# 21. sin

## mod:2-1-1

### relation of input:

$$X_{1,1} = 3.142 + X_{0,1}$$

$$X_{2,1} = -9.425 - X_{0,1}$$

$$X_{3,1} = -9.425 + X_{0,1}$$

$$X_{4,1} = 9.425 + X_{0,1}$$

$$X_{5,1} = 6.283 - X_{0,1}$$

$$X_{6,1} = 9.425 - X_{0,1}$$

$$X_{7,1} = X_{0,1}$$

$$X_{8,1} = -3.142 + X_{0,1}$$

$$X_{9,1} = -6.283 - X_{0,1}$$

$$X_{10,1} = -3.142 - X_{0,1}$$

$$X_{11,1} = 6.283 + X_{0,1}$$

$$X_{12,1} = -X_{0,1}$$

$$X_{13.1} = 3.142 - X_{0.1}$$

$$X_{14,1} = -6.283 + X_{0,1}$$

$$Y_{0,1} = sin(X_{0,1})$$

$$Y_{1,1} = sin(3.142 + X_{0,1})$$

$$Y_{2,1} = sin(-9.425 - X_{0,1})$$

$$Y_{3,1} = sin(-9.425 + X_{0,1})$$

$$Y_{4,1} = sin(9.425 + X_{0,1})$$

$$Y_{5,1} = sin(6.283 - X_{0,1})$$

$$Y_{6,1} = sin(9.425 - X_{0,1})$$

$$1_{0,1} = 8000(0.120)$$

$$Y_{7,1}=sin(X_{0,1})$$

$$Y_{8,1} = sin(-3.142 + X_{0,1})$$

$$Y_{9,1} = sin(-6.283 - X_{0,1})$$

$$Y_{10,1} = sin(-3.142 - X_{0,1})$$

$$Y_{11,1} = sin(6.283 + X_{0,1})$$

$$Y_{12,1} = sin(-X_{0,1})$$

$$Y_{13,1} = sin(3.142 - X_{0,1})$$

$$Y_{14.1} = sin(-6.283 + X_{0.1})$$

MR1: 
$$-Y_{0,1} - Y_{1,1} = 0$$
;

MR2: 
$$-Y_{0,1} + Y_{2,1} = 0$$
;

MR3: 
$$Y_{0,1} + Y_{3,1} = 0$$
;

MR4: 
$$-Y_{0,1} - Y_{4,1} = 0$$
;

MR5: 
$$Y_{0,1} + Y_{5,1} = 0$$
;

MR6: 
$$-Y_{0,1} + Y_{6,1} = 0$$
;

MR7: 
$$Y_{0,1} - Y_{7,1} = 0$$
;

MR8: 
$$-Y_{0,1} - Y_{8,1} = 0$$
;

MR9: 
$$2Y_{0,1} + 2Y_{9,1} = 0$$
;

MR10: 
$$Y_{0,1} - Y_{10,1} = 0$$
;

MR11: 
$$Y_{0,1} - Y_{11,1} = 0$$
;

MR12: 
$$-Y_{0.1} - Y_{12.1} = 0$$
;

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\begin{aligned} & \text{MR13:} \ Y_{0,1} - Y_{13,1} = 0; \\ & \text{MR14:} \ Y_{0,1} - Y_{14,1} = 0; \end{aligned}
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### relation of input:

$$egin{aligned} X_{1,1} &= 6.283 - X_{0,1} \ X_{2,1} &= 7.854 - 2X_{0,1} \ X_{3,1} &= X_{0,1} \ X_{4,1} &= -6.283 - X_{0,1} \ X_{5,1} &= -3.142 - X_{0,1} \ X_{6,1} &= -3.142 + X_{0,1} \ X_{7,1} &= -9.425 - X_{0,1} \ X_{8,1} &= 6.283 + X_{0,1} \ X_{9,1} &= 4.712 + 2X_{0,1} \ X_{10,1} &= 3.142 + X_{0,1} \ X_{11,1} &= -9.425 + X_{0,1} \ X_{12,1} &= -X_{0,1} \ X_{13,1} &= 9.425 + X_{0,1} \ X_{14,1} &= -6.283 + X_{0,1} \ X_{15,1} &= 9.425 - X_{0,1} \ X_{15,1} &= 9.425 - X_{0,1} \ X_{16,1} &= 3.142 - X_{0,1} \end{aligned}$$

