Security Selection: Handle Combinatorial Explosion

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Abstract: Fixed income portfolio construction and rebalance can often be defined as a task to enhance an index with a set of return signals while restricting the number of positions held in the portfolio. The task is multi-objective: maximizing signals, tracking index, complying constraints and selecting only subset of securities from the investment universe. The particular hard objective to implement is security selection due to the combinatorial search space. In this document we propose a novel heuristics algorithm to perform security selection and build close-to-optimal portfolios in a reasonable amount of time. The new algorithm enables us to replicate Bloomberg US Credit Index (>7000 bonds) using 100 liquid cash bonds in 4 minutes with tracking error 30bps. The algorithm also enables us to build index-enhancing portfolios against US IG Technology sub-index using return signals, achieving 1.5 Sharpe Ratio compared with benchmark 0.6 in the backtest period. In this document, we also propose new methods to handle categorical signal such as analyst recommendations, and combine signals using optimization model sensitivity analysis.

Keywords: Portfolio Optimization, Integer Programming, Heuristics, Index-replication, Index-enhancing, Signal Combining

1 Introduction

In the context of portfolio optimization, integer or discrete constraints refer to a special class of constraints that control the number of assets or trades in a portfolio, or set a minimum requirement on the holding levels for assets or the transaction levels for trades. Definition and use case of various integer constraints are given in Appendix A.

Quantitative portfolio managers frequently employ integer constraints to select securities from a large investment universe to construct or rebalance their portfolios. Security selection can be formulated by introducing a number of 0-1 integer variables. However these constraints also change the nature of an optimization problem from a well-behaved convex quadratic programming problem to a more complicated discrete quadratic programming problem. Commercial solvers have implemented various branch-and-bound and branch-and-cut (G. Cornuejols, R. Tütüncü 2007) algorithms for integer constraints, but take exponential running time and can handle only small-scale problems (usually less than 200 securities in the investment universe). As a result, various heuristics have been proposed in the literatures: Diving and Rounding (T. Berthold 2006); Genetic Algorithm, Simulated Annealing and Tabu Search(T.J. Chang, N. Meade, J.E. Beasley and Y.M. Sharaiha 2000); Lagrangian Relaxation (S. Diamond, R. Takapoui, and S. Boyd 2017). Some of the heuristics have been incorporated into commercial solvers, but none of them enables commercial solvers to solve discrete portfolio optimization problems of practical size in deterministic time.

In this document, we propose a novel heuristics based on the principle of diversification specific to the field of portfolio optimization. The proposed approach exploits correlations of an investment universe in order to group similar securities together and turn security selection into cluster selection. Commercial solvers usually can solve to optimality for the cluster selection problem that has reduced dimensionality. Result of cluster selection is then used to prune and restrict the investment universe to achieve a close-to-optimal solution to the original security selection problem in a reasonable amount of time.

The rest of the document is organized into four sections. Section 2 presents the clustering heuristics through a small example. Section 3 illustrates the use of heuristics in passive and active investment strategies. Backtest results are reported in section 4. Concluding remarks follow in Section 5

2 Clustering Heuristics for Integer Constraints

Investment themes can be broadly classified into passive and active management. The goal in passive portfolio management is to track a benchmark as closely as possible with a subset of securities in the index. This approach can be translated into a portfolio optimization problem that minimizes tracking error (TE) while restricting the number of positions held in the portfolio (See model 1).

$\underset{x}{Minimize}\ \ (x - x_b)^T \Sigma (x - x_b)$ *(1)*

Subject to: $lb * z \leq x \leq ub * z$ *(Selection and Bound)* *(1.1)*

$\sum x = 1$ *(Holding)* *(1.2)*

$\sum z = k$ *(Number of Assets)* *(1.3)*

$AX \leq b$ *(Any linear constraints)* *(1.4)*

x continuous, z binary

Where:

x: *Decision variable: weight of bond to be added to the replicating portfolio*

z: *0-1 decision variable.* $z_i = 1$ *indicates bond i will be added to the replicating portfolio*

x_b: *Weight of bond in benchmark*

Σ: *Bond-by-bond covariance matrix*

lb, ub: *Lower and upper bound for x*

k: *Number of bonds in the replicating portfolio*

In contrast, active portfolio management is to outperform the benchmark by optimally trading off return signals against portfolio risk to enable a wider space for return generation. In fixed-income portfolio management, most strategies fall somewhere between passive and active management, the so-called index-enhancing strategy. Index-enhancing can be translated into portfolio optimization problems in which the objective is to optimize a set of return signals while restricting the number of positions held in the portfolio and satisfying TE budget (see model 2)

$$\underset{x}{Maximize} \quad (signal)^T * x - \lambda * (penalty)^T * x \qquad (2)$$

Subject to: $lb * z \leq x \leq ub * z$ *(Selection and Bound)* *(2.1)*

$\sum x = 1$ *(Holding)* *(2.2)*

$\sum z = k$ *(Number of Assets)* *(2.3)*

$AX \leq b$ *(Any linear constraints)* *(2.4)*

$(x - x_b)^T \Sigma (x - x_b) <= TE\_MAX$ *(TE budget)* *(2.5)*

x continuous, z binary

Where:

