Laboratory 2

Trigonometric Fourier series

Student: Фам Куок Ань

Group: R3138

Tasks:

For a function f(x) given on a segment, construct three Fourier series: a general trigonometric series, a Fourier series in terms of sines and cosines.

The task report contains:

1) Analysis part

2) Construct graphs using computer of several partial sums (for example S5, S10, S50) and a graph of the original function. Make sure (visually) that partial sums approximate the original function.

3) Conclusion

1. Analysis part

Given function:

$$f(x) = \begin{cases} \sin(x), x \in [\,0, \frac{\pi}{2}) \\ 1, x \in [\,\frac{\pi}{2}, 3\pi) \end{cases}$$

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1, General trigonometric series

We choose a general function $f(x) = a_0 + \sum_{n=1}^{\infty} \left(a_n \cos \frac{2nx}{3} + b_n \sin \frac{2nx}{3}\right)$

Coefficients:

$$a_0 = \frac{1}{3\pi} \int_0^{3\pi} f(x)\,dx = \frac{1}{3\pi}\left[\int_0^{\pi/2} \sin x\,dx + \int_{\pi/2}^{3\pi/2} dx\right]$$

$$a_n = \frac{2}{3\pi} \int_0^{3\pi} f(x) \cos\frac{2nx}{3}\,dx = \frac{2}{3\pi}\left[\int_0^{\pi/2} \sin x \cos\frac{2nx}{3}\,dx + \int_{\pi/2}^{3\pi/2} \cos\frac{2nx}{3}\,dx\right]$$

Similarly:

$$b_n = \frac{2}{3\pi}\left[\int_0^{\pi/2} \sin x \sin\frac{2nx}{3}\,dx + \int_{\pi/2}^{3\pi/2} \sin\frac{2nx}{3}\,dx\right]$$

After several calulation

$$\rightarrow \begin{cases} a_0 = \frac{1}{3\pi} + \frac{5}{6} \\ a_n = \frac{-6n + 9\sin(\frac{\pi n}{3})}{\pi n(4n^2-9)} \\ b_n = \frac{-4n^2 - 9\cos(\frac{n\pi}{3}) + 9}{\pi n(4n^2-9)} \\ S_n = a_0 + \sum_{n=1}^{\infty} (a_k \cdot \cos kx + b_k \sin kx) \end{cases}$$

2, Fourier sine serie

$$f(x) = \begin{cases} -1, & x \in [-3\pi, -\frac{\pi}{2}] \\ \sin x, & x \in [-\frac{\pi}{2}, \frac{\pi}{2}] \\ 1, & x \in [\frac{\pi}{2}, 3\pi] \end{cases}$$

$$\rightarrow \begin{cases} a_n = 0 \\ b_n = \dfrac{-2\left[(n^2-9)(-1)^n + 9\cos(\frac{\pi n}{6})\right]}{\pi n(n^2-9} \\ S_n = \sum_{k=1}^{n} b_k \sin kx \end{cases}$$

3, Fourier cosine serie

$$f(x) = \begin{cases} 1, & x \in [-3\pi, -\frac{\pi}{2}] \\ -\sin x, & x \in [-\frac{\pi}{2}, 0] \\ \sin x, & x \in [0, \frac{\pi}{2}] \\ 1, & x \in [\frac{\pi}{2}, 3\pi] \end{cases}$$

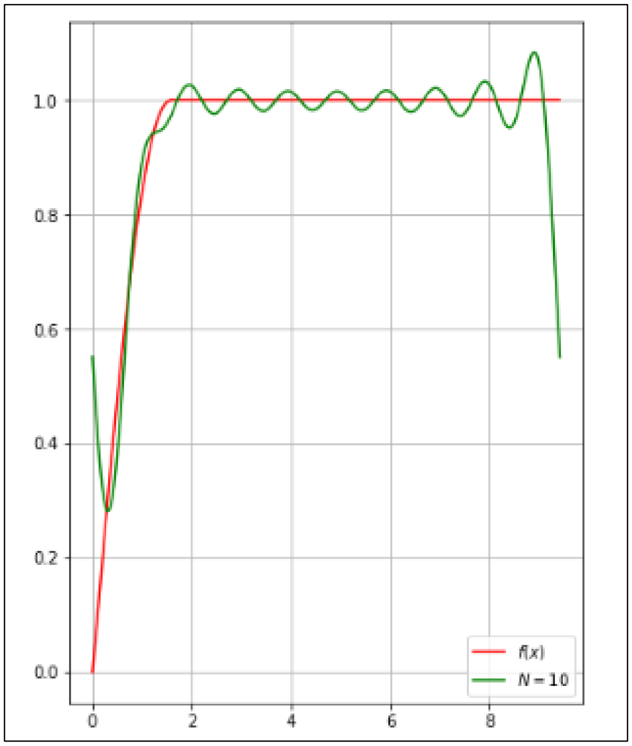
$$\rightarrow \begin{cases} a_n = \dfrac{6n - 18\sin(\frac{\pi n}{6})}{\pi n(9-n^2)} \\ a_0 = \dfrac{1}{3\pi} + \dfrac{5}{6} \\ b_n = 0 \\ S_n = a_0 + \sum_{k=1}^{n} a_k \cos kx \end{cases}$$