 $X_{17.1} = -4.712 + 2X_{0.1}$ 

$$\begin{split} Y_{0,1} &= sin(X_{0,1}) \\ Y_{1,1} &= sin(6.283 - X_{0,1}) \\ Y_{2,1} &= sin(7.854 - 2X_{0,1}) \\ Y_{3,1} &= sin(X_{0,1}) \\ Y_{4,1} &= sin(-6.283 - X_{0,1}) \\ Y_{5,1} &= sin(-3.142 - X_{0,1}) \\ Y_{6,1} &= sin(-3.142 + X_{0,1}) \\ Y_{7,1} &= sin(6.283 + X_{0,1}) \\ Y_{9,1} &= sin(6.283 + X_{0,1}) \\ Y_{10,1} &= sin(3.142 + X_{0,1}) \\ Y_{11,1} &= sin(3.142 + X_{0,1}) \\ Y_{12,1} &= sin(-X_{0,1}) \\ Y_{13,1} &= sin(9.425 + X_{0,1}) \\ Y_{14,1} &= sin(9.425 + X_{0,1}) \\ Y_{15,1} &= sin(9.425 - X_{0,1}) \\ Y_{15,1} &= sin(3.142 - X_{0,1}) \\ Y_{16,1} &= sin(3.142 - X_{0,1}) \\ Y_{17,1} &= sin(-4.712 + 2X_{0,1}) \\ \mathbf{MR1:} &-2Y_{0,1}^2 - 2Y_{0,1}Y_{1,1} + 2Y_{0,1} + 2Y_{1,1} = 0; \\ \mathbf{MR2:} &1 - 2Y_{0,1}^2 - Y_{2,1} = 0; \\ \mathbf{MR3:} &-Y_{0,1}^2 + 2Y_{0,1}Y_{3,1} - Y_{3,1}^2 = 0; \\ \mathbf{MR4:} &Y_{0,1}^2 + 2Y_{0,1}Y_{4,1} + Y_{4,1}^2 = 0; \\ \mathbf{MR5:} &-Y_{0,1}^2 + Y_{0,1}Y_{5,1} = 0; \\ \mathbf{MR6:} &Y_{0,1}^2 + 2Y_{0,1}Y_{6,1} + Y_{6,1}^2 = 0; \\ \mathbf{MR7:} &-Y_{0,1}Y_{7,1} - Y_{0,1} + Y_{7,1}^2 + Y_{7,1} = 0; \\ \mathbf{MR7:} &-Y_{0,1}Y_{7,1} - Y_{0,1} + Y_{7,1}^2 + Y_{7,1} = 0; \\ \mathbf{MR7:} &-Y_{0,1}Y_{7,1} - Y_{0,1} + Y_{7,1}^2 + Y_{7,1} = 0; \\ \mathbf{MR7:} &-Y_{0,1}Y_{7,1} - Y_{0,1} + Y_{7,1}^2 + Y_{7,1} = 0; \\ \mathbf{MR7:} &-Y_{0,1}Y_{7,1} - Y_{0,1} + Y_{7,1}^2 + Y_{7,1} = 0; \\ \mathbf{MR7:} &-Y_{0,1}Y_{7,1} - Y_{0,1} + Y_{7,1}^2 + Y_{7,1} = 0; \\ \mathbf{MR7:} &-Y_{0,1}Y_{7,1} - Y_{0,1} + Y_{7,1}^2 + Y_{7,1} = 0; \\ \mathbf{MR7:} &-Y_{0,1}Y_{7,1} - Y_{0,1} + Y_{7,1}^2 + Y_{7,1} = 0; \\ \mathbf{MR7:} &-Y_{0,1}Y_{7,1} - Y_{0,1} + Y_{7,1}^2 + Y_{7,1} = 0; \\ \mathbf{MR7:} &-Y_{0,1}Y_{7,1} - Y_{0,1} + Y_{7,1}^2 + Y_{7,1} = 0; \\ \mathbf{MR7:} &-Y_{0,1}Y_{7,1} - Y_{0,1} + Y_{7,1}^2 + Y_{7,1} = 0; \\ \mathbf{MR7:} &-Y_{0,1}Y_{7,1} - Y_{0,1} + Y_{7,1}^2 + Y_{7,1} = 0; \\ \mathbf{MR7:} &-Y_{0,1}Y_{7,1} - Y_{0,1} + Y_{7,1}^2 + Y_{7,1} = 0; \\ \mathbf{MR7:} &-Y_{0,1}Y_{7,1} - Y_{0,1} + Y_{7,1}^2 + Y_{7,1} = 0; \\ \mathbf{MR7:} &-Y_{0,1}Y_{7,1} - Y_{0,1} + Y_{7,1}^2 + Y_{7,1} = 0; \\ \mathbf{MR7:} &-Y_{0,1}Y_{7,1} - Y_{0,1} + Y_{7,1}^2 + Y_{7,1} + Y_{7,1}^2 + Y$$

