BRAC UNIVERSITY

Merul Badda, Dhaka, Bangladesh

CSE331 : Automata and Computability

Assignment 4

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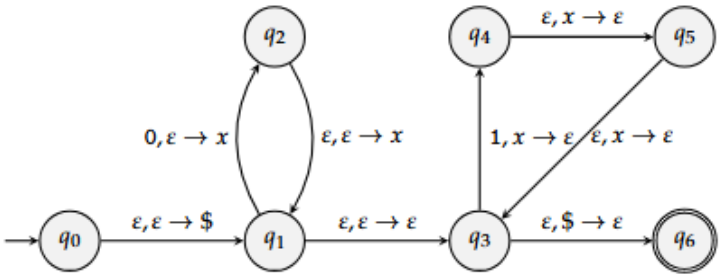
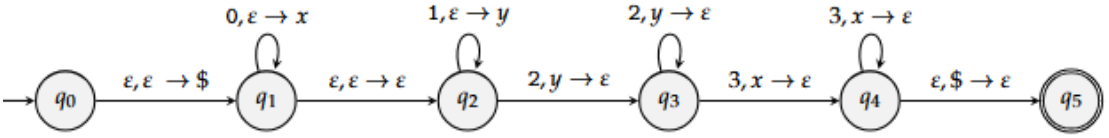
1. Draw the state diagram of a PDA for the following CFL:

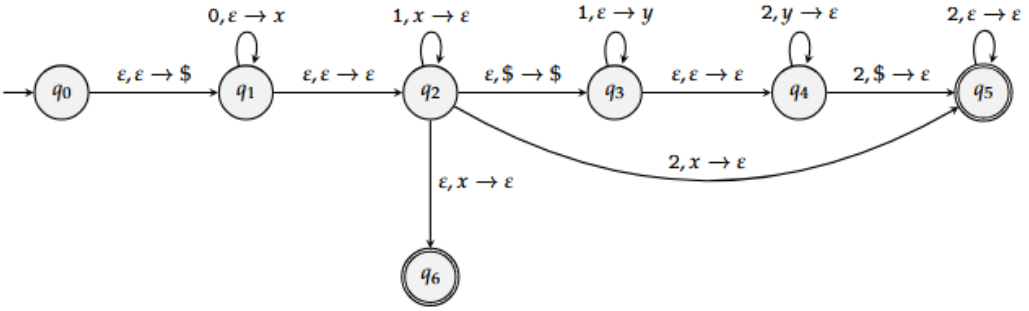
A. $L(M) \rightarrow \{0^n1^m \mid n, m \geq 0 \; and \; 2n = 3m\}, \; where \; \Sigma = \{0, 1\}$



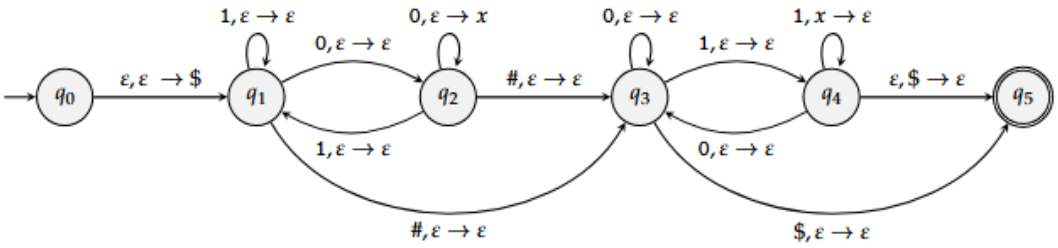
B. $L(M) \rightarrow \{0^n1^m2^m3^n \mid n, m > 0\}, \; where \; \Sigma = \{0, 1, 2, 3\}$



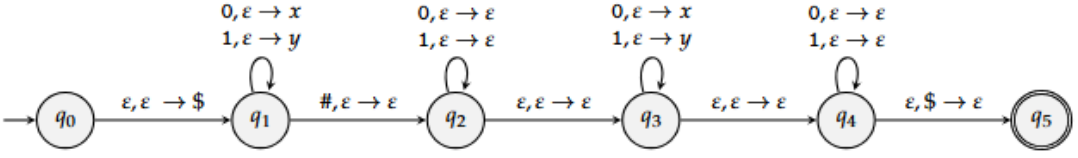
C. $L(M) \rightarrow \{w = 0^i1^j2^k \mid i, j, k \geq 0 \; and \; j < i + k\}, \; where \; \Sigma = \{0, 1, 2\}$



D. $L(M) \rightarrow \{w_1\#w_2 \mid the \; number \; of \; 00 \; in \; w_1 \; is \; equal \; to \; the \; number \; of \; 11 \; in \; w_2\}, \; where \; \Sigma = \{0, 1\}$

