18RMM2

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§1 Solution

Solution. No. Assume yes, then $\deg P = 2.1 \deg Q$. Derivating the equation

$$P'(x)P(x)^8 \cdot (10P(x) + 9) = Q'(x)Q^{19}(x) \cdot (21Q(x) + 20)$$

$$10P(x) + 9 \mid Q'(x)Q^{10}(x) \cdot (21Q(x) + 20)$$

as $\deg P > \deg Q' + \deg Q = 2 \cdot \deg Q - 1$, we would be done if we show

$$\gcd(10P(x) + 9, Q^{10}(x)) | \gcd(10P(x) + 9, Q(x)^{21} + Q(x)^{20})$$

$$= \gcd(10P(x) + 9, P(x)^{10} + P(x)^9) = C,$$

a constant by the euclidean algorithm. But this means there cannot exist such P, Q. $\square$