$$\begin{array}{l} \operatorname{MR8:} -2Y_{0,1}^2 + 2Y_{0,1}Y_{8,1} - 2Y_{0,1} + 2Y_{8,1} = 0; \\ \operatorname{MR9:} -1 + 2Y_{0,1}^2 - Y_{9,1} = 0; \\ \operatorname{MR10:} 2Y_{0,1}^2 + Y_{0,1}Y_{10,1} - Y_{0,1} - Y_{10,1}^2 - Y_{10,1} = 0; \\ \operatorname{MR11:} -Y_{0,1}^2 + Y_{0,1}Y_{11,1} - 2Y_{0,1} + 2Y_{11,1}^2 - 2Y_{11,1} = 0; \\ \operatorname{MR12:} -Y_{0,1}^2 - 2Y_{0,1}Y_{12,1} - Y_{12,1}^2 = 0; \\ \operatorname{MR13:} 2Y_{0,1}Y_{13,1} + 2Y_{13,1}^2 = 0; \\ \operatorname{MR14:} -Y_{0,1}^2 + 2Y_{0,1}Y_{14,1} - Y_{14,1}^2 = 0; \\ \operatorname{MR15:} -Y_{0,1}Y_{15,1} - Y_{0,1} + Y_{15,1}^2 + Y_{15,1} = 0; \\ \operatorname{MR16:} -2Y_{0,1}^2 + Y_{0,1}Y_{16,1} + 2Y_{0,1} + Y_{16,1}^2 - 2Y_{16,1} = 0; \\ \operatorname{MR17:} -1 + 2Y_{0,1}^2 + Y_{17,1} = 0; \end{array}$$

## relation of input:

$$X_{1,1} = 3.142 + X_{0,1}$$
 $X_{2,1} = -9.425 + X_{0,1}$ 
 $X_{3,1} = -3.142 - X_{0,1}$ 
 $X_{4,1} = 9.425 + X_{0,1}$ 
 $X_{5,1} = 7.854 - X_{0,1}$ 
 $X_{6,1} = -6.283 + X_{0,1}$ 
 $X_{7,1} = -7.854 - X_{0,1}$ 
 $X_{8,1} = -3.142 + X_{0,1}$ 
 $X_{9,1} = 7.854 + 2X_{0,1}$ 
 $X_{10,1} = -X_{0,1}$ 
 $X_{11,1} = -1.571 - 2X_{0,1}$ 
 $X_{12,1} = 6.283 + X_{0,1}$ 
 $X_{13,1} = -7.854 + 2X_{0,1}$ 
 $X_{14,1} = X_{0,1}$ 
 $X_{14,1} = X_{0,1}$ 
 $X_{15,1} = 3.142 - X_{0,1}$ 

$$egin{aligned} Y_{0,1} &= sin(X_{0,1}) \ Y_{1,1} &= sin(3.142 + X_{0,1}) \ Y_{2,1} &= sin(-9.425 + X_{0,1}) \ Y_{3,1} &= sin(-3.142 - X_{0,1}) \ Y_{4,1} &= sin(9.425 + X_{0,1}) \ Y_{5,1} &= sin(7.854 - X_{0,1}) \ Y_{6,1} &= sin(-6.283 + X_{0,1}) \ Y_{7,1} &= sin(-7.854 - X_{0,1}) \ Y_{8,1} &= sin(-7.854 - X_{0,1}) \ Y_{9,1} &= sin(7.854 + 2X_{0,1}) \ Y_{10,1} &= sin(-X_{0,1}) \ Y_{11,1} &= sin(-1.571 - 2X_{0,1}) \ Y_{12,1} &= sin(6.283 + X_{0,1}) \ Y_{13,1} &= sin(-7.854 + 2X_{0,1}) \ Y_{14,1} &= sin(X_{0,1}) \ Y_{15,1} &= sin(3.142 - X_{0,1}) \ Y_{15,1} &= sin(3.142 - X_{0,1}) \ \end{array}$$

