# 05SLN3

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## §1 Solution

*Proof.* Simple?

> **Claim —** $((a,b,c),(d,e,f))$ works if and only if $((a+1,b+1,c+1),(d-1,e-1,f-1))$ does.

*Proof.* Because

$$(a+1)(b+1)(c+1)+(d-1)(e-1)(f-1)=abc+def+ab+bc+ca-de-ef-fd$$

and

$$(a+1)(b+1)+(b+1)(c+1)+(c+1)(a+1)-(d-1)(e-1)-(e-1)(f-1)-(f-1)(d-1)$$
$$=ab+bc+ca-de-ef-fd+2(a+b+c+d+e+f)$$

□

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This implies that $S|(a+d)(b+d)(c+d)$ where $d=\min(d,e,f)$, assume $S$ is prime, $a+b+c+d+e+f=a+d$, which is absurd because positive integers. □