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Sec: 06

1. a) $2x+3y=7$ — (I)

$5x+3y=10$ — (II)

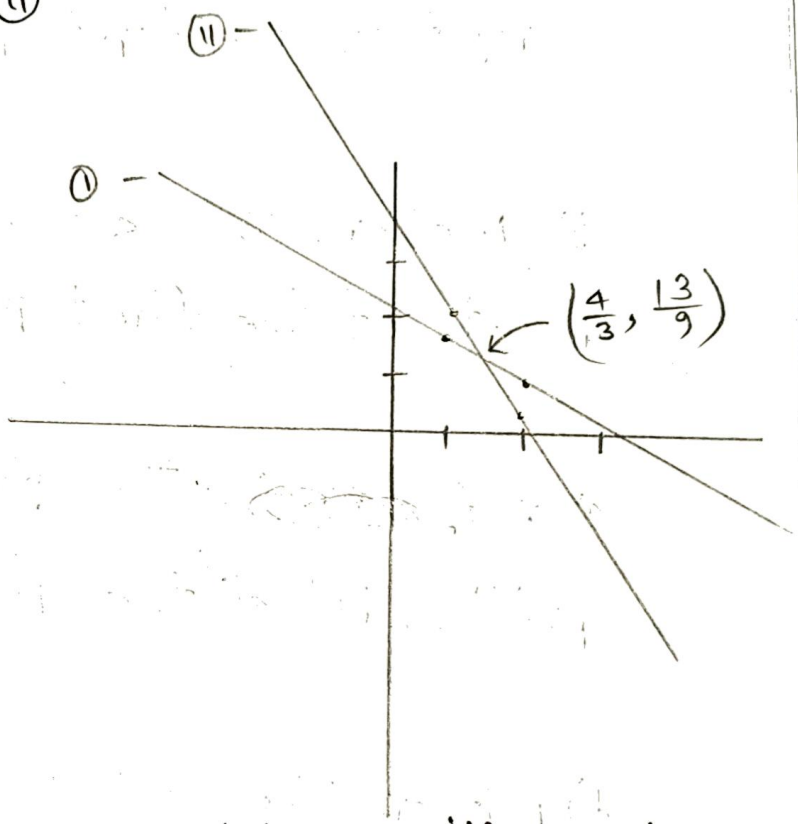
(I) $\Rightarrow$

$x=1,\ y=\frac{5}{3}$

$x=2,\ y=1$

(II) $\Rightarrow$ $x=1,\ y=2$

$x=2,\ y=\frac{1}{3}$



b) If we draw a row picture with 3 unknowns and 3 equation we will get 3 surface and they will cross at point if there is a solution.

2.

a) Neither. The second pivot is not to the right of the first pivot.

b) Neither. The second pivot is not to the right of the first pivot.

c) ref. The pivots are in correct position. Not all the pivots are 1.

d) Neither. The second pivot is not to the right of the first pivot.

e) ref. The pivots are in correct position. Not all the pivots are 1.

f) Neither. All zero row must be at bottom.

g) ref and rref. All the zero rows at bottom and the only pivot is 1.

h) ref and rref. All the zero rows at bottom and the only pivot is 1.

i) ref. All the zero rows at bottom and pivot is not one.

j) ref and rref. All the zero rows at right position.

3. a) $-7x_2 - 4x_3 = 2$

$2x + 4x_2 + 6x_3 = 12$

$3x_1 + x_2 - x_3 = -2$

$$\left(\begin{array}{ccc|c} 0 & -7 & -4 & 2 \\ 2 & 4 & 6 & 12 \\ 3 & 1 & -1 & -2 \end{array}\right)$$

$$= \left(\begin{array}{ccc|c} 3 & 1 & -1 & -2 \\ 2 & 4 & 6 & 12 \\ 0 & -7 & -4 & 2 \end{array}\right) \quad [R_1 \leftrightarrow R_3]$$

$$= \left(\begin{array}{ccc|c} 3 & 1 & -1 & -2 \\ 0 & 10 & 20 & 40 \\ 0 & -7 & -4 & 2 \end{array}\right) \quad [R_2' = 3R_2 - 2R_1]$$

$$= \left(\begin{array}{ccc|c} 3 & 1 & -1 & -2 \\ 0 & 1 & 2 & 4 \\ 0 & -7 & -4 & 2 \end{array}\right) \quad \left[r_2' = \frac{r_2}{10}\right]$$

$$= \left(\begin{array}{ccc|c} 3 & 0 & -3 & -6 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & 10 & 30 \end{array}\right) \quad \begin{array}{l} \left[r_1' = r_1 - r_2\right] \\ \left[r_3' = r_3 + 7r_2\right] \end{array}$$

$$= \left(\begin{array}{ccc|c} 1 & 0 & -1 & -2 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & 1 & 3 \end{array}\right) \quad \begin{array}{l} \left[r_1' = \frac{r_1}{3}\right] \\ \left[r_3' = \frac{r_3}{10}\right] \end{array}$$

$$= \left(\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 3 \end{array}\right) \quad \begin{array}{l} \left[r_1' = r_1 + r_3\right] \\ \left[r_2' = r_2 - 2r_1\right] \end{array}$$

$\therefore$ $x_1 = 1$

$x_2 = -2$ } Ans.

