68PTNMA6

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§1 Solution

Solution. Let the roots be $\lambda_1, \cdots, \lambda_n$, then $\sum_i \lambda_i^2 = a_{n-1}^2 - 2a_{n-2} = 3$ but also by AMGM $\prod_i \lambda_i^2 = 1 \implies \sum_i \lambda_i^2 \geq n\sqrt{\prod_i \lambda_i^2}$. So, $n = \{3, 2, 1\}$. For $n = 3$, checking $\{x^3 \pm x^2 - x \pm 1\}$ we see that $P(x) = x^3 - x \pm (x^2 - 1)$ work. $n = 2, 1$ are easy. So the final set of solutions are $P(x) = \{x \pm 1, x^2 - 1 \pm x, x^3 - x \pm (x^2 - 1)\}$ □