# 14PTNMA4

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## §1 Solution

*Solution.* Denote by $p_i, i \in \mathbb{Z}$ the probablity of $X = i$.
Notice that

$$\mathbb{E}[(X-1)^2(X-3)] = \mathbb{E}[X^3 - 5X^2 + 7X - 3] = -1$$

$$= -3 \cdot p_0 + 2 \cdot p_2 + \cdots \implies -3 * p_0 \leq -1 \implies p_0 \geq \frac{1}{3}.$$

The **construction** is $p_0 = \frac{1}{3}, p_1 = \frac{1}{2}, p_2 = 0, p_3 = \frac{1}{6}, p_4 = p_5 = \cdots = 0$.

□