Department of Mathematics and Natural Sciences
MAT216: Linear Algebra & Fourier Analysis
Summer 2023

**ASSIGNMENT 2**

**Faculty Name: Shovan Sourav Datta (SVP) Mark:** 50 *Make a **Front Page** by yourself, mentioning your #name, #ID, and #section. (Compulsory)*

1. Determine which sets are vector spaces under the given operations. For those that are not vector spaces, list at least one axiom that fail to hold. (2.5 × 5 = 12.5)

   (a) The set of all triples of real numbers $(x, y, z)$ with the operations $(x, y, z) + (x', y', z') = (x + x', y + y', z + z')$ and $k(x, y, z) = (0, 0, 0)$.

   (b) The set of all pairs of real numbers of the form $(x, y)$, where $x \geq 0$; with the standard operations on $R^2$.

   (c) The set of all pairs of real numbers $(x, y)$ with the operations $(x, y) + (x', y') = (x + x' + 1, y + y' + 1)$ and $k(x, y) = (kx, ky)$.

   (d) The set of all pairs of real numbers of the form $(1, x)$ with the operations $(1, y) + (1, y') = (1, y + y')$ and $k(1, y) = (1, ky)$.

   (e) The set of all positive real numbers with the operations $x + y = xy$ and $kx = x^k$.

2. Determine which of the following are subspace of the vector space $V$. (1.5 × 5 = 7.5)

   (a) All vectors of the form $(a, 0, 0)$, where $V = R^3$

   (b) All vectors of the form $(a, b, c)$ with $c = a - b$, where $V = R^3$

   (c) All vectors of the form $(a, b, c)$ with $c = a + b + 3$, where $V = R^3$

   (d) All matrices $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ with $a + b + c + d = 0$, where $V = M_{2\times2}$

   (e) All matrices $\begin{bmatrix} a & a \\ -a & -a \end{bmatrix}$, where $V = M_{2\times2}$

3. Consider $A =$ 1 0 3 and **b** = 2 3 2

2 2 3

1 2      2      1

(a) Do the vector **b** is in the column space of $A$? (3) (b) Do the column vectors of $A$ are linearly independent? What will be *Null*($A$)? (3)

4. Consider four vectors, $\mathbf{v_1}$ = 1 2, $\mathbf{v_2}$ = 3 4, $\mathbf{v_3}$ = 5 4 5, $\mathbf{v_4}$ = 6 6 7 8

(a) Do the vectors $\mathbf{v_1}$, $\mathbf{v_3}$, $\mathbf{v_4}$ are linearly independent? (3)

(b) Do $\mathbf{b_1}$, $\mathbf{b_2}$ ∈ *span*{$v_1$, $v_2$, $v_3$, $v_4$}? where, $\mathbf{b_1}$ = 0 1 −2, $\mathbf{b_2}$ = 11 7 0 (3+3)

5. Calculate the *Row*($A$), *Col*($A$), *Null*($A$) and *Null*($A^T$) (left nullspace of $A$) of the following matrix. (3 × 5 = 15)

(a) $A$ = 1 4 −2 5 2 9 −1 8    (b) $A$ = 1 4 5 2    (c) $A$ = 1 −1 3

2 9 −1 9 −1 −4 2 −5

2 1 3 0 −1 3 2 2

2

5 −4 −4 7 −6 2