Week 4 Assignment

There are three problem statements this week. All of them are based on the 2D Heat Equation and the simulation shown in the demonstration session. In this set of questions, you just have to play around with various boundary conditions. By doing these questions, you'll truly be able to appreciate the beauty of the heat equation and how it can lead to different solutions, just by changing the boundary conditions!

Problem 1 - Dirichlet Boundary Conditions:
For a rectangular plate of length "l=2m" (horizontal) and width "w=1m" (vertical), simulate the steady state temperature contours for the following boundary conditions:

- Temperature at the top edge is (100 + 50sin(2*pi*x/l)) degree Celsius (constant in time)
- At all the other edges, the temperature is maintained at 100 degree Celsius

Hint: Instead of initializing the top row of the array to all constant values, set them in accordance with the given sinusoidal formula. Also keep in mind the rectangular shape of the plate (Do not get rid of (dx)^2 and (dy)^2 in the FDM formula)

Problem 2 - Neumann Boundary Conditions:
For a square plate of edge length "l=1m", simulate the steady state temperature contours for the following boundary values:

- Temperature at three edges is held constant (100 degree Celsius at the top edge, 50 degree celsius at the bottom edge, and 0 degree celsius at the right edge)
- The left edge is perfectly insulating, i.e. there is no heat flux. (This is to say that the derivative of the temperature wrt the horizontal direction is 0 at the left edge)

Hint: In each iteration, first calculate the temperatures at the interior grid points as usual. Then update the temperatures at the left edge according to the temperatures at their horizontal right hand side neighbours.

Problem 3 - Point Source-Sink(wait, what!?!?) (Oscillating Temperature):
For a square plate of edge length "l=1m", simulate the continuous evolution of the temperature map (with a time step of 0.01s, for a total of 2s) with the following conditions:

- Temperature at all four edges is 0 degree Celsius
- There is a point source (occasionally sink, you'll realise soon why!) at the center, with a time varying temperature given as 10sin(pi*t) (Assume that steady state is achieved instantaneously with each time step, so you need not worry about the transient period. That is to say, that at each iteration, the heat map plotted represents the steady state)

Hint: At the beginning of each iteration, first set the temperature of the center grid point, using the sinusoidal formula. Then proceed to update the temperatures of all the ***other*** *interior grid points. Also, produce the plot at each iteration.*

Note: For all the questions, you may take 0.1m as your step size in both the x and y directions. Also after you're done with the assignment submission, we encourage you to tinker with the temperature values at the boundaries and enjoy pleasing visualisations!