

13SMTA9

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§1 Solution

Solution.

$$\begin{aligned} & \frac{a^4}{(a-b)(a-c)} + \frac{b^4}{(b-a)(b-c)} + \frac{c^4}{(c-a)(c-b)} \\ &= \frac{-a^4(b-c) + -b^4(c-a) + -c^4(a-b)}{(a-b)(b-c)(c-a)} \end{aligned}$$

$P(a, b, c) = -1 \cdot (a^4(b-c) + b^4(c-a) + c^4(a-b))$, which is clearly cyclic.

Also, $P(a, a, c) = 0 \implies (a-b)(b-c)(c-a) \cdot Q(a, b, c) = P(a, b, c)$, where Q is of degree 2 and cyclic. Thus, $Q = k(a^2 + b^2 + c^2) + l(ab + bc + ca)$, after some trivial checking we can conclude that $k = l = 1$. So, the answer is 30. □