

## **Practice Problems Chapter 4: Non-Linear Equations**

1. Consider a fixed point function  $g(x) = (9x - 1)^{\frac{1}{3}}$ . The corresponding nonlinear function  $f(x)$  has a solution  $x_* \in \mathbb{R}$ .
  - a. Show that  $g(x)$  would lead to linear convergence if  $x > \frac{1}{9}(1 + \sqrt{27})$ . *\*\* hint: use the condition for linear convergence \*\**
  - b. Starting from  $x_0 = 2.5$ , find the value of  $x_*$  after 5 iterations while keeping up to 5 significant figures.
2. Use Secant method to estimate the root of the following function  $f(x) = x^3 - 3x^2 + x$  with initial the initial values of  $x_0 = 0.35$  &  $x_1 = 0.3$ . Show your result with the error in a tabular format for the first 5 iterations.
3. Consider the following function  $f(x) = x^2\sqrt{(2x + 3)}$ . Now, find the solution of  $f(x) = 0$  by doing 5 iterations using the Newton's Raphson method starting with  $x_0 = 1.5$ . Keep your answers up to 3 significant figures.
4. Consider the function,  $f(x) = x^3 - x^2 - 9x + 9$ . Answer the following:
  - a. State the exact roots of  $f(x)$ .
  - b. Construct three different fixed point functions  $g(x)$  such that  $f(x) = 0$ . (Make sure that one of the  $g(x)$ 's that you constructed converges to at least a root).
  - c. Find the convergence rate/ratio for each  $g(x)$  constructed in the previous part and also find which root it is converging to.
  - d. Find the approximate root,  $x^*$ , of the above function using fixed point iterations up to 4 significant figures within the error bound of  $1 \times 10^{-3}$  using  $x_0 = 0$  and any fixed point function  $g(x)$  from part(b) that converges to the root(s).
5. Use Newton's method to find the root,  $x_*$ , of the equation,  $f(x) = x^2 e^{-x} - 0.6$ , up to machine epsilon of  $1 \times 10^{-4}$  starting with  $x_0 = 0.2$ .
6. Let  $f(x) = x^3 + 4x^2 - 10 = 0$ , which can be written as  $g(x) = x$  for some function  $g(x)$ .
  - a. By manipulating  $f(x) = 0$ , find at least three expressions for  $g(x)$  such that  $g(x) = x$ .
  - b. The given function,  $f(x)$ , has one real root which is  $x_* = 1.36523$ . The other two roots are complex and ignore those two roots. Now evaluate the rate  $\lambda$  for the three function,  $g(x)$ , you evaluated in the previous part using the real root. Are these three function,  $g(x)$ , converging or diverging?
7. Use Newton's method to find the solutions for (i)  $f(x) = \sqrt{x} - \cos(x)$  and (ii)  $f(x) = x^2 - 2xe^{-x} + e^{2x}$  starting with  $x_0 = 2.0000$  within  $10^{-5}$ .
8. Consider the nonlinear equation,  $f(x) = x^3 - 7x^2 + 4x + 12$ . Answer the following:
  - a. Find the roots of the given function. Note all three distinct roots are real in this case.
  - b. Construct two different fixed point functions for the given function.
  - c. Find out if the fixed point functions you evaluated in the previous part are converging or diverging. If converging, which root it is converging to.
  - d. Construct a superlinear converging function  $g(x)$  for the given function and computer six iterations starting from (i)  $x_0 = 4$  and (ii)  $x_0 = 0$ . Which root the  $g(x)$  seems to be converging to?
9. A function is given by  $f(x) = x^3 + 2x^2 - x - 2$ . Answer the following:
  - a. By manipulating  $f(x) = 0$ , find at least three expressions for  $g(x)$  such that  $g(x) = x$ .
  - b. The three roots of  $f(x)$  are  $\pm 1$  and  $-2$ . For all your  $g(x)$ 's, compute the rate  $\lambda$  to find if it is converging to any of the roots.

10. Use Newton's method to find the solution of  $f(x) = 1 - 4x \cos(x) + 2x^2 + \cos(2x) = 0$  within  $10^{-5}$  for  $0 \leq x \leq 1$  starting with  $x_0 = 0.25$ .
11. Use Newton's method to find the root,  $x_*$ , of the function  $f(x) = x^2 e^{-x} - 0.5$  up to machine epsilon  $1.0 \times 10^{-4}$  starting with  $x_0 = 0.2$ .
12. A function is given by:  $f(x) = x^6 - x^3 - 2$  which has two real roots and the other roots are complex. Answer the following:
- Construct two fixed point function  $g(x)$  such that  $f(x) = 0$ .
  - Compute the rate  $\lambda$  for the fixed point functions constructed above, and which root it is converging to or diverging
  - Starting from  $x_0 = 60$ , and the converging fixed point function  $g(x)$  that you constructed in the previous part to find the root of the above function accurate up to 3 decimal places.
13. Find the root of the equation,  $f(x) = xe^x - 1$  using fixed point iteration accurate up to machine epsilon of  $1.0 \times 10^{-5}$ . Use the fixed point function  $g(x) = e^{-x}$  and start with  $x_0 = 0$ .
14. Use a secant method to find the root of the equation,  $f(x) = 2x^3 + 7x^2 - 14x + 5$ . Find the root accurate up to 4 decimal places starting with  $x_0 = -5.5$  and  $x_1 = -4.5$ .