$$\begin{split} & \operatorname{MR1:} -Y_{0,1}^3 - Y_{0,1}Y_{1,1} + Y_{0,1}Y_{1,1}^2 - Y_{1,1}^2 = 0; \\ & \operatorname{MR2:} -Y_{0,1}^2 + 2Y_{0,1}^3 - 2Y_{0,1}^2Y_{2,1} - Y_{0,1}Y_{2,1} - 2Y_{0,1}Y_{2,1}^2 + 2Y_{2,1}^3 = 0; \\ & \operatorname{MR3:} Y_{0,1}^3 - 2Y_{0,1}^2Y_{3,1} + Y_{0,1}Y_{3,1} + 2Y_{0,1}Y_{3,1}^2 - Y_{3,1}^2 - Y_{3,1}^3 = 0; \\ & \operatorname{MR4:} -2Y_{0,1}^3 - 2Y_{0,1}^2Y_{4,1} + 2Y_{0,1}Y_{4,1} + 2Y_{0,1}Y_{4,1}^2 + 2Y_{0,1} + 2Y_{4,1}^2 + 2Y_{4,1}^3 + 2Y_{4,1} = 0; \\ & \operatorname{MR5:} -2 + 2Y_{0,1}^2 + 2Y_{0,1}^3 + 2Y_{0,1}^2Y_{5,1} + 2Y_{0,1}Y_{5,1}^2 - 2Y_{0,1} + 2Y_{5,1}^2 + 2Y_{5,1}^3 - 2Y_{5,1} = 0; \\ & \operatorname{MR6:} -Y_{0,1}^2 + 2Y_{0,1}^3 + 2Y_{0,1}Y_{6,1} - 2Y_{0,1}Y_{6,1}^2 - Y_{6,1}^2 = 0; \\ & \operatorname{MR7:} -2Y_{0,1}^2Y_{7,1} - 2Y_{7,1}^3 + 2Y_{7,1} = 0; \\ & \operatorname{MR8:} -Y_{0,1}^2Y_{8,1} + 2Y_{0,1}Y_{8,1} - 2Y_{0,1}Y_{8,1}^2 + 2Y_{8,1}^2 - Y_{8,1}^3 = 0; \\ & \operatorname{MR9:} 1 - 2Y_{0,1}^2 - 2Y_{0,1}^3 - Y_{0,1}Y_{9,1} + Y_{0,1} - Y_{9,1} = 0; \\ & \operatorname{MR10:} -Y_{0,1}Y_{10,1} - Y_{0,1}Y_{10,1}^2 + Y_{0,1} - Y_{10,1}^2 - Y_{10,1}^3 + Y_{10,1} = 0; \\ & \operatorname{MR11:} 1 - 2Y_{0,1}^2 - 2Y_{0,1}^2Y_{11,1} + Y_{11,1}^2 + 2Y_{11,1} = 0; \\ & \operatorname{MR12:} -2Y_{0,1}^2 + Y_{0,1}^3 + Y_{0,1}^2Y_{12,1} + 2Y_{0,1}Y_{12,1} - Y_{0,1}Y_{12,1}^2 - Y_{12,1}^3 = 0; \\ & \operatorname{MR13:} 1 - 2Y_{0,1}^2 - 2Y_{0,1}^3 + 2Y_{0,1}^2Y_{13,1} + Y_{0,1}Y_{13,1} + Y_{0,1} - Y_{13,1}^2 = 0; \\ & \operatorname{MR14:} 2Y_{0,1}^3 - 2Y_{0,1}^2Y_{14,1} - 2Y_{0,1}Y_{14,1}^2 + 2Y_{14,1}^3 = 0; \\ & \operatorname{MR14:} 2Y_{0,1}^3 - 2Y_{0,1}^2Y_{14,1} - 2Y_{0,1}Y_{14,1}^2 + 2Y_{14,1}^3 = 0; \\ & \operatorname{MR15:} 2Y_{0,1}^2 - 2Y_{0,1}^3 + 2Y_{0,1}^2Y_{15,1} - 2Y_{0,1}Y_{15,1} + 2Y_{0,1}Y_{15,1}^2 - 2Y_{15,1}^3 = 0; \\ & \operatorname{MR15:} 2Y_{0,1}^3 - 2Y_{0,1}^2Y_{14,1} - 2Y_{0,1}Y_{14,1}^2 + 2Y_{14,1}^3 = 0; \\ & \operatorname{MR15:} 2Y_{0,1}^3 - 2Y_{0,1}^3 + 2Y_{0,1}^2Y_{15,1} - 2Y_{0,1}Y_{15,1} + 2Y_{0,1}Y_{15,1}^2 - 2Y_{15,1}^3 = 0; \\ & \operatorname{MR15:} 2Y_{0,1}^3 - 2Y_{0,1}^3 + 2Y_{0,1}^2Y_{15,1} - 2Y_{0,1}Y_{15,1} + 2Y_{0,1}Y_{15,1}^2 - 2Y_{15,1}^3 = 0; \\ & \operatorname{MR15:} 2Y_{0,1}^3 - 2Y_{0,1}^3 + 2Y_{0,1}^2Y_{15,1} - 2Y_{0,1}Y_{15,1} + 2Y_{0,1}Y_{15,1}^2 - 2Y_{15,1}^3 = 0; \\ & \operatorname{MR15:} 2Y_{0,1}^3$$

