MAT1856/APM466 Assignment 1

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Fundamental Questions - 25 points

1.

   (a) The reason that the government issues bonds instead of simply printing money is that simply printing money would increase money supply in the market and so cause unnecessary inflation.

   (b) One hypothetical example of the long-term part of yield curve being flatten is the US Treasury 10-year bond is 5%, and 30-year bond with 5.1% when the investors expect the inflation would decrease in the future, which also reflects their worries about the macroeconomics outlook.

   (c) Quantitative easing is a form of unconventional monetary policy where the central bank purchases longer-term securities from the open market in order to increase the money supply and encourage lending and investment. Since the beginning of the COVID-19 pandemic, the US Fed introduced new weekly recurring one- and three-month term repo operations besides its updated monthly schedule of repo operations, and the US Fed increased its holdings of Treasury and agency mortgage-backed securities by at least 500 billion and 200 billion, respectively.

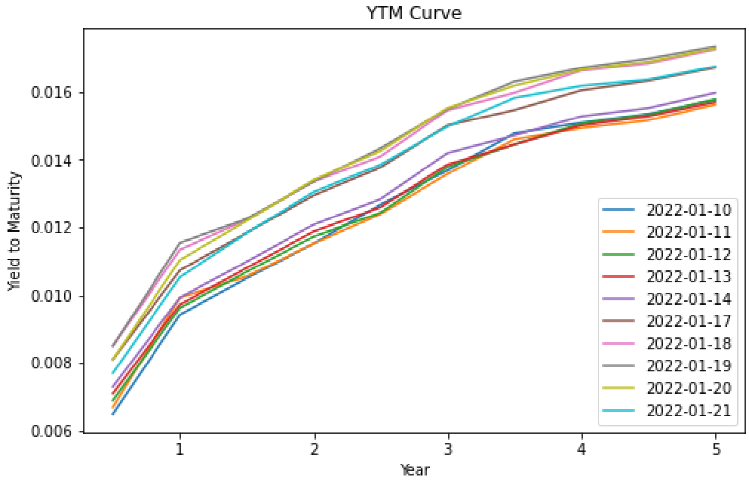
2. I have chosen 10 bonds as follows: CAN 0.25 Aug 1, CAN 0.25 Feb 1, CAN 0.25 Aug 1, CAN 0.75 Feb 1, CAN 1.50 Sep 1, CAN 1.25 Mar 1, CAN 0.50 Sep 1, CAN 0.25 Mar 1, CAN 1.00 Sep 1, CAN 1.25 Mar 1. The reasons for choosing such 10 bonds are: Firstly, since we are trying to find the spot curve with terms ranging from 1-5 years, and the Canadian government bonds issue the coupon semi-annually, we would choose the bonds with maturity dates from the current time (where I choose February 1st as the current time) ranging from 0.5 to 5 years, with the time gap between two bonds being six months. Secondly, the issue dates of the chosen bonds should not vary too much since otherwise the yield curve would vary a lot, and here the issue dates of my bonds are ranging from 2019 to 2021, which are quite close. Linear interpolation technique is involved when the time gap of two bonds is not six months apart exactly.

3. The eigenvalues and eigenvectors of the covariance matrix would tell us the variance among those stochastic processes, and for the specific case of stochastic curves, the largest eigenvalue multiplying with its eigenvector would represent the direction of the axis explaining the largest variance among the curves, say for situations in the yield curve, it could be the parallel shift of the curve among different days.

Empirical Questions - 75 points

4.

   (a) After calculating the yield for 10 chosen bonds, the 5-year yield curve would look like the following graph:



Notice that my fourth bond's maturity on Feb 1st and my fifth bond's maturity on Sep 1st, and so to make sure the bond's time gap is six months, I used the linear interpolation here as it is relatively simple. Since we are interpolating the bonds with a relatively short time gap of six months, it would be reasonable to use linear interpolation to account for all the possible yield values between two year without too many errors.

(b) The pseudo-code for spot curve would be:

- Initializing a dictionary whose keys are days observed and values are spot rates on each day
- Loop over the observed days:
  - Initializing the spot rates list on this day
  - Loop over the chosen bonds
    - If the bond's time to maturity from the current time $T = 0.5$:

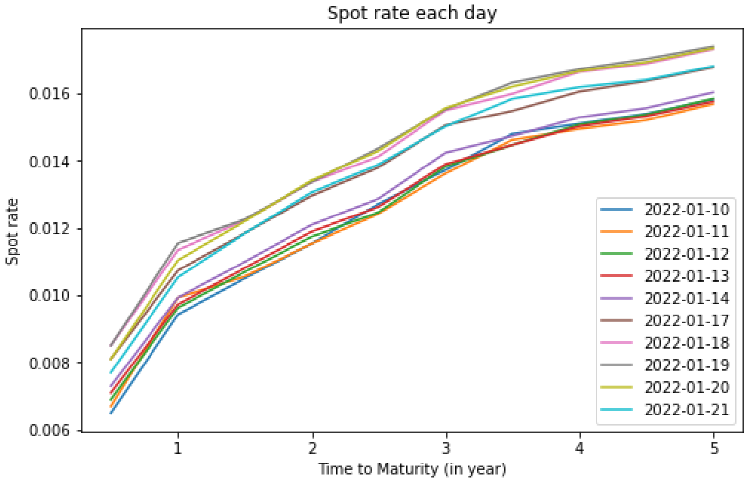
      $$\text{spot rate} = -\frac{\log(\text{Price of the bond}/(\text{Notional value} + \text{last coupon payment}))}{T}$$