$signal$*: Return signal, e.g. carry*

$penalty$*: Any undesirable performance measures, e.g. transaction cost*

λ*: Tradeoff parameter between signal and penalty*

TE_MAX*: TE budget*

As mentioned in the previous section, both index-tracking Model (1) and index-enhancing Model (2) introduce integer constraints and lead to combinatorial explosion in the search space. For instance, there are 10^{85}different ways of choosing 50 bonds out of 1000, whereas the estimated number of atoms in the observable universe is only around10^{82}. In order to reduce the search space, we merge the bonds in the investment universe into clusters based on the following observations (P. Glabadanidis 2012):

- Selected securities that are highly correlated with each other tend to increase the TE
- Selected securities that correlate more highly with the benchmark tend to reduce the TE

A natural idea is to group securities into clusters based on the security correlation matrix, then solve the original models by substituting the bonds with clusters:

$$\underset{x^g}{Maximize} \quad (score)^T * x^g \qquad (3)$$

Subject to: $x^g \leq z^g$ *(Selection)* *(3.1)*

$\sum z^g = k$ *(Number of Assets)* *(3.2)*

$\left(x^g - x_b^g\right)^T \Sigma^g (x^g - x_b^g) <= TE\_MAX$ *(TE budget)* *(3.3)*

x^g *continuous,* z^g *binary representing cluster selection*

The cluster selection problem, Model (3), has much lower dimension compared with Model (2). For instance, we may group 1000 securities in the investment universe into 100 clusters. Model (3) selects 50 clusters out of 100, reducing the searching space to10^{29}. Commercial solvers (CPLEX/GUROBI/MOSEK) usually can solve problem of this size in seconds. Note there is a new symbol *score* in the objective function. We will explain it in subsection 2.1.

Solutions to Model (3) are the selected clusters. The selected clusters are first used to prune the investment universe. Assuming equal number of securities in each cluster, the unselected 50 clusters (500 securities) will be dropped, leaving 500 securities in the final investment universe. Model (2) with additional pick-one-from-each-cluster constraints in the reduced universe is solved finally:

$$\sum_{j \text{ in cluster } i} z_j = 1, i = 1,2, \ldots k. \qquad (4)$$

This type of constraints is called SOS1 (CPLEX 2017), and can effectively reduce the searching space to 10^{50}(select 1 out of 10 in each 50 clusters). We refer to the final model as Model (4).

2.1 Cluster Selection

We have illustrated the steps for the proposed heuristics, but there are still a few questions that remain unanswered:

- How to cluster the securities?
- How to build cluster-by-cluster covariance matrix Σ^g used in Model (3)?
- How to decide the cluster score in the objective function of Model (3)?

2.1.1 Clustering Algorithms

Cluster analysis is the task of grouping a set of objects such that the objects within a group are more similar to each other than the objects in other groups. It is commonly used in statistical data analysis and classified as unsupervised learning in the machine learning community (T. Hastie, R. Tibshirani, and J. Friedman 2009). Clustering methods can be divided into two basic types: hierarchical and partitional. Hierarchical clustering either merges smaller clusters into larger clusters or splits larger clusters into smaller ones. The recently proposed Hierarchical Risk Parity approach for building diversified portfolios (M. L´opez de Prado 2016) is based on hierarchical clustering. The best known partitional method is k-means clustering, which partitions objects into clusters so that each object belongs to the cluster with the nearest mean. In addition to these two systematic approaches, we can also simply group bonds by appropriate bond attributes such as issuer.

k-means clustering requires a distance function to ensure the distance between two highly correlated securities is small. We use the same distance function in (M. L´opez de Prado 2016):

$$d_{ij} = \sqrt{\frac{1}{2}(1-\rho_{ij})}$$

Both hierarchical and partitional clustering implementations are available from open sources such as scikit-learn.

2.1.2 Cluster Covariance Matrix

Risk attribution breaks down TE in terms of its contributions from a given set of factors. However portfolio managers often need to monitor portfolio exposures in term of custom linear combined factors such as PCA shift movement or 2-10 slope of a curve. Mathematically custom factors can be defined as:

$\hat{F} = PF$ where $F^{M\times 1}$ is the original factor realization, $P^{K\times M}$ specifies linear combination of F

The number of customized factor K is required to be no more than M, the number of original factors. There are three cases:

1. *K* = *M*. Full risk attribution to customized factors is possible.
2. *K* < *M and weights in P is from the same universe to which risk attribution is performed*. Full risk attribution to customized factors is possible.
3. K < M. Only partial risk attribution is possible, i.e. residual risk exists.

If we aggregate securities to clusters using the capitalization-weighted method and index is also capitalization-weighted, the cluster aggregation falls into case 2. Therefore we can turn bond covariance matrix Σ into cluster covariance matrix Σ^g using bond capitalization weight (same as index weight).