# 4. arcsin

# mod:2-1-1

### relation of input:

$$X_{1,1} = X_{0,1} X_{2,1} = -X_{0,1}$$

### output:

$$egin{aligned} Y_{0,1} &= arcsin(X_{0,1}) \ Y_{1,1} &= arcsin(X_{0,1}) \ Y_{2,1} &= arcsin(-X_{0,1}) \end{aligned}$$

$$\begin{aligned} & \text{MR1: } 2Y_{0,1} - 2Y_{1,1} = 0; \\ & \text{MR2: } -2Y_{0,1} - 2Y_{2,1} = 0; \end{aligned}$$

## mod:2-1-2

### relation of input:

$$X_{1,1} = X_{0,1}$$
  
 $X_{2,1} = -X_{0,1}$ 

#### output:

$$egin{aligned} Y_{0,1} &= arcsin(X_{0,1}) \ Y_{1,1} &= arcsin(X_{0,1}) \ Y_{2,1} &= arcsin(-X_{0,1}) \end{aligned}$$

MR1: 
$$2Y_{0,1}^2-2Y_{0,1}Y_{1,1}+2Y_{0,1}-2Y_{1,1}=0;$$
 MR2:  $-2Y_{0,1}^2-2Y_{0,1}Y_{2,1}-2Y_{0,1}-2Y_{2,1}=0;$ 

# mod:2-1-3

### relation of input:

$$egin{aligned} X_{1,1} &= -X_{0,1} \ X_{2,1} &= X_{0,1} \end{aligned}$$

### output:

$$egin{aligned} Y_{0,1} &= arcsin(X_{0,1}) \ Y_{1,1} &= arcsin(-X_{0,1}) \ Y_{2,1} &= arcsin(X_{0,1}) \end{aligned}$$

$$\begin{array}{l} \mathbf{MR1:}\ 2Y_{0,1}^2+2Y_{0,1}^3+2Y_{0,1}^2Y_{1,1}+2Y_{0,1}Y_{1,1}-2Y_{0,1}Y_{1,1}^2+2Y_{0,1}-2Y_{1,1}^3+2Y_{1,1}=0;\\ \mathbf{MR2:}\ 2Y_{0,1}^2-2Y_{0,1}^3-2Y_{0,1}^2Y_{2,1}+2Y_{0,1}Y_{2,1}^2-Y_{0,1}-2Y_{2,1}^2+2Y_{2,1}^3+Y_{2,1}=0; \end{array}$$

# 5. arcsinh

# mod:2-1-1

## relation of input:

$$X_{1,1} = X_{0,1}$$
  
 $X_{2,1} = -X_{0,1}$ 

#### output:

 $egin{aligned} Y_{0,1} &= arcsinh(X_{0,1}) \ Y_{1,1} &= arcsinh(X_{0,1}) \ Y_{2,1} &= arcsinh(-X_{0,1}) \end{aligned}$ 

 $\begin{aligned} & \text{MR1:} -Y_{0,1} + Y_{1,1} = 0; \\ & \text{MR2:} -Y_{0,1} - Y_{2,1} = 0; \end{aligned}$ 

# mod:2-1-2

### relation of input:

$$X_{1,1} = X_{0,1}$$
  
 $X_{2,1} = -X_{0,1}$ 

### output:

 $egin{aligned} Y_{0,1} &= arcsinh(X_{0,1}) \ Y_{1,1} &= arcsinh(X_{0,1}) \ Y_{2,1} &= arcsinh(-X_{0,1}) \end{aligned}$ 

 $\begin{aligned} & \text{MR1:}\ Y_{0,1}^2-Y_{0,1}Y_{1,1}+2Y_{0,1}-2Y_{1,1}=0;\\ & \text{MR2:}\ -Y_{0,1}^2-Y_{0,1}Y_{2,1}-2Y_{0,1}-2Y_{2,1}=0; \end{aligned}$ 

# mod:2-1-3

### relation of input:

$$egin{aligned} X_{1,1} &= X_{0,1} \ X_{2,1} &= -X_{0,1} \end{aligned}$$

#### output:

 $Y_{0,1} = arcsinh(X_{0,1}) \ Y_{1,1} = arcsinh(X_{0,1}) \ Y_{2,1} = arcsinh(-X_{0,1})$ 

$$\begin{array}{l} \mathbf{MR1:} -Y_{0,1}^2 + Y_{0,1}^3 - Y_{0,1}^2 Y_{1,1} + Y_{0,1} Y_{1,1} - Y_{0,1} Y_{1,1}^2 + 2 Y_{0,1} + Y_{1,1}^3 - 2 Y_{1,1} = 0; \\ \mathbf{MR2:} -2Y_{0,1}^2 - Y_{0,1}^3 + 2 Y_{0,1}^2 Y_{2,1} + Y_{0,1} Y_{2,1}^2 + 2 Y_{0,1} + 2 Y_{2,1}^2 - 2 Y_{2,1}^3 + 2 Y_{2,1} = 0; \end{array}$$

