

CSE 330 Assignment-2

Deadline: 17 November 2025

Question 1

Consider the following table of data points (nodal points):

Time (s)	Velocity (m/s)
2	10
4	20
6	25

- (a) Find an interpolating polynomial of velocity that goes through the above data points using the **Vandermonde Matrix method**. Also compute an approximate value of acceleration at time $t = 7$ seconds.
- (b) Find an interpolating polynomial of velocity that goes through the above data points using the **Lagrange method**.
- (c) If a new data point is added in the above scenario, which method should be used to find a new interpolating polynomial? Also, what will be the degree of that new polynomial?

Question 2

Consider the nodes $[-\pi/2, 0, \pi/2]$. Find an interpolating polynomial of appropriate degree using **Newton's divided-difference method** for the function $f(x) = x \sin(x)$.

Add a new node π to the above nodes, and find the interpolating polynomial of appropriate degree.

Question 3

Consider a function $f(x) = 2(\sin x + \cos x)$ with the given nodes $[-\pi/4, 0, \pi/4]$, where $x_0 = 2$. Keep up to five decimal places in all calculations.

- (i) Calculate the truncation error for differentiating $f'(x_0)$ of the above function using **backward** and **central difference methods** for the step sizes $h = 0.1, 0.01$, and 0.001 .
- (ii) Calculate the upper bound of the truncation error for **forward** and **central difference methods** using $h = 0.1$.
- (iii) Considering the given function and nodes, evaluate the **maximum upper bound of the interpolation error** using Cauchy's theorem. Keep up to five significant figures.

Question 4

The function $f(x) = \frac{1}{1+15x^2}$ shows **Runge's phenomenon** when interpolated by equally spaced nodes over some interval. To overcome this phenomenon, a specific method of choosing nodes can be used.

- (i) Name the method of choosing the nodes to avoid Runge's phenomenon.
- (ii) Calculate the **Chebyshev nodes** over the interval $[-2, 2]$ for degree 3.
- (iii) Consider a function $f(x) = \cos x$ with the given nodes $[0, \pi/2]$. Calculate the **Hermite polynomial interpolation** at $x = 4$.