2.1.3 Cluster Score Function

Note for the cluster selection problem Model (3), signal and penalty terms in the objective, and holding and general linear constraints are dropped. Instead a term called *score* appears in the objective, representing how attractive the cluster is.

We first compute the score for each *security*:

- Contribution to objective (O_i): as in Model (2) it is computed as signal adjusted by penalty
- Contribution to infeasibility (f_i): let vector s denote the vector of shadow prices for constraints $AX \leq b$, then the contribution to infeasibility from bond i is $f_i = s^T A_{.,i}$. Please refer to (G. Cornuejols, R. Tütüncü 2009) for shadow price definition. We can understand f_i in the following way: optimization model constraints define how much resources are available for securities, and f_i is the amount of resources consumed by security i.

We then can define the score for a *cluster* as $Decile(O_i - f_i)$, i.e. the first of 10-quantiles of security scores sorted in decreasing order in a cluster. The final step in the heuristics, Model (4), chooses only one security from each group. Therefore the attractiveness of a cluster is decided by only a few top-ranked securities in the cluster.

If the optimization problem to solve has many feasible solutions, use just contribution to objective O_i as security score; If it is hard to find a feasible solution for the optimization problem, use only contribution to infeasibility $-1 * f_i$; Use $O_i - f_i$ to balance optimality and feasibility.

2.1.4 Variations of the Heuristics

The heuristics can be implemented with a couple of variations in addition to the original algorithm (call it **H0**):

- **H1**: If we group securities into k (the desired cardinality) clusters, we don't need to solve cluster selection problem model (3). Instead just solve the final Model (4).
- **H2**: Instead of running cluster selection Model (3), we can select one security from each cluster to come up with a reduced universe and solve Model (2). This variation has computational advantage with the cost of solution quality.

2.2 Illustrative Example:

We construct an index-enhancing portfolio for the below 5-bond benchmark using 2 bonds and 50bps TE budget. The objective is to maximize carry while matching benchmark PMV and MWS. Solution to replicating portfolio (Model 5) is *02342TAE9 (0.14)* and *11134LAF6 (0.86).*

SSM_ID	ISSUER	CLUSTER	PMV(x_b)	MWS	CARRY
02342TAE9	DOX	2	0.1	11.9	172.0
11134LAF6	EDM30167952	1	0.2	1.7	52.0
11135FBC4	EDM30167952	1	0.3	3.6	97.2
458140AM2	INTC	3	0.3	0.5	3.0
882508AW4	TXN	3	0.1	0.6	10
Benchmark			1.0	3.1	61.1

Table 1: Example Benchmark

	02342TAE9	11134LAF6	11135FBC4	458140AM2	882508AW4
02342TAE9	1.00	0.44	0.53	0.34	0.34
11134LAF6	0.44	1.00	0.96	0.59	0.59
11135FBC4	0.53	0.96	1.00	0.50	0.50
458140AM2	0.34	0.59	0.50	1.00	0.74
882508AW4	0.34	0.59	0.50	0.74	1.00

Table 2: Benchmark Security Correlation

The example optimization model can be specified as below:

$$\underset{x}{Maximize} \quad (CARRY)^T * x \qquad (5)$$

Subject to: $x_i \le z_i$ *(Selection)* *(5.1)*

$$\sum x = 1 \qquad \textit{(Holding)} \qquad (5.2)$$

$$\sum z = 2 \qquad \textit{(Cardinality=2)} \qquad (5.3)$$

$$(MWS)^T * x = 3.1 \qquad \textit{(Matching Benchmark MWS)} \qquad (5.4)$$

$$(x - x_b)^T \Sigma (x - x_b) <= 50 \qquad \textit{(TE budget=50)} \qquad (5.5)$$

x continuous, z binary

Step 1: Clustering

The benchmark is grouped into 3 clusters using k-mean clustering. The clustering result (Table 1) is intuitive. Cluster 1 has two Broadcom bonds; Cluster 2 has one Intel and one Texas Instrument bond; Cluster 3 has only one bond from AMD.

CLUSTER	Contrib to Obj (O)	Contrib to infeasibility (f)	O-f	PMV(x_b^g)	1	2	3
1	171.97	10.48	161.49	0.13	1.00	0.51	0.55
2	97.17	13.36	83.80	0.46	0.51	1.00	0.36
3	10.00	9.93	0.08	0.41	0.55	0.36	1.00

Table 3: Cluster Score and Correlation

Cluster scores are given in table 3. We take cluster 2 as an example to explain how the cluster score is calculated. We first solve the integer-relaxed Model (5), i.e. no binary variable z or constraints (5.1) and (5.3). The shadow prices of constraint (5.2) and (5.4) are 7.95 and 19.22 respectively. The infeasibility contribution (f_i)for bond 11135FBC4 is 7.95*1 + 19.22*0.3=13.4, and the objective contribution (O_i) is its carry 97.17. The total score ($O_i - f_i$) is 97.17 – 13.4 = 83.8. Similarly we have the total score for 11134LAF6 40.7. The cluster score is simply the larger one 83.8.

