

# H1883820

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## §1 Solution

*Solution.* Let  $P(x) = x^3 + a_2x^2 + a_1x + a_0$  be a polynomial which has roots  $a, b, c$ .  
 $p_i = a^i + b^i + c^i$ , so we have

$$p_3 = -a_2^3 + 3a_2a_1 - 3a_0$$

$$p_5 = -a_2^5 + 5a_2^3a_1 - 5a_2^2a_0 - 5a_1^2a_2 + 5a_1a_0$$

If  $p_3 = p_1^3 = -a_2^3 \implies a_2a_1 = a_0$ , thus,

$$p_5 = -a_2^5 + 5(a_2^3a_1 - a_2^2a_0) - 5(a_1^2a_2 - a_1a_0) = -a_2^5$$

On the other hand, if  $p_5 = p_1^5 \implies a_1a_2(a_2^2 - a_1) = a_0(a_2^2 - a_1)$

Thus,  $a_1a_2 = a_0$  or  $a_2^2 = a_1$ , first case is trivial. For the second case, we have

$(a + b + c)^2 = ab + bc + ca \implies a = -b, b = -c, c = -a \implies a = b = c = 0$ , for which the statement obviously holds true.  $\square$