3. Graphics obtained as a result of the program

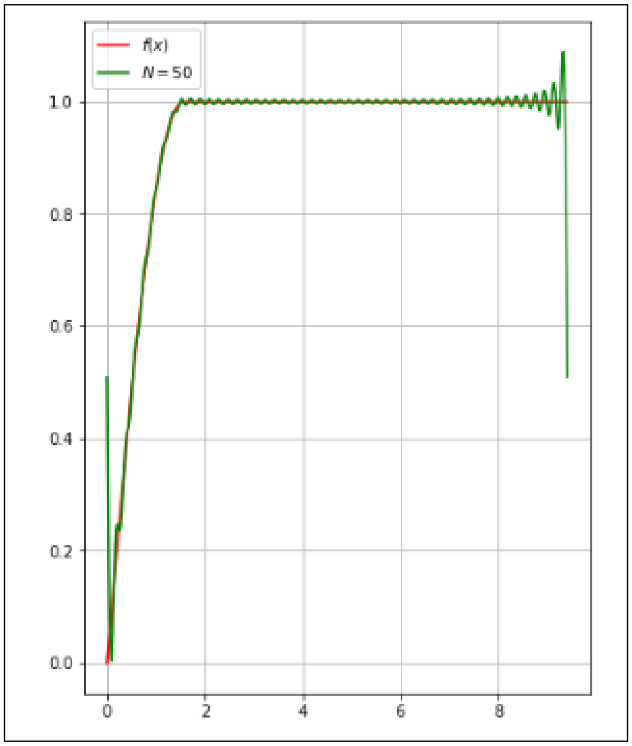
Link on github: https://github.com/quocanh34/ITMO-Mathematics-Laboratory