Step 2: Cluster Selection

$$\underset{x^g}{Maximize} \quad O^T * x^g \qquad (6)$$

Subject to: $x^g \leq z^g$ *(Selection)* (6.1)

$\sum z^g = 2$ *(Select 2 clusters out of 3)* (6.2)

$\left(x^g - x_b^g\right)^T \Sigma^g (x^g - x_b^g) <= 50$ *(TE budget)* (6.3)

x^g *continuous,* z^g *binary representing cluster selection*

Since there are many feasible solutions to this example, we use O column in table 3 in the objective and solve Model (6). The selected clusters are 1 and 2.

Step 3: Feed Cluster Selection to the Original Model

Since cluster 3 is not selected, two bonds (INTC and TXN) are excluded from the investment universe. In addition two SOS1 type constraints are added to the original model (5):

$$z_{02342TAE9} = 1$$

$$z_{11134LAF6} + z_{11135FBC4} = 1$$

We got the same solution as that from solving the original model (5): 02342TAE9 (0.14) and 11134LAF6 (0.86)

3 Case Studies

In this section we illustrate use of the proposed clustering heuristics to fixed income portfolio construction. Under passive portfolio management, we cover the fundamental problem of tracking a large index with a portfolio of small number of securities.
Under active portfolio management, we show how the heuristics can be used in conjunction with alpha signals to construct index-enhancing portfolios.

3.1 Index Replication

Motivated by the efficient capital market hypothesis, a passive investment strategy aims to replicate the performance of a benchmark index rather than outperform it. The simplest approach is full-replication, but such a strategy is expensive to implement due to the high cost of holding a large number of securities, both liquid and illiquid, with very small weights. In practice, portfolio managers construct portfolios containing a limited number of securities to replicate the index.

3.1.1 Computational Study

We construct a replicating portfolio for Bloomberg US Credit Index (bogie 286) as of July 30, 2021, and compare the computing performance of the clustering heuristics with that of CPLEX. The US Credit is the most popular benchmark index in the US investment grade (IG) corporate bond market. The index universe as of July 30, 2021 consists of 7617 bonds. Our objective is to construct a 50-bond portfolio from the most liquid 1977 constituents (liquidity score<=3 M. Sharif, A. Zheng, R. Nambimadom, R. Mattu 2018) so that its TE to the index is minimized.

The security selection search space is $\binom{1977}{50} = 10^{100}$. We group the universe into 400 clusters with 5 bonds in each cluster in average, so that bonds in the same cluster are similar to each other enough. In the cluster-selection step, 50 clusters are chosen from 400 with a search space $\binom{400}{50} = 10^{64}$. We limit the search time to 1 minute in this step. In the security-selection step, the search space becomes 5^{50} and we limit the solving time to 3 minutes. We only impose the holding constraint, i.e. full and no-leverage investment. For a sparse index-replicating model, the more number of constraints, the easier it is to find the optimal solution.
We solve the same sparse index-replicating model using CPLEX 12.6.3.0 with default configurations and time limit 30 minutes. The computational results are reported in Figure 1. The clustering heuristics found the first feasible solution TE=42 in 61 seconds (60 seconds for cluster selection and 1 second for security selection) and stayed at TE=40 after 80 seconds until the time limit 240 seconds (4 minutes) reached. CPLEX didn't find the first feasible solution until 720 seconds, and found three feasible solutions TE=72, 53, 49 in the entire solution process (1800 seconds or 30 minutes).

We also performed a number of exercises by varying cardinality in the replicating portfolio (Figure 2). As expected TE between the replicating portfolio and index decreases as the number of selected bonds increases. The speed of decreasing however slows down because it becomes harder and harder to match idiosyncratic risk in the index after the systematic risk is hedged away. The 30bps annual TE between the 100-bond replicating portfolio and the 7617-bond index intuitively suggests that the clustering heuristics has already found a close-to-optimal solution in the 4-minute time limit.