# **15.** log

### relation of input:

$$egin{aligned} X_{1,1} &= X_{0,1} \ X_{2,1} &= 2X_{0,1} \end{aligned}$$

### output:

$$Y_{0,1} = log(X_{0,1}) \ Y_{1,1} = log(X_{0,1}) \ Y_{2,1} = log(2X_{0,1})$$

MR1: 
$$Y_{0,1} - Y_{1,1} = 0$$
;

MR2: 
$$-1.386 - 2Y_{0,1} + 2Y_{2,1} = 0$$
;

# mod:2-1-2

### relation of input:

$$egin{aligned} X_{1,1} &= X_{0,1} \ X_{2,1} &= 2X_{0,1} \end{aligned}$$

### output:

$$egin{aligned} Y_{0,1} &= log(X_{0,1}) \ Y_{1,1} &= log(X_{0,1}) \ Y_{2,1} &= log(2X_{0,1}) \end{aligned}$$

MR1: 
$$-2Y_{0,1}^2 + 2Y_{0,1} + 2Y_{1,1}^2 - 2Y_{1,1} = 0$$
;

MR2: 
$$0.906 + Y_{0,1}^2 - 2Y_{0,1}Y_{2,1} + 2Y_{0,1} + Y_{2,1}^2 - 2Y_{2,1} = 0$$
;

# mod:2-1-3

### relation of input:

$$X_{1,1} = X_{0,1}$$

### output:

$$Y_{0,1} = log(X_{0,1})$$
  
 $Y_{1,1} = log(X_{0,1})$ 

MR1: 
$$2Y_{0,1}^2 + 2Y_{0,1}^3 - 2Y_{0,1}^2Y_{1,1} - 2Y_{0,1}Y_{1,1}^2 - Y_{0,1} - 2Y_{1,1}^2 + 2Y_{1,1}^3 + Y_{1,1} = 0$$
;

# 12. exp

# mod:2-1-2

## relation of input:

$$X_{1,1} = -10 - X_{0,1}$$
  
 $X_{2,1} = 1.608 - X_{0,1}$   
 $X_{3,1} = -1.712 - X_{0,1}$ 

$$egin{aligned} Y_{0,1} &= exp(X_{0,1}) \ Y_{1,1} &= exp(-10-X_{0,1}) \ Y_{2,1} &= exp(1.608-X_{0,1}) \ Y_{3,1} &= exp(-1.712-X_{0,1}) \end{aligned}$$

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\begin{aligned} & \text{MR1: } 2Y_{0,1}Y_{1,1}=0;\\ & \text{MR2: } -9.988+2Y_{0,1}Y_{2,1}=0;\\ & \text{MR3: } -0.361+2Y_{0,1}Y_{3,1}=0; \end{aligned}
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### relation of input:

$$X_{1,1} = -10 - X_{0,1}$$

#### output:

$$egin{aligned} Y_{0,1} &= exp(X_{0,1}) \ Y_{1,1} &= exp(-10-X_{0,1}) \end{aligned}$$

MR1: 
$$2Y_{0,1}Y_{1,1} + 2Y_{0,1}Y_{1,1}^2 = 0$$
;

# 14. hypot

# mod:2-1-1

# relation of input:

$$\begin{split} (X_{0,1},X_{0,2}) &= (X_{0,2},X_{0,1}) \\ (X_{1,1},X_{1,2}) &= (-X_{0,1},X_{0,2}) \\ (X_{2,1},X_{2,2}) &= (2X_{0,2},2X_{0,1}) \\ (X_{3,1},X_{3,2}) &= (-2X_{0,2},2X_{0,1}) \\ (X_{4,1},X_{4,2}) &= (X_{0,1},X_{0,2}) \\ (X_{5,1},X_{5,2}) &= (-2X_{0,1},2X_{0,2}) \\ (X_{6,1},X_{6,2}) &= (X_{0,2},-X_{0,1}) \\ (X_{7,1},X_{7,2}) &= (-X_{0,1},-X_{0,2}) \\ (X_{8,1},X_{8,2}) &= (2X_{0,2},-2X_{0,1}) \\ (X_{9,1},X_{9,2}) &= (-X_{0,2},X_{0,1}) \\ (X_{10,1},X_{10,2}) &= (-2X_{0,1},-2X_{0,2}) \\ (X_{11,1},X_{11,2}) &= (2X_{0,1},-2X_{0,2}) \end{split}$$