General trigonometric serie

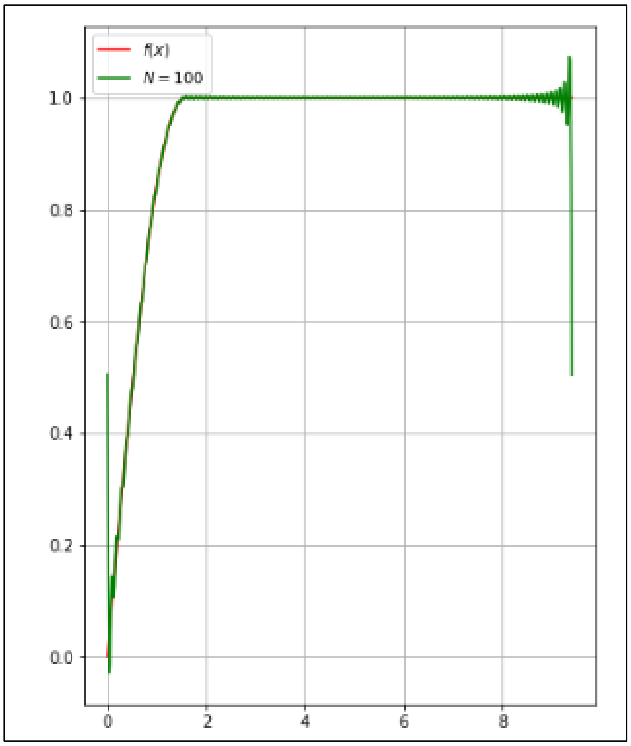
N = 10



N = 50

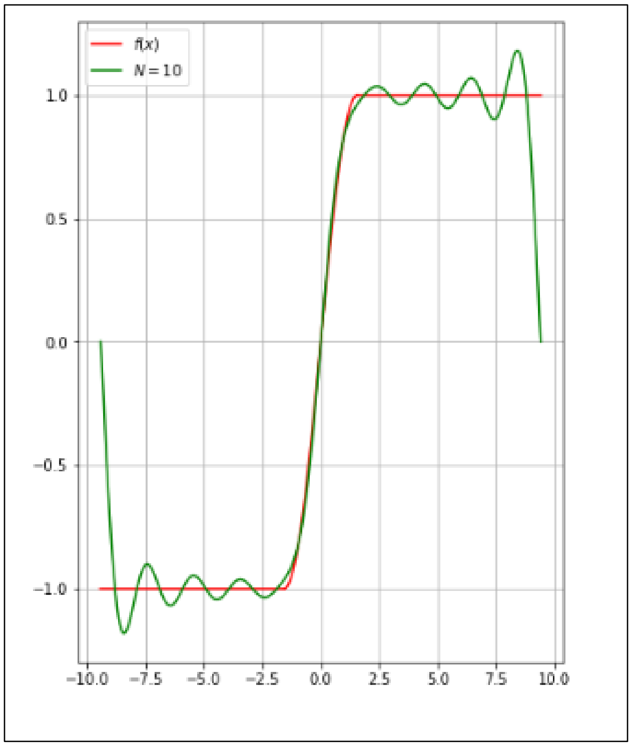


N = 100

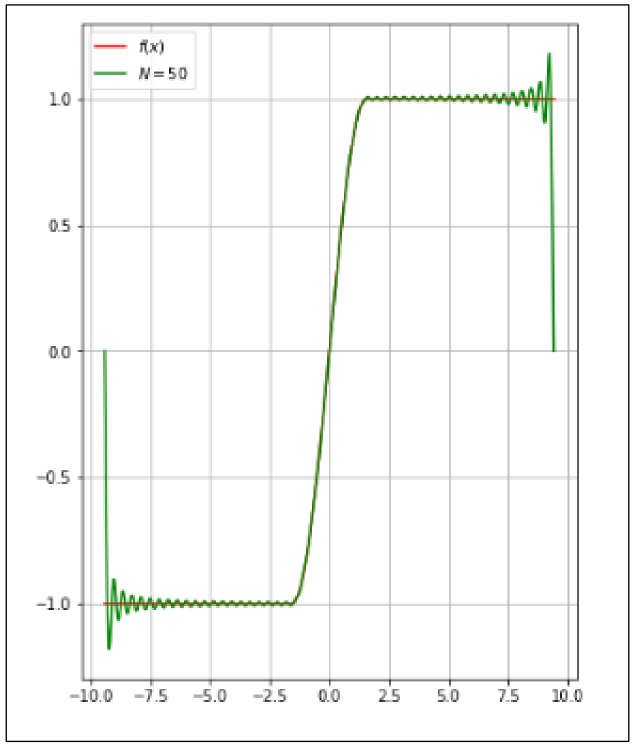


Fourier sine serie

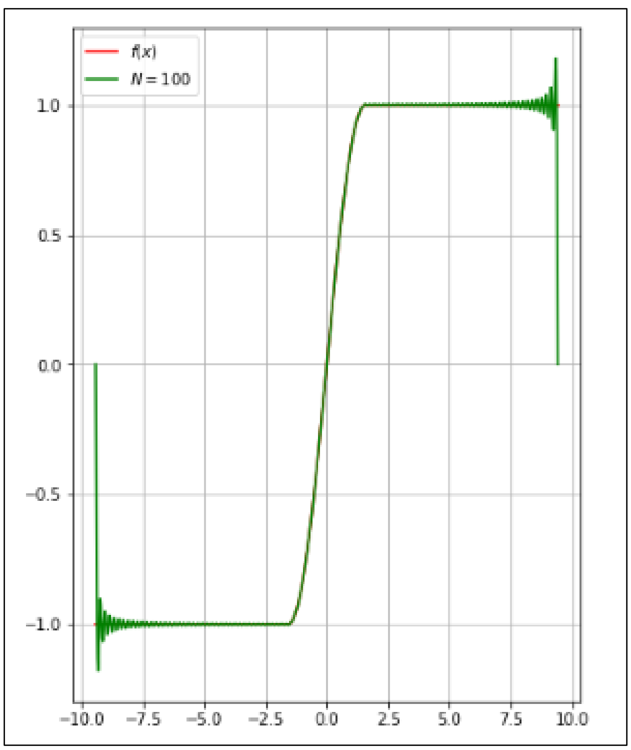
N = 10



N = 50

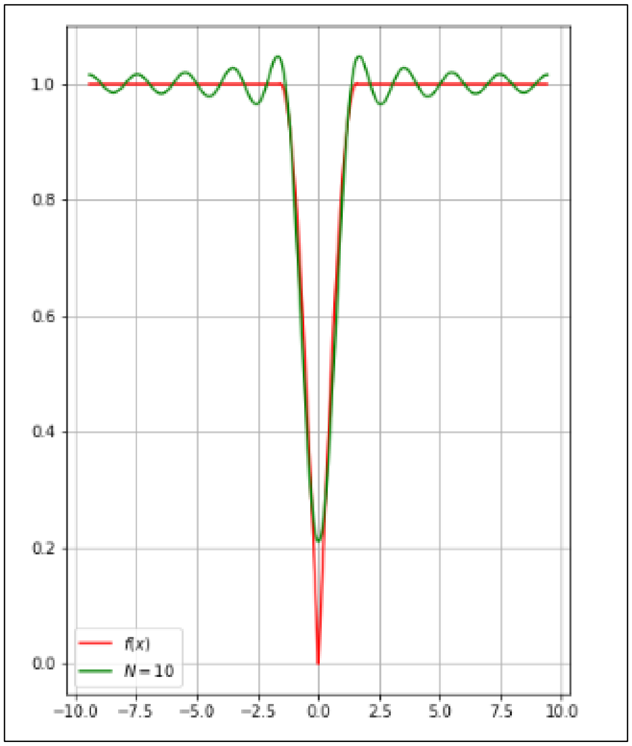


N = 100



Fourier cosine serie

N = 10



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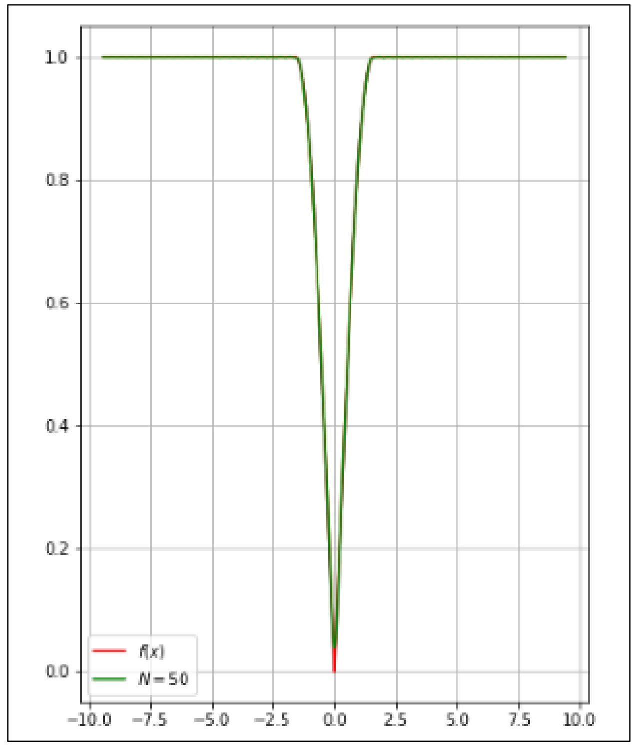

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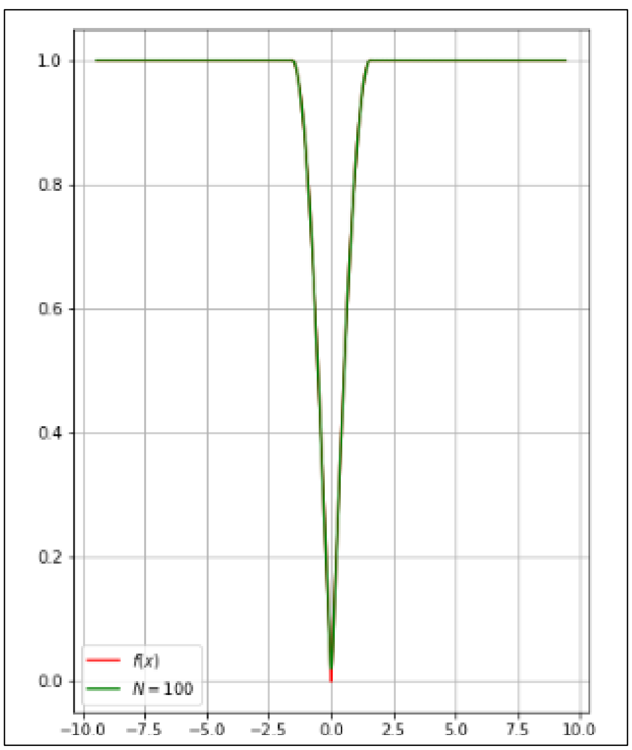

<u>4 .Conclusion</u>

We can guarantee that the approximation error decreases with increasing n as visualization from the graphs

This remark confirms the theory: because the function satisfies the conditions of Dirichlet's theorem (or can satisfy them due to some transformations) the Fourier series converges to it.