ $\left(\begin{array}{c|ccc} 0 & 0 & 0 & 1 \\ \hline 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 \end{array}\right)$ $x^2 - y^2 + 2z = 0$