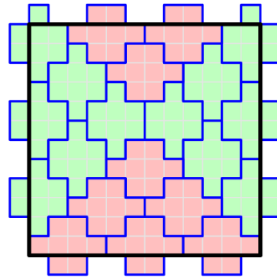# 19EGMO2

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§1 Solution

Solution. The answer is $\frac{n(n+1)}{2}$.

> **Claim 1.1 —** Construction

Proof. Consider this easily generalisable construction: (credits to @vEnhance for image)

$\square$

Call the cells adjacent to a domino its **cloak**. Note that a domino's cloak might have $\{5, 6, 7, 8\}$ cells, let there be a cells with cloak 5, b with 6, c with 7 and d with 8. We have $5a + 6b + 7c + 8d = 4n^2$.

> **Claim 1.2 —**
> $$4a + 4b + 3c \le 4(2n - 1)$$

Proof. Pretty straight forward, cells with cloak 5 cover 4, 6 cover 4 and 7 cover 5, cells with cloak 8 *don't* lie on the border. $\square$

To finish,

$$4n^2 + 2(2n - 1) \ge (5a + 6b + 7c + 8k) + \frac{1}{2}(4a + 4b + 3c)$$

$$= 8\sum_{\text{cyc}}(a) + \frac{1}{2}c - a \ge 8\sum_{\text{cyc}}a - 4 \implies \sum_{\text{cyc}}(a) \le \frac{n(n + 1)}{2} + \frac{1}{4}$$

$\square$