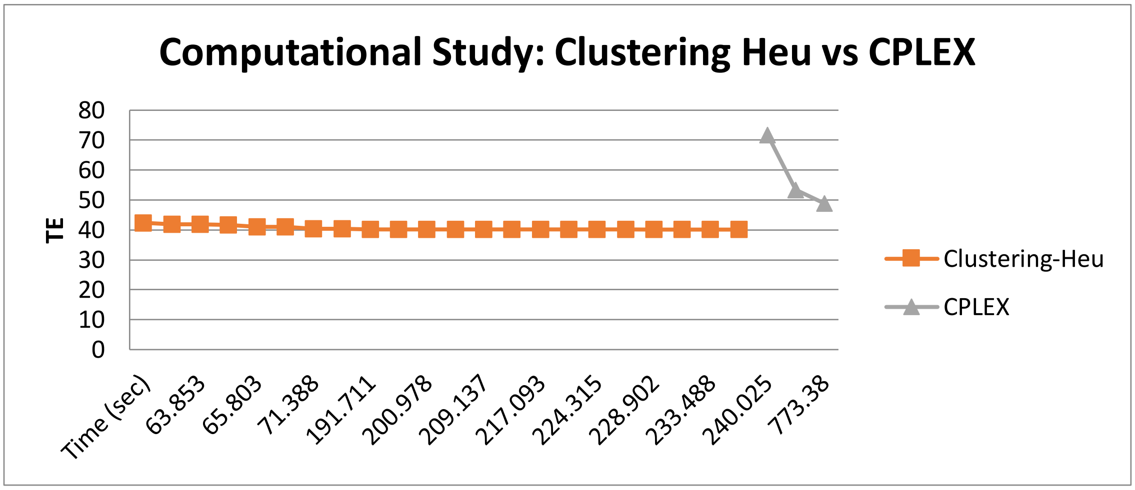

Figure 1: Computational study: clustering heuristics vs CPLEX

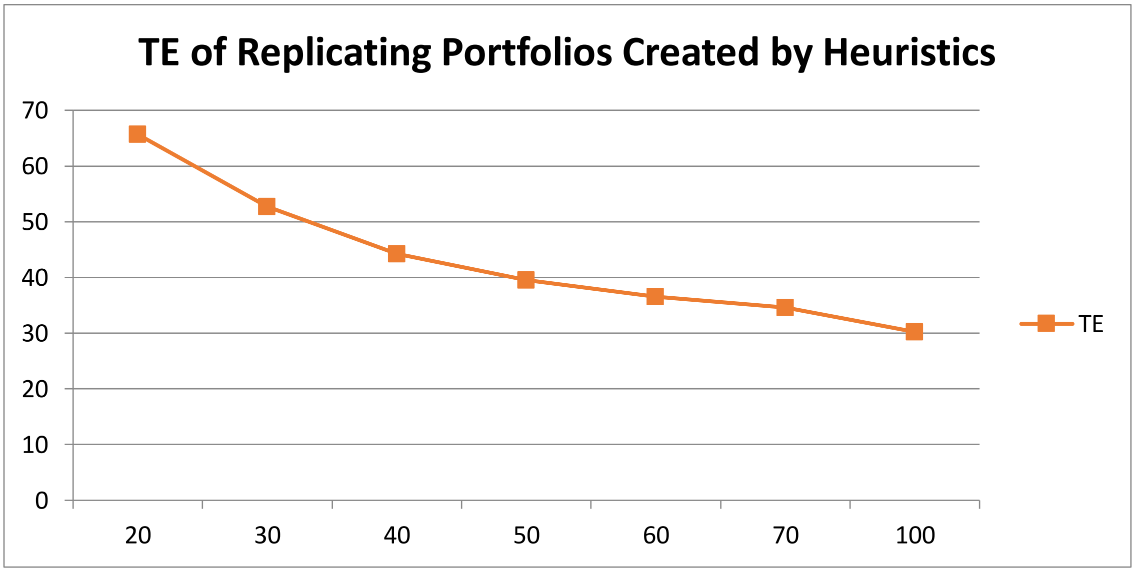

Figure 2: TE of replicating portfolios by varying number of holding bonds

3.1.2 Performance of Tracking

We backtested the tracking performance of the portfolios created by the clustering heuristics. The universe of interest is: IG Technology sub-index. We shall use the same universe for active management studies. The reason to choose this sub-index is that there is a good coverage for two important alpha signals: Analyst Recommendations and Distance-to-Default (DD). The test specifications are as below:

- Model (1) with the following parameters:
    - Number of bonds selected = 50 (number of bonds in the sub-index is around 400)
    - For each bond, the low and up bounds of weight are 0.2% and 20%
    - Full investment without leverage (sum of weights = 1)
    - Matching MWS between the replicating portfolio and sub-index
- Only credit risk/return is considered. Duration risk is hedged away from the covariance matrices.
- Test period: monthly between 1/31/2018 to 8/31/2021, where dates are end of month.

The ex-ante TEs are plotted in figure 3. The clustering heuristics creates replicating portfolios with ex-ante TE under 10 bps for the majority of months, except a few including the COVID period.

