Week 5 Assignment

Problem 1 - Phase Plots:

You are given the force F as function of position x, acting on a point particle of mass m.

$$F(x) = -4\,e^{-0.2(x-5)}(1 - e^{-0.2(x-5)})\ , \quad U(5) = 10$$

$$m = 1kg$$

1. You are expected to plot the potential energy U of the particle, as a function of its position x. (Note that $F(x) = -\frac{dU(x)}{dx}$. Take x as $np.arange(0, 50, 0.1)$)

2. If the total energy of the particle is constant and represented by E, from the $U(x)\ vs\ x$ curve, identify two different values E_1 and E_2 (satisfying $E_1 < E_2$), such that:

- $\forall E \in (E_1, E_2)$: The phase plot is a closed curve (Well, that seems familiar xD)
- $\forall E \in (E_2, \infty)$: The phase plot is an open curve (You might wonder why!)
- $\forall E \in (-\infty, E_1)$: The phase plot is not real (What does that even mean? Well, in this region, E represents a classically forbidden value, which makes the kinetic energy $K < 0\ \forall x \in (-\infty, \infty)$)

3. Plot the phase curves for the first two cases, in a single plot, taking any valid value of E in the corresponding ranges. Don't forget to plot the legend, mentioning the values of E chosen.

Hints: For calculating $U(x)$ from $F(x)$, use the Runge-Kutta method. Since the initial condition is given at $x = 5$, you would need to apply the method in two seperate for loops: one for setting the U values for $x \in (5, 50)$, and other for setting those for $x \in [0, 5)$
For the phase plots, you need the velocity v of the particle as a function of x. For this purpose, first find out the kinetic energy $K(x)$, using E and $U(x)$. Be careful to only use the points having a positive value in the $K(x)$ array, to generate the $v(x)$ array (Since negative values are classically forbidden). Further, use only the corresponding points from the x array, for plotting the phase plot.

Problem 2 - Concept of Damping in RLC circuits:

You are given a simple series RLC circuit, with no power supply. The specifications are :

$$R = 2\Omega,\ L = 1H,\ C = xF$$

1. You have to find the value of the unknown capacitance x, such that you achieve a critically damped circuit (Note that once you have the x value for the critically damped case, just changing it would take you to either the overdamped or the underdamped case, depending on the direction you make the change in).

2. For the initial conditions $q_0 = 50.0C,\ i_0 = 0.0A$, and the time t array taken as $np.arange(0, 20, 0.05)$ produce the following 6 plots:

- The first two plots should correspond to the '$q\ vs\ t$' and '$i\ vs\ t$' plots respectively, for the overdamped case.
- The next two plots should correspond to the '$q\ vs\ t$' and '$i\ vs\ t$' plots respectively, for the critically damped case.
- The last two plots should correspond to the '$q\ vs\ t$' and '$i\ vs\ t$' plots respectively, for the underdamped case.

Note: For each of the 6 plots, mention which damping case it represents, along with the x value used, in the title.

Hint: First write the differential equation, using Kirchoff's circuit laws. Then identify the relation between R, L and C, for the three different damped cases. Choose the x values accordingly. Use the odeint package for solving this problem.