## output:

$$egin{aligned} Y_{0,1} &= hypot(X_{0,1}, X_{0,2}) \ Y_{1,1} &= hypot(X_{0,2}, X_{0,1}) \ Y_{2,1} &= hypot(-X_{0,1}, X_{0,2}) \ Y_{3,1} &= hypot(2X_{0,2}, 2X_{0,1}) \ Y_{4,1} &= hypot(-2X_{0,2}, 2X_{0,1}) \ Y_{5,1} &= hypot(X_{0,1}, X_{0,2}) \ Y_{6,1} &= hypot(-2X_{0,1}, 2X_{0,2}) \ Y_{7,1} &= hypot(X_{0,2}, -X_{0,1}) \ Y_{8,1} &= hypot(-X_{0,1}, -X_{0,2}) \ Y_{9,1} &= hypot(2X_{0,2}, -2X_{0,1}) \ Y_{10,1} &= hypot(-X_{0,2}, X_{0,1}) \ Y_{11,1} &= hypot(-2X_{0,1}, -2X_{0,2}) \ Y_{12,1} &= hypot(2X_{0,1}, -2X_{0,2}) \end{aligned}$$

$$\begin{split} & \text{MR1:}\ Y_{0,1}-Y_{1,1}=0;\\ & \text{MR2:}\ -Y_{0,1}+Y_{2,1}=0;\\ & \text{MR3:}\ -2Y_{0,1}+Y_{3,1}=0;\\ & \text{MR4:}\ 2Y_{0,1}-Y_{4,1}=0;\\ & \text{MR5:}\ -Y_{0,1}+Y_{5,1}=0; \end{split}$$