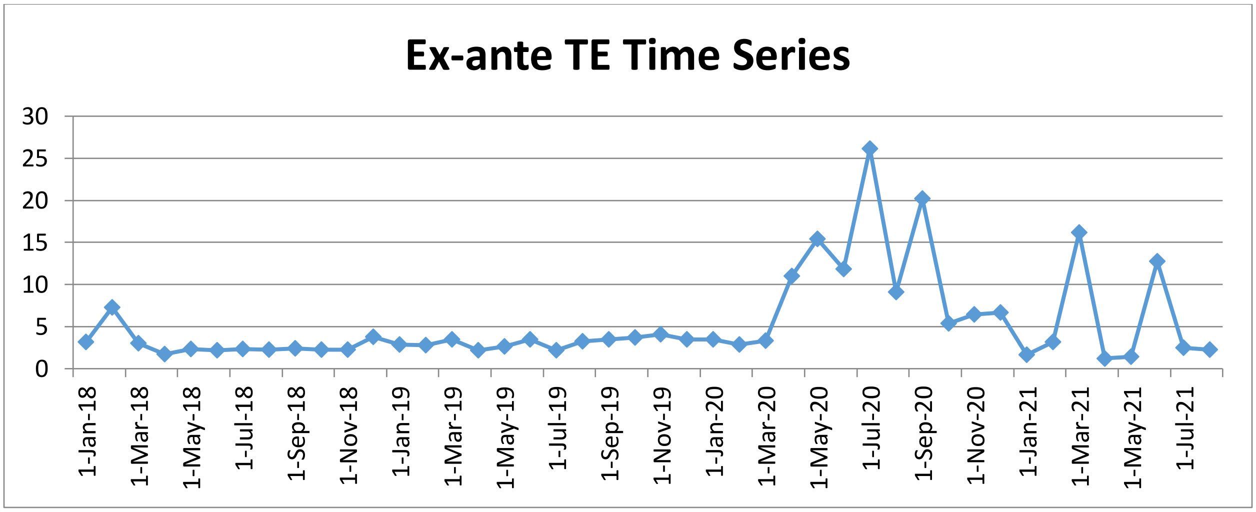

Figure 3: IG Technology Replicating Portfolios ex-ante TE time series

The realized excess return time series is plotted in Figure 4. The replicating portfolios beautifully tracked the performance of the benchmark with realized excess return TE 9.55bps/month.

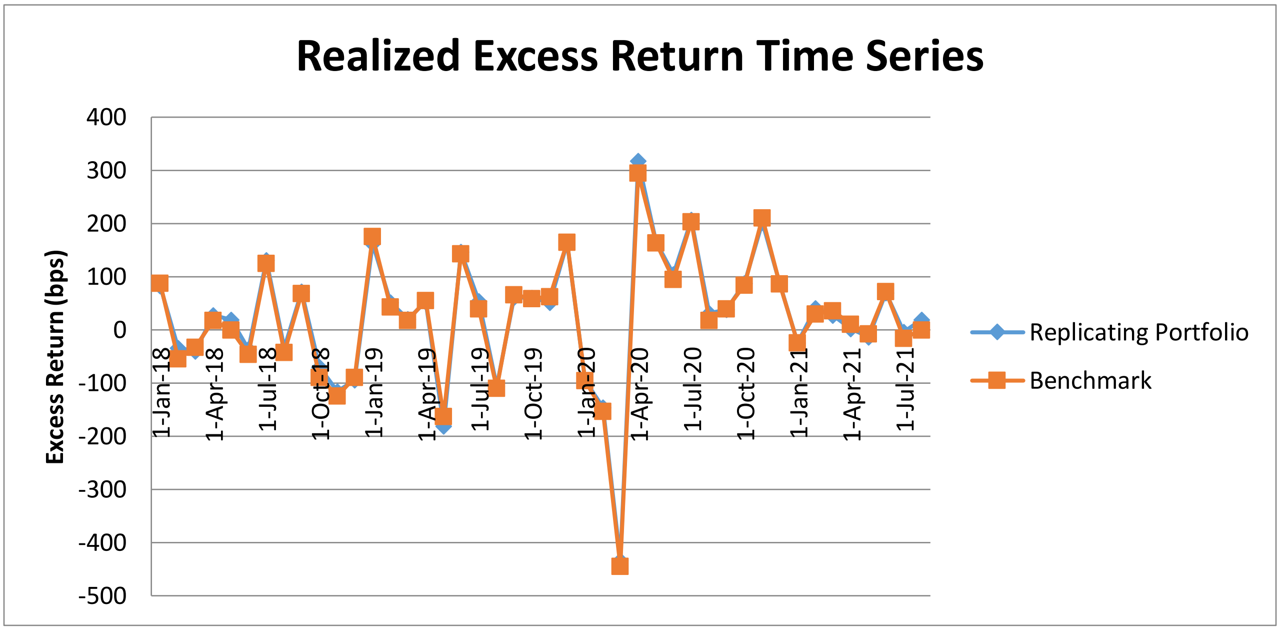

Figure 4: IG Technology Replicating Portfolios realized excess return time series

3.2 Index Enhancing

In this section we illustrate the use of the clustering heuristics as an active portfolio management tool when alpha signals are available. We first construct IG Technology sub-index enhancing model portfolios with three alpha signals: Credit Carry, Analyst Recommendations and Distance-to-Default. We then discuss the approach to combine multiple signals.

3.2.1 Single Signal

We build index-enhancing portfolios (Model 2) as of July 30 2021 using three alpha signals individually in this subsection with the following parameters (similar to index-replicating parameters given in subsection 3.1.2):

- TE budget 50bps
- Number of bonds selected = 50 (number of bonds in the sub-index is around 400)
- For each bond, the low and up bounds of weight are 0.2% and 20%
- Full investment without leverage (sum of weights = 1)
- Matching MWS between replicating portfolio and sub-index
- No penalty such as transaction cost is imposed. The TE budget is tight, so the turnovers are low for all signal models.

