16HMMTA7

Himadri Mandal

November 12, 2021

§1 Solution

Solution. $A(n+1) = A(n)10^{\lfloor 10(n+1)\cdot\log_{10} 2\rfloor+1} + 2^{10(n+1)}$
Notice that

$$\lfloor 10(n+1)\log_{10}(2)\rfloor + 1 \approx \lfloor (n+1)\cdot 3.01\rfloor + 1 \approx 3(n+1)+1$$

and the equality holds for small $n(\approx 40)$. Therefore,

$$A(n+1) = A(n)\cdot 10^{3(n+1)+1} + 2^{10(n+1)}$$

for small n. Now, notice that $\nu_2(A(n+1)) \geq \min(10(n+1), 3(n+1)+1+\nu_2(A(n)))$
Working some values out we see that,

n	1	2	3	4	5	6	7	...	n
$\nu_2(A(n))$	10	17	27	$\geq$40	50	60	70	...	$10n$

As we want n such that $A(n) \equiv 2^{10n} \pmod{2^{170}}$, we want the smallest n such that $\nu_2(A(n-1)) + 3n + 1 = 10(n-1) + 3n + 1 = 13n - 9 > 170 \implies n \geq 14$ for $n \geq 5$. But after checking we see that $n = 5$ doesnt work. So the answer is 14. $\square$