18BAMO4

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§1 Solution

Solution. Take a prime $p \mid abc$ and let $x = \nu_p(a)$,s $y = \nu_p(b)$, $z = \nu_p(c)$. It is enough to prove $x + y + z \equiv 0 \pmod 3$.
Notice that if $x = y = z$ we are done, so assume this is not true. Then $\nu_p(a/b) = x - y$, $\nu_p(b/c) = y - z$, $\nu_p(c/a) = z - x$. One of them have to be negative, which is not possible. But we have $\nu_p(a/b + b/c + c/a) \geq 0$, so we conclude that the two smallest numbers among $\{x - y, y - z, z - x\}$ must be equal.
To finish WLOG assume that if $x - y = y - z$, then $2y = x + z$ and so $x + y + z \equiv 0 \pmod 3$.

$\square$