    - ELSE: set **value** equals to the price of the bond minus sum up all the discounted coupon payments except the last one, and:

      $$\text{spot rate} = -\frac{\log(\text{value}/(\text{Notional value} + \text{last coupon payment}))}{T}$$

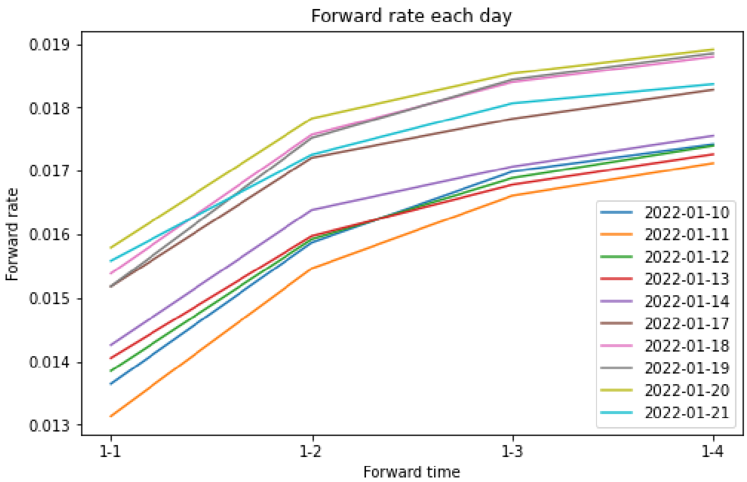
    - Add this spot rate to the spot rates list on this day
  - Set the value of the key being this day as the spot rates list on this day

The corresponding graph is:



(c) The pseudo-code for forward curve would be:

- Initializing a dictionary whose keys are days observed and values are forward rates on each day
- Loop over the observed days:
  - Initializing the forward rates list on this day
  - Loop over the forward rates required to compute
    - $$f_{1\text{-}i} = \frac{\text{spot rate}_{1+i} \times (1+i) - \text{spot rate at year 1}}{i}$$
    - Add this forward rate to the forward rate list on this day
  - Set the value of the key being this day as the forward rates list on this day



5. The covariance matrix of yield is:

$$\begin{bmatrix} 0.00205392 & 0.00082329 & 0.00077153 & 0.00075260 & 0.00072205 \\ 0.00082329 & 0.00068826 & 0.00065362 & 0.00058847 & 0.00056493 \\ 0.00077153 & 0.00065362 & 0.00065796 & 0.00059417 & 0.00057516 \\ 0.00075260 & 0.00058847 & 0.00059417 & 0.00056649 & 0.00054292 \\ 0.00072205 & 0.00056493 & 0.00057516 & 0.00054292 & 0.00052372 \end{bmatrix}$$

and the covariance matrix of forward is:

$$\begin{bmatrix} 0.00108327 & 0.00077570 & 0.00057856 & 0.00052860 \\ 0.00077570 & 0.00070255 & 0.00057689 & 0.00054971 \\ 0.00057856 & 0.00057689 & 0.00053297 & 0.00050556 \\ 0.00052860 & 0.00054971 & 0.00050556 & 0.00048617 \end{bmatrix}$$

6. The first eigenvalue of the covariance matrix of yield is approximately 0.0037592 with associated eigenvector being

$$[-0.66934629, -0.39383128, -0.38291881, -0.36059567, -0.34672239]$$

The first principal component would be:

$$[-0.00251619, -0.00148048, -0.00143946, -0.00135554, -0.00130339]$$

It would approximate the shape of the 5-year yield curve since it explains approximately 83.72% of the total variance according to its eigenvalue.
Similarly, the first eigenvalue of the covariance matrix of forward is approximately 0.0025154 with associated eigenvector being

$$[0.60935609, 0.52278311, 0.43415682, 0.40852274]$$

The first principal component would be:

$$[0.00153275, 0.00131499, 0.00109206, 0.00102758]$$

It would approximate the shape of the forward curve since it explains approximately 89.68% of the total variance according to its eigenvalue.

References and GitHub Link to Code

1. Quantitative Easing: https://www.investopedia.com/terms/q/quantitative-easing.asp
2. Flatten Yield Curve: https://www.investopedia.com/terms/f/flatyieldcurve.asp
3. PCA in yield curve: https://towardsdatascience.com/applying-pca-to-the-yield-curve-4d2023e555b3
4. The COVID-19 Crisis and the Federal Reserve's Policy Response: https://www.federalreserve.gov/econres/feds/files/2021035pap.pdf
5. Github Code link: https://github.com/qunoujiejingti/APM466/blob/main/Assignment%201/466%20a1-update.ipynb