

BRAC UNIVERSITY
CSE331 : Automata and Computability
Summer 2025

Duration: 30 minutes

Quiz 3

Total: 20 marks

Name:

ID:

0.5 Points

Section:

0.5 Points

Pumping Lemma

Use the pumping lemma to prove that the following language is not regular: (8 points)

$L(M) \rightarrow \{w \in \{0,1\}^* \mid w \text{ contains more numbers of 0s than 1s}\}.$

Solution:

$$w = 1^p 0^{p+1}$$

For $i = 2$,

$$xy^2z = 1^a 1^{2b} 1^{p-a-b} 0^{p+1}$$

$$\Rightarrow b < 1$$

CFG

$L_1 \rightarrow \{w \in \{0, 1\}^* \mid w \text{ contains equal numbers of 0s and 1s}\}$

$$R_1 = 0\Sigma^*1 \mid 1\Sigma^*0$$

$$L_2 = L_1 \cap L(R_1)$$

A. Write a CFG for L_1 . (3 Marks)

Answer:

$$S \rightarrow 0S1 \mid 1S0 \mid SS \mid \varepsilon$$

B. Convert R_1 into an equivalent CFG. (3 Marks)

Answer:

$S \rightarrow A \mid B$

$A \rightarrow 0X1$

$B \rightarrow 1X0$

$X \rightarrow 0X \mid 1X \mid \epsilon$

C. Write all four-letter strings in L_2 . [Note: $L(R_1)$ denotes the language generated by the regular expression R_1] (2 Marks)

Answer: 0011, 0101, 1010, 1100.

D. Write a CFG for L_2 . (3 Marks)

Answer:

$S \rightarrow 0A1 \mid 1A0$

$A \rightarrow 0A1 \mid 1A0 \mid AA \mid \epsilon$

Bonus

Use the pumping lemma to prove that the following language is not regular: (2 points)

$L(M) \rightarrow \{w_1w_2 \mid w_1 \text{ is a substring of } w_2 \text{ and } |w_1| \neq |w_2|\}$, where $\Sigma = \{0, 1\}$.

Solution: