Proof of Lemma 1 and Theorem 1 in "Achieving Scalable and Efficient Routing in LEO Constellations"

Lemma 1. *Starting from satellite* $\langle p, s \rangle$*, after traversing* N *continuous inter-orbit hops and getting back to source orbit again, i.e., the* p*-th orbit, the satellite number of the reached satellite in the* p*-th orbit is* $((s-(N-F)+c*M) \bmod M)$*, where* c *is a non-negative integer to ensure satellite number* $\in [0, M-1]$.

Proof. According to the definition of phase factor F, if satellite $\langle p, s \rangle$ is connected to satellite $\langle p+1, s-1 \rangle$ via an inter-ISL, the phase offset caused by one inter-orbit hop is $(2\pi/M/N)*(N-F)$. So after N continuous inter-orbit hops, the accumulative phase offset equals $(2\pi/M)*(N-F)$. Because $(2\pi/M)$ equals the angle between two adjacent satellites in the same orbit, the number of intra-orbit hops between the source satellite and the satellite that returns to the source orbit after N hops is $N-F$. Therefore the satellite number equals $((s-(N-F)+c*M) \bmod M)$, where c is a non-negative integer to make sure the satellite number $\in [0, M-1]$ $\square$

Theorem 1. *Assume an inter-ISL connecting two satellites from the two adjacent orbits* 0*-th and* $(N-1)$*-th orbit, the satellite numbers of these two satellites,* s_0 *and* s_{N-1}*, satisfy:*

$$s_0 = (s_{N-1} + F - 1) \mod M, \tag{1}$$

$$s_{N-1} = (s_0 - F + 1 + c*M) \mod M. \tag{2}$$

Proof. We first suppose s_{N-1} is known and derive the s_0 from s_{N-1}. Considering an extended data forwarding path which travels the sequence of orbits $0 \to 1 \to 2 \to \cdots \to (N-2) \to (N-1) \to 0$. The last hop from the $(N-1)$-th orbit to the 0-th orbit is the inter-ISL of our interests and we need to derive the satellite number s_0 of the satellite on the second 0-th orbit in the sequence. According to *Observation 1*, the satellite number of the satellite on the first 0-th orbit can be derived as $s_{N-1}+(N-1)$. Based on *Lemma 1*, s_0 is obtained as:

$$\begin{aligned} s_0 &= (s_{N-1} + (N-1) - (N-F)) \mod M && (3)\\ &= (s_{N-1} + F - 1) \mod M. && (4) \end{aligned}$$

The derivation of s_{N-1} given s_0 is similar to the above process, which is omitted here. $\square$