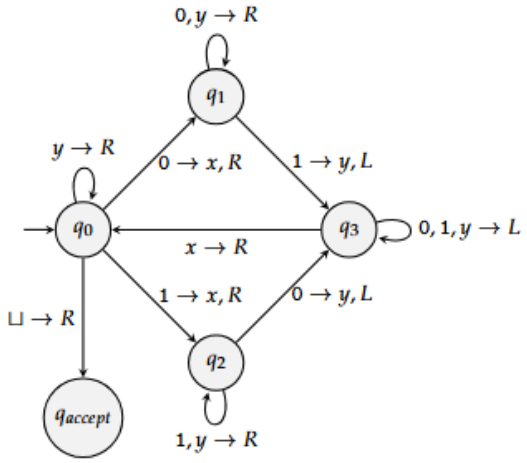


E. $L(M) \rightarrow \{w\#x \mid w^R \; is \; a \; substring \; of \; x\}, \; where \; \Sigma = \{0, 1\}$

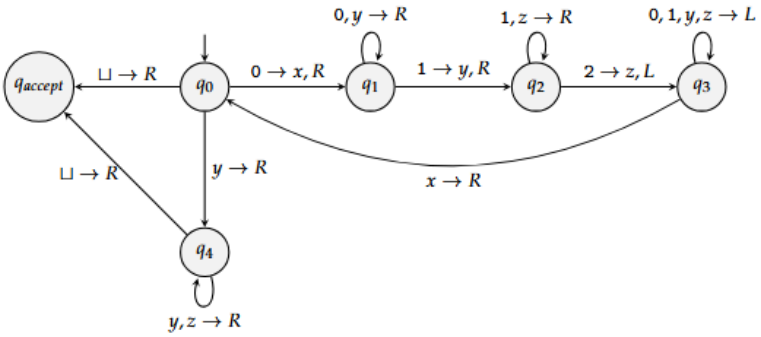


2. Draw the state diagram of a TM that decides the following languages:

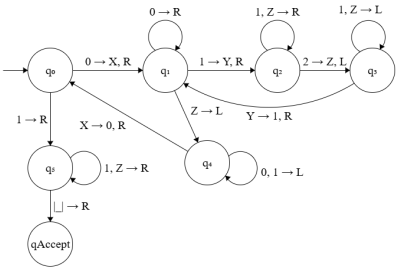
A. $L(M) \rightarrow \{w \in \Sigma^* \mid w \text{ contains equal numbers of } 0s \text{ and } 1s\}, \text{ where } \Sigma = \{0, 1\}$



B. $L(M) \rightarrow \{0^n 1^n 2^n \mid n, m \geq 0\}, \text{ where } \Sigma = \{0, 1, 2\}$



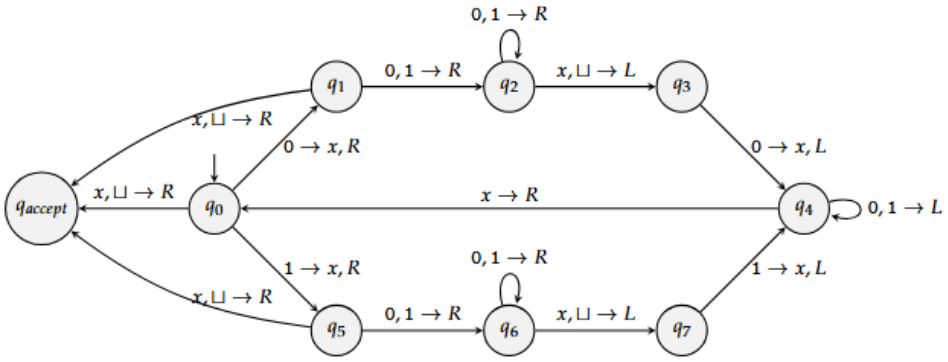
C. $L(M) \rightarrow \{0^i 1^j 2^k \mid i, j, k > 0 \text{ and } k = i * j\}, \text{ where } \Sigma = \{0, 1, 2\}$



D. $L(M) \rightarrow \{0^{2^n} \mid n \geq 0\}, \text{ where } \Sigma = \{0\}$

See example 3.7 from the book.

E. $L(M) \rightarrow \{w \in \Sigma^* \mid w \text{ is a palindrome}\}, \text{ where } \Sigma = \{0, 1\}$



3. Prove that the following languages are decidable.

A. L = {⟨M, w⟩ | The Turing machine M halts on input w within N steps}
   M_N: "On input ⟨M,w⟩:
   1. simulate M on w for at most N steps.
   2. If M halts within N steps, accept. Otherwise reject."
   Since N is finite, M_N is a decider.

B. L = {⟨G⟩ | G is a connected undirected graph}
   See example 3.23 from the book.

4. Show that the following statements are true:

A. The collection of decidable languages is closed under Union.
   See problem 3.15.a from the book.

B. The collection of decidable languages is closed under Complement.
   For any decidable languages L, let M be the TM that decides L. We construct a TM M' that decides the complement of L.
   M'="On input w:
   1. Simulate M on w.
   2. If M rejects, accept. Otherwise, reject."
   Since M is a decider, M' must be a decider.