$x_3 = 3$

b) $2x_1 + x_2 + 12x_3 = 1$

$x_1 + 2x_2 + 9x_3 = -1$

$$\left(\begin{array}{ccc|c} 2 & 1 & 12 & 1 \\ 1 & 2 & 9 & -1 \end{array}\right)$$

$$= \left(\begin{array}{ccc|c} 2 & 1 & 12 & 1 \\ 0 & 3 & 6 & -3 \end{array}\right) \quad [r_2' = 2r_2 - r_1]$$

$$= \left(\begin{array}{ccc|c} 2 & 1 & 12 & 1 \\ 0 & 1 & 2 & -1 \end{array}\right) \quad \left[r_2' = \frac{r_2}{3}\right]$$

$$= \left(\begin{array}{ccc|c} 2 & 0 & 10 & 2 \\ 0 & 1 & 2 & -1 \end{array}\right) \quad [r_1' = r_1 - r_2]$$

$$= \left(\begin{array}{ccc|c} 1 & 0 & 5 & 1 \\ 0 & 1 & 2 & -1 \end{array}\right) \quad \left[r_1' = \frac{r_1}{2}\right]$$

$\therefore$ $x_1 + 5x_3 = 1$

$x_2 + 2x_3 = -1$

here x_3 is a free variable.

Let $x_3 = t$

$\therefore x_1 = 1 - 5t$

$\therefore x_2 = -1 - 2t$

$\therefore x_3 = t$

} As.

c) $-3x_1 - 2x_2 + x_3 = 15$

$5x_1 + 3x_2 + 2x_3 = 0$

$3x_1 + x_2 + 3x_3 = 11$

$-6x_1 - 4x_2 + 2x_3 = 30$

$$\left(\begin{array}{ccc|c} -3 & -2 & 1 & 15 \\ 5 & 3 & 2 & 0 \\ 3 & 1 & 3 & 11 \\ -6 & -4 & 2 & 30 \end{array}\right)$$

$$= \left(\begin{array}{ccc|c} -3 & -2 & 1 & 15 \\ 5 & 3 & 2 & 0 \\ 3 & 1 & 3 & 11 \\ -3 & -2 & 1 & 15 \end{array}\right) \quad \left[r_4' = \frac{r_4}{2}\right]$$

$$= \left(\begin{array}{ccc|c} -3 & -2 & 1 & 15 \\ 0 & -1 & 11 & 75 \\ 0 & -1 & 4 & 26 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$\left[r_2' = 3r_2 + 5r_1\right]$

$\left[r_3' = r_3 + r_1\right]$

$\left[r_4' = r_4 - r_1\right]$

$$= \left(\begin{array}{cccc|c} -3 & 0 & -21 & | & -135 \\ 0 & -1 & 11 & | & 75 \\ 0 & 0 & -7 & | & -49 \\ 0 & 0 & 0 & | & 0 \end{array}\right) \qquad \left[r_1' = r_1 - 2r_2\right] \quad \left[r_3' = r_3 - r_2\right]$$

$$= \left(\begin{array}{cccc|c} 1 & 0 & 7 & | & 45 \\ 0 & 1 & -11 & | & -75 \\ 0 & 0 & 1 & | & 7 \\ 0 & 0 & 0 & | & 0 \end{array}\right) \qquad \left[r_1' = \frac{r_1}{-3}\right] \quad \left[r_2' = (-1)r_2\right] \quad \left[r_3' = \frac{r_3}{-7}\right]$$

$$= \left(\begin{array}{ccc|c} 1 & 0 & 0 & -4 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 7 \\ 0 & 0 & 0 & 0 \end{array}\right) \qquad r_1' = r_1 - 7r_3 \quad r_2' = r_2 + 11r_3$$

$\therefore x_1 = -4$

$\therefore x_2 = 2$

$\therefore x_3 = 7$

} Ans.