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\begin{array}{l} \operatorname{MR6:} -2Y_{0,1} + Y_{6,1} = 0; \\ \operatorname{MR7:} Y_{0,1} - Y_{7,1} = 0; \\ \operatorname{MR8:} -Y_{0,1} + Y_{8,1} = 0; \\ \operatorname{MR9:} 2Y_{0,1} - Y_{9,1} = 0; \\ \operatorname{MR10:} Y_{0,1} - Y_{10,1} = 0; \\ \operatorname{MR11:} 2Y_{0,1} - Y_{11,1} = 0; \\ \operatorname{MR12:} 2Y_{0,1} - Y_{12,1} = 0; \end{array}
```

### relation of input:

$$(X_{0,1},X_{0,2})=(-X_{0,2},X_{0,1}) \ (X_{1,1},X_{1,2})=(-X_{0,1},-X_{0,2}) \ (X_{2,1},X_{2,2})=(X_{0,1},X_{0,2}) \ (X_{3,1},X_{3,2})=(X_{0,1},-X_{0,2}) \ (X_{4,1},X_{4,2})=(-0.002+X_{0,2},-0.006+X_{0,1}) \ (X_{5,1},X_{5,2})=(-X_{0,2},-X_{0,1}) \ (X_{6,1},X_{6,2})=(-X_{0,1},X_{0,2}) \ (X_{7,1},X_{7,2})=(X_{0,2},-X_{0,1})$$

$$\begin{aligned} & \text{output:} \\ & Y_{0,1} = hypot(X_{0,1}, X_{0,2}) \\ & Y_{1,1} = hypot(-X_{0,1}, -X_{0,2}) \\ & Y_{2,1} = hypot(-X_{0,1}, -X_{0,2}) \\ & Y_{3,1} = hypot(X_{0,1}, X_{0,2}) \\ & Y_{4,1} = hypot(X_{0,1}, -X_{0,2}) \\ & Y_{5,1} = hypot(-0.002 + X_{0,2}, -0.006 + X_{0,1}) \\ & Y_{6,1} = hypot(-X_{0,2}, -X_{0,1}) \\ & Y_{7,1} = hypot(-X_{0,1}, X_{0,2}) \\ & Y_{8,1} = hypot(X_{0,2}, -X_{0,1}) \\ & \text{MR1:} \ Y_{0,1}^2 - Y_{0,1}Y_{1,1} - 2Y_{0,1} + 2Y_{1,1} = 0; \\ & \text{MR2:} \ -Y_{0,1}^2 + 2Y_{0,1}Y_{2,1} - 2Y_{0,1} - Y_{2,1}^2 + 2Y_{2,1} = 0; \\ & \text{MR3:} \ Y_{0,1}^2 - 2Y_{0,1}Y_{3,1} + Y_{0,1} + Y_{3,1}^2 - Y_{3,1} = 0; \\ & \text{MR4:} \ -Y_{0,1}^2 + 2Y_{0,1}Y_{4,1} - Y_{4,1}^2 = 0; \\ & \text{MR5:} \ Y_{0,1}^2 - 2Y_{0,1}Y_{5,1} + Y_{5,1}^2 = 0; \\ & \text{MR6:} \ Y_{0,1}^2 - 2Y_{0,1}Y_{6,1} + Y_{6,1}^2 = 0; \\ & \text{MR7:} \ -Y_{0,1}^2 + 2Y_{0,1}Y_{7,1} - 2Y_{0,1} - Y_{7,1}^2 + 2Y_{7,1} = 0; \\ & \text{MR8:} \ Y_{0,1}^2 - 2Y_{0,1}Y_{8,1} + Y_{0,1} + Y_{8,1}^2 - Y_{8,1} = 0; \end{aligned}$$

### mod:2-1-3

### relation of input:

$$(X_{0,1},X_{0,2})=(X_{0,1},-X_{0,2})\ (X_{1,1},X_{1,2})=(X_{0,2},X_{0,1})\ (X_{2,1},X_{2,2})=(X_{0,2},-X_{0,1})\ (X_{3,1},X_{3,2})=(-X_{0,1},X_{0,2})\ (X_{4,1},X_{4,2})=(-X_{0,1},-X_{0,2})\ (X_{5,1},X_{5,2})=(-X_{0,2},X_{0,1})\ (X_{6,1},X_{6,2})=(X_{0,1},X_{0,2})\ (X_{7,1},X_{7,2})=(-X_{0,2},-X_{0,1})$$

$$Y_{0,1} = hypot(X_{0,1}, X_{0,2})$$

$$Y_{1,1} = hypot(X_{0,1}, -X_{0,2})$$

$$Y_{2,1} = hypot(X_{0,2}, X_{0,1})$$

$$Y_{3,1} = hypot(X_{0,2}, -X_{0,1})$$

$$Y_{4,1} = hypot(-X_{0,1}, X_{0,2})$$

$$Y_{5,1} = hypot(-X_{0,1}, -X_{0,2})$$

$$Y_{6,1} = hypot(-X_{0,2}, X_{0,1})$$

$$Y_{7,1} = hypot(X_{0,1}, X_{0,2})$$

$$Y_{8,1} = hypot(-X_{0,2}, -X_{0,1})$$

MR1: 
$$-2Y_{0,1}^2 + Y_{0,1}^3 - 2Y_{0,1}^2Y_{1,1} + 2Y_{0,1}Y_{1,1} + Y_{0,1}Y_{1,1}^2 + 2Y_{0,1} - 2Y_{1,1} = 0;$$

MR2: 
$$-Y_{0,1}^3 + 2Y_{0,1}^2Y_{2,1} + Y_{0,1}Y_{2,1} - Y_{0,1}Y_{2,1}^2 + 2Y_{0,1} - Y_{2,1}^2 - 2Y_{2,1} = 0;$$

MR3: 
$$0.002 + Y_{0.1}^2 - Y_{0.1}^2 Y_{3,1} + 2Y_{0,1} Y_{3,1}^2 - 2Y_{0,1} - Y_{3,1}^2 - Y_{3,1}^3 + 2Y_{3,1} = 0;$$

MR4: 
$$-2Y_{0,1}^2 - Y_{0,1}^2 Y_{4,1} + 2Y_{0,1} Y_{4,1} + 2Y_{0,1} Y_{4,1}^2 + 2Y_{0,1} - Y_{4,1}^3 - 2Y_{4,1} = 0;$$

MR5: 
$$-Y_{0,1}^2 + Y_{0,1}^3 - Y_{0,1}^2 Y_{5,1} + 2Y_{0,1} Y_{5,1} - 2Y_{0,1} Y_{5,1}^2 + 2Y_{0,1} - Y_{5,1}^2 + 2Y_{5,1}^3 - 2Y_{5,1} = 0;$$

MR6: 
$$Y_{0,1}^2 - Y_{0,1}^3 + Y_{0,1}^2 Y_{6,1} + Y_{0,1} Y_{6,1} + Y_{0,1} Y_{6,1}^2 + 2Y_{0,1} - 2Y_{6,1}^2 - Y_{6,1}^3 - 2Y_{6,1} = 0;$$

$$\mathsf{MR7:} -2Y_{0,1}^2 - 2Y_{0,1}^3 + 2Y_{0,1}^2Y_{7,1} + 2Y_{0,1}Y_{7,1}^2 + 2Y_{0,1} + 2Y_{7,1}^2 - 2Y_{7,1}^3 - 2Y_{7,1} = 0;$$

MR8: 
$$Y_{0,1}^2 + Y_{0,1}^3 - Y_{0,1}^2 Y_{8,1} - 2 Y_{0,1} Y_{8,1} - Y_{0,1} Y_{8,1}^2 + Y_{0,1} + Y_{8,1}^2 + Y_{8,1}^3 - Y_{8,1} = 0;$$