# 16SLC5

Himadri Mandal

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## §1 Solution

Sorry for the wording Evan, even though I know you wouldn't mind, couldn't resist myself >_<.

*Solution.* I claim the answer is $n-3$ when $n$ is odd and $n-2$ otherwise. $n-3$ is quite easy to prove, just notice that there can be no intersecting diagonals, so the trivial triangulation is the best we can do. The construction for evens is to triangulate in order from a vertex and then join the farthest 2 vertices.

Now lets prove the statement for evens. Call any diagonal which intersects in the polygon "hot". If we have 2 intersecting hot diagonals $m_1, m_2$, then clearly every other hot diagonal is parallel to either $m_1$ or $m_2$.

> **Claim 1.1** — The longest hot diagonal in an orientation cannot be a part of a hot rectangle.

*Proof.* If it were then, the arc *not* in the region of the biggest hot rectangle would have lesser no. of diagonals because we can always make a new hot diagonal which doesn't disturb the triangulation of the arc. □

Assume there are $\gamma$ hot diagonals. Now, notice that there are atmost $n-(\gamma+2)$ virgin vertices which don't touch a hot diagonal.

> **Claim 1.2** — Each virgin vertex contribute atmost 1 diagonal.

*Proof.* If there are $\geq 2$ virgin vertices in that virgin arc then clearly a triangulation does the job. There cannot be a single virgin vertex in any such arc because if there was one it would mean the adjacent vertices are hot-touching but that would contradict Claim 1.1. □

So we have atmost $\gamma + (n-(\gamma+2)) = n-2$ diagonals. □