Formula Sheet

MAT215: Complex Variables & Laplace Transformations

BRAC University

1. **Laplace Transformation of the function** $f(t)$:

$$F(s) = \mathcal{L}\{f(t)\} = \int_0^{\infty} f(t)e^{-st}dt$$

2. **Laplace Transformation table:**

$f(t)$	$\mathcal{L}\{f(t)\} = F(s)$
1	$\frac{1}{s} \quad Re(s) > 0$
t	$\frac{1}{s^2} \quad Re(s) > 0$
$t^n; \quad n = 0, 1, 2, ...$	$\frac{n!}{s^{n+1}} \quad Re(s) > 0$
e^{at}	$\frac{1}{s-a} \quad Re(s) > a$
$\sin(at)$	$\frac{a}{s^2+a^2} \quad Re(s) > \|a\|$
$\cos(at)$	$\frac{s}{s^2+a^2} \quad Re(s) > \|a\|$
$\sinh(at)$	$\frac{a}{s^2-a^2} \quad Re(s) > \|a\|$
$\cosh(at)$	$\frac{s}{s^2-a^2} \quad Re(s) > \|a\|$

Here s is a complex variable and $Re(s)$ indicates the real part of s.

3. **Laplace Transformations of derivatives:**

$$\mathcal{L}\{y^n(t)\} = s^n\mathcal{L}\{y(t)\} - s^{n-1}y(0) - s^{n-2}y'(0) - .... - sy^{n-2}(0) - y^{n-1}(0)$$

4. **First Translation Theorem:** If $F(s) = \mathcal{L}\{f(t)\}$, then
   - $\mathcal{L}\{e^{at}f(t)\} = F(s-a)$
   - $\mathcal{L}^{-1}\{F(s-a)\} = e^{at}f(t) \quad$ or, $\quad \mathcal{L}^{-1}\{F(s)\} = e^{at}\mathcal{L}^{-1}\{F(s+a)\}$

5. **Unit Step Function:**

$$u(t-a) = \begin{cases} 0 & t < a \\ 1 & t \geq a \end{cases}$$

6. **Second Translation Theorem:** If $F(s) = \mathcal{L}\{f(t)\}$ and $a > 0$, then
   - If $\mathcal{L}\{f(t-a)u(t-a)\} = e^{-as}F(s) \quad$ or, $\quad \mathcal{L}\{f(t)u(t-a)\} = e^{-as}\mathcal{L}\{f(t+a)\}$
   - $\mathcal{L}^{-1}\{e^{-as}F(s)\} = f(t-a)u(t-a)$

7. If $F(s) = \mathcal{L}\{f(t)\}$, then $\mathcal{L}\{t^n f(t)\} = (-1)^n \frac{d^n}{ds^n}\{F(s)\}, \qquad$ for $n = 1, 2, 3, ...$

Line Integral

1. **Complex line integral:**
   - $\int_a^b f(z)\,dz = \oint_C f(z)\,dz = \int_{t_1}^{t_2} f(z(t))z'(t)\,dt \quad t_1 \leq t \leq t_2$
   - $\int_{t_1}^{t_2} f(z(t))z'(t)\,dt = \int_a^b \sqrt{[x'(t)]^2 + [y'(t)]^2}\,dt$

2. **Cauchy's Integral Formula:**

   Let $f(z)$ be analytic inside and on a simple closed curve C and let $z = z_0$ be any point inside C. Then,

   $$f^n(z_0) = \frac{n!}{2\pi i}\oint \frac{f(z)}{(z-z_0)^{n+1}}dz$$

3. **Residue Theorem:**

   Let $f(z)$ be single-valued and analytic inside and on a simple closed curve C except at the singularities $z_1, z_2, z_3, ..., z_k$ inside C. Then the residue theorem states that,

   $$\oint f(z)dz = 2\pi i \sum_{i=1}^{k} Re(z = z_i)$$

   where the residues $Re(z = z_i)$ can be calculated by,

   $$Re(z = z_i) = \lim_{z \to z_i} \frac{1}{(m-1)!}\frac{d^{m-1}}{dz^{m-1}}\{(z-z_i)^m f(z)\}$$

   where, $z = z_i$ is a pole of order m.