02AMO3

Himadri Mandal

December 27, 2021

§1 Solution

Solution. We can guarantee n real roots with n points by IVT easily. Now, consider sequences a_i, b_i with $a_i + b_i = 2f(i)$, such that $\operatorname{sgn}(a_i) = (-1)^i, \operatorname{sgn}(b_i) = (-1)^{i+1}$, with $|a_i|, |b_i|$ being sufficiently large. Then interpolating on $\{a_1, \cdots, a_n\}$ we would get some g_1 with $\deg g_1 = n - 1$, to get a monic polynomial now $g = (x-1)\cdots(x-n) + g_i$. We can do something similar for h. Now taking $f = \frac{1}{2} \cdot (g + h)$ finishes.

□