4. a)

$$x_1 + 3x_3 = 1$$

$$2x_1 + 3x_2 + 2x_3 = 1$$

$$2x_1 + 2x_2 + 3x_3 = 1$$

$$\begin{pmatrix} 1 & 0 & 3 \\ 2 & 3 & 2 \\ 2 & 2 & 3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 3 \\ 2 & 3 & 2 \\ 2 & 2 & 3 \end{pmatrix}^{-1} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 3 \\ 2 & 3 & 2 \\ 2 & 2 & 3 \end{pmatrix}^{-1} = \left(\begin{array}{ccc|ccc} 1 & 0 & 3 & 1 & 0 & 0 \\ 2 & 3 & 2 & 0 & 1 & 0 \\ 2 & 2 & 3 & 0 & 0 & 1 \end{array}\right)$$

$$= \left(\begin{array}{ccc|ccc} 1 & 0 & 3 & 1 & 0 & 0 \\ 0 & 3 & -4 & -2 & 1 & 0 \\ 0 & 2 & -3 & -2 & 0 & 1 \end{array}\right) \quad \begin{matrix} [R_2' = R_2 - 2R_1] \\ [R_3' = R_3 - 2R_1] \end{matrix}$$

$$= \left(\begin{array}{ccc|ccc} 1 & 0 & 3 & 1 & 0 & 0 \\ 0 & 3 & -4 & -2 & 1 & 0 \\ 0 & 0 & -1 & -2 & -2 & 3 \end{array}\right) \quad [r_3' = 3r_3 - 2r_2]$$

$$= \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -5 & -6 & 9 \\ 0 & 3 & 0 & 6 & 9 & -12 \\ 0 & 0 & -1 & -2 & -2 & 3 \end{array}\right) \quad \begin{matrix} [r_1' = r_1 + 3r_3] \\ [r_2' = r_2 - 4r_3] \end{matrix}$$

$$= \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -5 & -6 & 9 \\ 0 & 1 & 0 & 2 & 3 & -4 \\ 0 & 0 & 1 & 2 & 2 & -3 \end{array}\right) \quad \begin{matrix} [r_2' = \frac{r_2}{3}] \\ [r_3' = (-1)r_3] \end{matrix}$$

$$\therefore \begin{pmatrix} -5 & -6 & 9 \\ 2 & 3 & -4 \\ 2 & 2 & -3 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix}$$

$\therefore x_1 = -2$

$\therefore x_2 = 1$

$\therefore x_3 = 1$

} As.

b) $x_1 + 4x_3 = 11$

$x_2 + 2x_3 = 12$

$-3x_2 - 4x_3 = 13$

$$\therefore \begin{pmatrix} 1 & 0 & 4 \\ 0 & 1 & 2 \\ 0 & -3 & -4 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 11 \\ 12 \\ 13 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 4 \\ 0 & 1 & 2 \\ 0 & -3 & -4 \end{pmatrix}^{-1} \begin{pmatrix} 11 \\ 12 \\ 13 \end{pmatrix}$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 4 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 & 1 & 0 \\ 0 & -3 & -4 & 0 & 0 & 1 \end{array}\right)$$

$$= \left(\begin{array}{ccc|ccc} 1 & 0 & 4 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 & 1 & 0 \\ 0 & 0 & 2 & 0 & 3 & 1 \end{array}\right) \quad \boxed{r_3' = r_3 + 3r_2}$$

$$= \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -6 & -7 \\ 0 & 1 & 0 & 0 & -2 & -1 \\ 0 & 0 & 2 & 0 & 3 & 1 \end{array}\right) \quad \begin{array}{l} r_1' = r_1 - 2r_3 \\ r_2' = r_2 - r_3 \end{array}$$

$$= \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -6 & -7 \\ 0 & 1 & 0 & 0 & -2 & -1 \\ 0 & 0 & 1 & 0 & \frac{3}{2} & \frac{1}{2} \end{array}\right) \quad \left[r_3' = \frac{r_3}{2}\right]$$

$$\therefore \begin{pmatrix} 1 & -6 & -7 \\ 0 & -2 & -1 \\ 0 & \frac{3}{2} & \frac{1}{2} \end{pmatrix} \begin{pmatrix} 11 \\ 12 \\ 13 \end{pmatrix} = \begin{pmatrix} -87 \\ -37 \\ \frac{49}{2} \end{pmatrix}$$

$\therefore x_1 = -87$

$\therefore x_2 = -37$

$\therefore x_3 = \frac{49}{2}$

Ans.

5.

Elementary Matrix: A square matrix obtained by performing a single elementary row operation on an identity matrix.

Permutation Matrix: A square matrix having only one value of 1 in each row and column and rest is 0 is a permutation matrix.

$$I = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

i. $\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ $[r_1 \leftrightarrow r_2]$

ii. $\begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$ $[r_1 \leftrightarrow r_3]$

iii. $\begin{pmatrix} 10 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ $[r_1' = 10r_1]$

iv. $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix}$ $[r_2' = r_2 + 2r_3]$

v. $\begin{pmatrix} 1 & 0 & -10 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ $[r_1' = r_1 - 10r_3]$

vi. $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \frac{1}{2} & 0 & 0 \end{pmatrix}$ $[r_3' = r_3 + \frac{1}{2} r_1]$

vii. $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 100 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ $[r_2' = 100 r_2]$

viii. $\begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$ $[r_2' = \frac{1}{2} r_2]$