Credit Carry

Carry is the return of a security if time passes but market conditions do not change. Credit carry includes two components: OAS and credit curve rolldown. Most issuers have upward sloping credit spread term structures, implying that credit curve rolldown is positive.

Analyst Recommendations

Credit research analysts study various sectors and companies, and make categorical recommendations: Market Overweight (OW), Market Underweight (UW) and Market Neutral (MW). It is not straightforward to incorporate categorical signals into portfolio optimization. People sometimes simply exclude UW bonds from the investment universe. This approach is suboptimal, since some UW bonds may carry other signals and contribute to diversification. We propose to use bond ex-ante betas to the OW/UW sub-indices in the benchmark and maximize/minimize the OW/UW betas in the objective.

Distance-to-Default (DD)

Credit analyst group calibrates the Distance-to-Default measure which predicts financial distress for a company. The DD measure is also conveniently translated into Fair Value Spread (FVS) for bonds. We use *Spread Duration * (FVS – OAS)* as the DD signal for a bond.

Results have been omitted to protect proprietary data!

3.2.2 Signal Combining

Given more than one signal, another natural question therefore is how to optimally combine them. One combining approach (Mixed) is to construct model portfolios for each alpha signal and solve for optimal weights using historical realized returns and covariance of the model portfolios. The other approach (Integrated) is combining signals in the security level and building the optimal portfolio using the combined signal utility in the objective. There are good amount of discussions about the benefits and challenges to the Mixed and Integrated approaches. In this document we will only explore the integrated approach, as advocated in this AQR research (S. Fitzgibbons, J. Friedman, L. Pomorski, and L. Serban 2018). Different from all the previous published works, we propose to integrate signals using the trade-off among signals suggested by the sensitivity analysis of optimization model (Model 2).

We illustrate our approach to combine DD and Carry signals using the example given in subsection 3.2.1. We first solve the DD index-enhancing model with an additional constraint: model portfolio carry must be larger than a threshold, see Model (7) below.

$f(CARRY\_MIN) = \underset{x}{Maximize} \quad (DD)^T * x$	*Maximize DD Signal*	*(7)*
Subject to: $0.2\% * z \le x \le 20\% * z$	*(Selection and Bound)*	*(7.1)*
$\sum x = 1$	*(Holding)*	*(7.2)*
$\sum z = 50$	*(Number of Assets)*	*(7.3)*
$(MWS)^T * x = 11.6$	*(Match Bench MWS)*	*(7.4)*
$(x - x_b)^T \Sigma (x - x_b) <= 50$	*(TE budget)*	*(7.5)*
$(CARRY)^T * x \ge CARRY\_MIN$	*(Minimum Carry)*	*(7.6)*
x continuous, z binary		

By varying $CARRY\_MIN$, we get a list of corresponding model portfolios. We plot CHP% of the model portfolios against CARRY_MIN in Figure 5. While model portfolio carry increases, the CHP% decreases: slow at the beginning and faster after carry=145. Recall CHP% is 33.4% and carry is 134bps in the DD model portfolio (Table 4). If the portfolio manager is willing to bump model portfolio's carry from 134bps to 140bps, by sacrificing CHP% from 33.4% to 33.3%, the model portfolio corresponding to $CARRY_{MIN} = 140$is the choice.

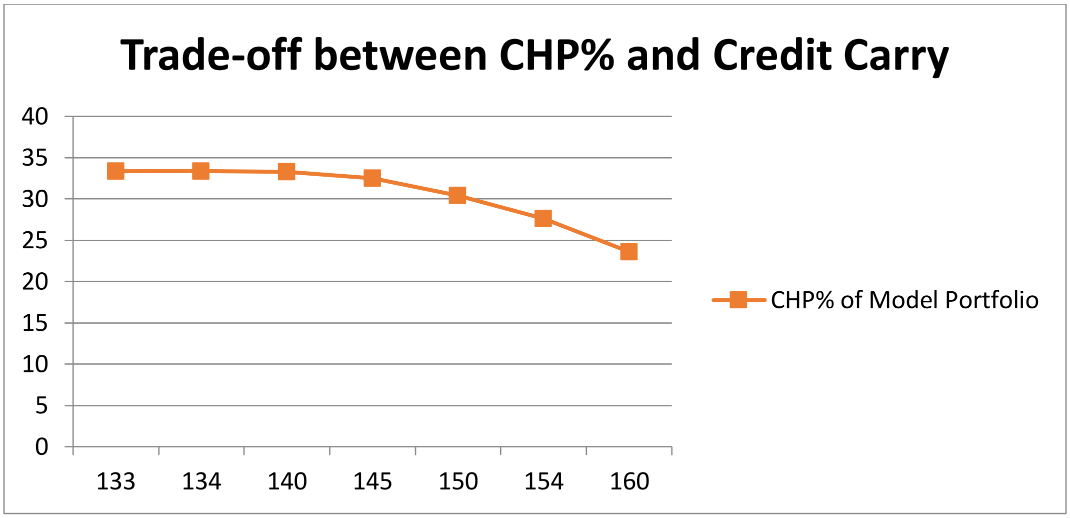

Figure 5: Trade-off between CHP% and Credit Carry

We want to have the combined signal expressed in the form: $Signal = DD + w_{carry} * Carry$. This can be done using the shadow price of constraint (7.6): $(CARRY)^T * x \geq CARRY\_MIN$. Shadow price of integer models is not directly available from solvers. We compute it using numerical method and got 0.7.

