

**BRAC UNIVERSITY**  
**CSE331 : Automata and Computability**  
**Summer 2025**

**Duration: 30 minutes**

**Quiz 3**

**Total: 20 marks**

**Name:**

**ID:**

0.5 Points

**Section:**

0.5 Points

**Pumping Lemma**

Use the pumping lemma to prove that the following language is not regular: (8 points)

$L(M) \rightarrow \{w \in \{0,1\}^* \mid w \text{ contains more numbers of 0s than 1s}\}.$

Solution:

$$w = 1^p 0^{p+1}$$

For  $i = 2$ ,

$$xy^2z = 1^a 1^{2b} 1^{p-a-b} 0^{p+1}$$

$$\Rightarrow b < 1$$

**CFG**

$L_1 \rightarrow \{w \in \{0, 1\}^* \mid w \text{ contains equal numbers of 0s and 1s}\}$

$$R_1 = 0\Sigma^* 1 \mid 1\Sigma^* 0$$

$$L_2 = L_1 \cap L(R_1)$$

A. Write a CFG for  $L_1$ . (3 Marks)

Answer:

$$S \rightarrow 0S1 \mid 1S0 \mid SS \mid \epsilon$$

B. Convert  $R_1$  into an equivalent CFG. (3 Marks)

Answer:

$$S \rightarrow A \mid B$$

$$A \rightarrow 0X1$$

$$B \rightarrow 1X0$$

$$X \rightarrow 0X \mid 1X \mid \epsilon$$

C. Write all four-letter strings in  $L_2$ . [Note:  $L(R_1)$  denotes the language generated by the regular expression  $R_1$ ] (2 Marks)

Answer: 0011, 0101, 1010, 1100.

D. Write a CFG for  $L_2$ . (3 Marks)

Answer:

$$S \rightarrow 0A1 \mid 1A0$$

$$A \rightarrow 0A1 \mid 1A0 \mid AA \mid \epsilon$$

### **Bonus**

Use the pumping lemma to prove that the following language is not regular: (2 points)

$L(M) \rightarrow \{w_1 w_2 \mid w_1 \text{ is a substring of } w_2 \text{ and } |w_1| \neq |w_2|\}$ , where  $\Sigma = \{0, 1\}$ .

Solution: