

11CSUR6

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§1 Solution

Solution. I claim the answer is $(2n + 1)(n + 1)$.

Construction is simple enough: color the board with black stripes alternately.

To prove this we will induct. Call the board $Q(n)$. Color $Q(n)$ as shown below, and call the yellow part $Q(n - 1)$ and the green part $L(n - 1)$.



Denote by $|Q(n)|, |L(n)|$ the maximum amount of black cells possible with the given coloring.

We know that

$$|Q(n)| \leq |Q(n - 1)| + |L(n - 1)|$$

.

Claim 1.1 — $|L(n - 1)| = 4n + 1$

Proof. This is actually pretty easy just induct. □

Now we will finish, assume $|Q(n - 1)| = (2(n - 1) + 1)((n - 1) + 1)$ note that

$$(2(n - 1) + 1)((n - 1) + 1) + (4n + 1) = (2n + 1)(n + 1).$$

But that means $|Q(n)| \leq (2n + 1)(n + 1)$. So we are done. □