11SMTA7

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§1 Solution

Solution. Define $Q(x) = P(2x) - P(x) - 1$, clearly $Q(1) = Q(2) = \cdots = Q(2^{2010}) = 0$, which are the only ones because Q has degree 2011. $Q(x) = k \cdot (x-1)(x-2) \cdots (x-2^{2010})$, comparing the constant term $k = \frac{1}{2^{1+\cdots+2010}}$. So the coefficient of x in P is $2 - \frac{1}{2^{2010}}$ $\quad\square$