JLAMMY

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§1 Solutions

Solution. I claim that 2026 is the actual lower limit.
Construction: Consider the set

$$A = \{-44, -43, ..., 45\}$$

It is easy to see that sum of elements of nonempty subsets of A achieves all numbers $x : -990 \leq x \leq 1035$.
Minimality:
Note that there are atleast $\frac{m(m+1)}{2}$ different sums in $\{a_1, ..., a_m\}$ with $0 < a_1 < a_2 < ... < a_m$ or the other way around. It is easy to see that if $0 \in A$ then we would have lower num of sums. So if there are a positives and b negatives in A then $S \geq \frac{a(a+1)}{2} + \frac{b(b+1)}{2}$ and $a + b = 89$. Which achieves minimum at 2026. □