$w_{carry}(shadow\ price) = \frac{f(CARRY\_MIN^-)-f(CARRY\_MIN^+)}{2*SHOCK}$ where

SHOCK: the change to CARRY_MIN

$f(CARRY\_MIN^-)$: *objective value with down shock*

$f(CARRY\_MIN^+)$: *objective value with up shock*

The model with combined signals of DD and Carry is specified as below (Model 8). Model (8) and Model (7) with CARRY_MIN=140 are equivalent. After we have Model (8), we can combine the Analyst UW recommendation similarly and reach the following combined signals: $DD + 0.7 * CARRY + 295 * BETA\_to\_UW$

$\underset{x}{Maximize}\quad (DD + 0.7 * CARRY)^T * x$	*Maximize Combined Signal (8)*	
Subject to: $0.2\% * z \leq x \leq 20\% * z$	*(Selection and Bound)*	*(8.1)*
$\sum x = 1$	*(Holding)*	*(8.2)*
$\sum z = 50$	*(Number of Assets)*	*(8.3)*
$(MWS)^T * x = 11.6$	*(Match Bench MWS)*	*(8.4)*
$(x - x_b)^T \Sigma (x - x_b) <= 50$	*(TE budget)*	*(8.5)*
x continuous, z binary		

4 Backtesting Model Portfolios

Results have been omitted to protect proprietary data!

4.1 Performance Attribution

In order to attribute the performance to alpha signals, we create the following factor model for bond excess return.

$$
\begin{aligned}
& \qquad\qquad \boxed{\text{Carry}} \qquad\qquad\qquad\qquad\qquad\qquad\qquad \boxed{\text{Systematic/Sector}} \\
r_{i,t-1\ to\ t}^{excess} = {} & CARRY + \left[-1 * SDUR_{i,t-1} * \left(F_{t-1,t}^{D} + OAS_{i,t-1} * F_{t-1,t}^{DTS}\right)\right] \\
& +[-1 * SDUR_{i,t-1} * OAS_{i,t-1} * (1^{OW} * F_{t-1,t}^{OW} + 1^{UW} * F_{t-1,t}^{UW} + 1^{MW} * F_{t-1,t}^{MW})] \quad \boxed{\text{Analyst Recommendations}} \\
& +[\min((SDUR_{i,t-1} - \overline{SDUR_{t-1}}), 0) * F_{t-1,t}^{SHORT} + \max((SDUR_{i,t-1} - \overline{SDUR_{t-1}}), 0) * F_{t-1,t}^{LONG}] \quad \boxed{\text{Curve}} \\
& +residual \quad \boxed{\text{Idio (DD etc.)}}
\end{aligned}
$$

Where :
$F_{t-1,t}^{D}$: is the risk factor realization for low or negative OAS bonds, return of which is not proportional to OAS percentage change
$F_{t-1,t}^{DTS}$: is the risk factor realization for bonds, return of which is proportional to OAS percentage change
$F_{t-1,t}^{OW}$: is the risk factor realization above $F_{t-1,t}^{DTS}$ for bonds in the OW sub-index. 1^{OW} is 1 if bond is in OW sub-index.
$F_{t-1,t}^{UW}$ and $F_{t-1,t}^{MW}$: are defined similarly as $F_{t-1,t}^{OW}$
$F_{t-1,t}^{SHORT}$ and $F_{t-1,t}^{LONG}$ are two curve slope factors. $\overline{SDUR_{t-1}}$ is the median SDUR at t-1

This model is similar to the Barclays POINT credit risk model (A.B. Silva 2009). Three OW/UW/MW analyst recommendation factors are added to the POINT model. We run capitalization-weighted regression against IG Technology sub-index from 1/31/2018 to 8/31/2021. The average r-squared is 0.632.

Results have been omitted to protect proprietary data!

5 Conclusion

Key takeaways:

- The proposed clustering heuristics is computational effective and capable of translating signals into portfolio alpha successfully.
- Other practical guides:
    - Beta can serve as a proxy for analyst categorical recommendations
    - Optimization model sensitivity analysis can be used to combine multiple signals
    - Factor model can be used to attribute return to alpha signals.
- Comments about signals Used in the study:
    - DD is the most powerful signal discovered in our study
    - May sacrifice a little bit DD signal to incorporate Carry
    - OW recommendation does not turn out to be useful
    - UW recommendation is useful, and correlated with DD signal
- Realized TE of model portfolios are unanimously higher than the ex-ante TE budget (50bps), suggesting there is still a room to enhance the risk model.
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