12SLC2

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October 26, 2021

§1 Solution

Solution. I claim the answer is $\left\lfloor \frac{2n-1}{5} \right\rfloor$.
Maximality: Call $f(n)$ the maximum number of possible disjoin pairs. Double counting on the sum of elements of pairs and bounding gives

$$\frac{n \cdot (n+1)}{2} - \frac{(n-f(n))(n-f(n)+1)}{2} - \frac{(4f(n)(f(n)+1)}{2} - f(n)) \geq 0$$

$$\implies f(n) \leq \left\lfloor \frac{2n-1}{5} \right\rfloor$$

Construction: For $n = 5q+3$, consider the following pairs

$$(4q+2, 1), (4q+3, 3) \cdots (3q+2, 2q+1) \text{ and } (3q+1, 2) \cdots (2q+2, 2q)$$

.
This clearly also works for $5m+4, 5m+5, 5m+6, 5m+7$. So we are done. □